International Journal of Enhanced Research in Science, Technology & Engineering
ISSN: 2319-7463, Vol. 11 Issue 3, March-2022, Impact Factor: 7.957

Theoretical and Numerical Investigation on Propulsive Configuration and Performance Characteristics for a Sounding Rocket

Ankit Kumar Mishra[1], Janani Kavipriya V.S.[2], Prasannalakshmi Thota[3], Kabhilesh G[4]

[1, 2, 3]Department of Research and Development, Astroex Research Assoctioian, Deoria, India
[4]Department of Mechanical engineering, Chennai Institute of Technology, Chennai, India

***Corresponding Author:** Ankit Kumar Mishra, email: ankitkumarm1998@gmail.com

ABSTRACT

This paper gives the overview of a sounding rocket which is developed to perform certain scientific experiments in low earth orbit. The propulsion characteristics and calculations related to nozzles for both the booster stage and the sustainer stage of two-stage sounding rocket have been discussed and calculated using isentropic relations and the graphical representation of the performance of the two stages with respect to expansion ratio is shown. Further, in this paper we deal with a critical feature of our rocket, which is used to change the orientation or direction of the rocket, the thrust vector control and the type of the thrust vector control we plan to implement on our rocket and reasons for choosing it.

Keywords: **Aerodynamics, Isentropic Relations, Nozzles, Propellants, Rockets.**

INTRODUCTION

Sounding rockets also referred as research /experimental rocket which contain instruments that are used to perform experiments in the space or the higher altitude. As the name suggest two stage, it consist of two stages having different purposes, the ground stage of the rocket is implement with solid propellant and the upper stage of the rocket is of liquid propellant, for normal rockets gimbal system is used to change the direction but for the sounding rocket we generally don't have any external mechanism to change the direction of the rocket hence the secondary thrust control system is been implemented on the sounding rocket, when a rocket is travelling through the atmosphere the wind that's act on the rocket and the higher gravitational acceleration of the rocket pulls it towards the earth creating the change in the direction and maximum deflection of the rocket would be in the lesser altitude. The thrust vector controlling of the rocket is been implemented to the upper stage of the rocket which is the liquid propellant based so that the maximum deflection can be directly changed in the upper stage.

LITERATURE REVIEW

Marco Pallone et.al described about sounding rockets are generally research rockets which carry instruments that are used measure the certain parameters while performing certain experiments. Sounding rockets are easily affordable used to test the new components or any type new prototype and used in the launch vehicles and sub systems [1].Singh et.al mentioned about the characteristics of solid motor in which the propellant is the mixture of the chemicals which consists of fuel and oxidizer and it burns rapidly when ignited the ratio of the fuel and oxidizer depends on the type of the fuel and oxidizer we used .and the oxidizer provides the oxygen for the fuel burning and the ratio of the two components alters the burn rate of the propellant also alters the amount of the thrust and the fuel consumption of the fuel and alters the chamber parameters [2].Suskila et.al this research is based on the design and construction of the solid propellant and the standard test to ensure the thrust and the total pressure produced inside the thrust chamber that were designed and constructed and also the ratios of the mass of the solid propellant is compared based on the maximum thrust that's been achieved and also the consideration of the mass thickness based on the temperature of combustion chamber is also considered [3].A theoretical study has also been presented which highlights the propellants being used in the solid rocket motor. The performance parameters are also being described along with the material selection as described by Ankit et.al [12].

Neumann et.al described about the combination advantages of solid and which is easy in handling and has long storage time and of liquid rocket propellant which are thrust modulation and shut off capabilities [4].Ankit et.al presented a study on nozzle flow partition that is carried out by simulation of rocket nozzle designed Fusion 360 and ANSYS to inspect the laminar as well as turbulent regime for deviating section of nozzle [5]. Ankit et.al reviewed about pintle injector used in rocket nozzle along with motor combination for generating higher amount of thrust. It show the influence of spray angles and characteristics such as flow as well as combustion on spray images, droplet size, momentum ratio, opening distance and SMD distributions which affect the injector geometry [6].Ankit et.al paper discussed a theoretical and conceptual design for compact size 2 stage sounding rocket by focusing on structural optimizations at various levels. The aim of the paper is to develop a two-stage sounding rocket with overall length constrained to 1 meter [7].

The aim of paper is to design a two stage sounding rocket and its nozzles using fusion 360 and analysis of different properties using simulation on ANSYS software. The rocket is designed to reach maximum apogee to perform scientific experiments and can be recovered safely after use [8].Ankit et.al presented a work caused by lack of data on aerodynamics for profile of elliptical nose cone and especially improved aerodynamic qualities that can be used in designing aircrafts. Flow phenomena observed in numerical simulations for different AOA for elliptical nose cone profile are highlighted, critical design aspects and performance characteristics of selected nose cone are presented [9]. Ankit et.al described about different classification of propellants used for launch vehicles. The cryogenic propellants taken for comparison are liquid hydrogen, liquefied methane and for semi cryogenic fuels considered are RP-1 (kerosene) and UDMH with liquid oxygen as the oxidizer. The scope of this work addresses the comparison among the propellants, on their chemical properties, overall efficiency and fatigue life which is a major criterion for launch vehicles [10].Aerodynamic thrust variation and performances play a key role in estimating forces along with the injected flow and their characteristics. This review paper deals with the aerodynamics characterization, its properties at different conditions in addition with the performance analysis of the aerospike nozzle study carried by Ankit et.al [11]. Ankit et.al discusses about composition of chemical substances that affects the launch vehicle in ground station as well as atmospheric conditions is been presented. During the rocket-launching the large amount of inhalation of an exhaust gas released and it majorly affects the surface of the launch pad as well as the atmosphere [13].

METHODOLOGY

The forthcoming calculations and analysis have been done considering the rocket design shown in figure 1 & 2.

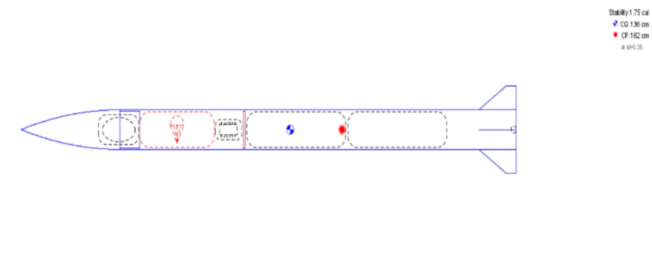

Figure.1 Upper rocket body design

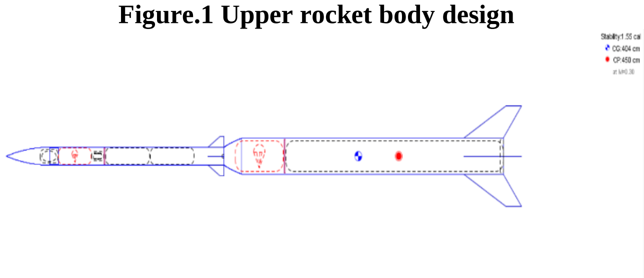

Figure.2 Full rocket body design

Rocket nozzle equations:
Nozzle plays a vital role in rocket propulsion as it allows the flow of fuel and oxidizer mixer to flow through it with required mass flow rate and producing enough thrust to reach a desired height. The amount of thrust a nozzle can produce depends upon the total mass flow of the propellant, the throat area from which it is flowing and exit area of the nozzle from which it will leave in the form of burning gasses.

The amount of thrust produced by a rocket is given by

$$F = \dot{m} * v_e + (p_e - p_0) * A_e \qquad (01)$$

In this the first term denotes the momentum thrust of the rocket; the second term denotes the pressure thrust of the rocket. From the above equation we can say that the thrust of the rocket depends mainly on the mass flow of the rocket $\dot{m}$, the exit velocity of the rocket v_e, and the exit pressure of the engine. Mostly, the exit pressure of the rocket is maintained slightly greater than that of the atmospheric pressure because generally, there will be a flow from higher pressure to the lower pressure so if you consider this condition there could be a chance of increase in the thrust of the rocket

Specific impulse of the rocket is:

$$I_{sp} = \frac{F}{\dot{m} * g_0} \quad (02)$$

The speed of sound of the Rocket in terms of the temperature and gas constant is given as:

$$a = \sqrt{\gamma R T} \quad (03)$$

The Mach number of the rocket can be found from the relation:

$$M_e{}^2 = \frac{2}{\gamma}\left[\left(\frac{P_c}{P_{atm}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \quad (04)$$

The isentropic relations of the various parameters of the flow can be given by the following relation. But basically, we will consider the condition to be isentropic because the stagnation properties are constant in isentropic flow, but while considering isentropic flow there are certain assumptions that need to be considered such as the ideal flow nothing but there is no exchange of the heat from the surroundings then these ratios can be related to that of the stagnation parameters at a given Mach number

$$\frac{T_0}{T} = [1 + \frac{(\gamma-1)}{2} * M^2] \quad (05)$$

$$\frac{p_0}{p} = [1 + \frac{(\gamma-1)}{2} * M^2]^{\frac{\gamma}{\gamma-1}} \quad (06)$$

$$\frac{\rho 0}{\rho} = [1 + \frac{(\gamma-1)}{2} * M^2]^{\frac{1}{\gamma-1}} \quad (07)$$

The ratio of exit area to that of the throat area for a given Mach number:

$$\frac{A}{A_t} = \frac{1}{M}[\frac{2}{\gamma+1}(1 + \frac{(\gamma-1)}{2} * M^2)]^{\frac{\gamma+1}{2(\gamma-1)}} \quad (08)$$

The flow per unit area will be maximum at the throat those relations are

$$\frac{p_t}{p_0} = [\frac{2}{(\gamma+1)}]^{\frac{\gamma}{\gamma-1}} \quad (09)$$

$$\frac{T_t}{T_0} = [\frac{2}{(\gamma+1)}] \quad (10)$$

$$\frac{\rho_t}{\rho_0} = [\frac{2}{(\gamma+1)}]^{\frac{1}{\gamma-1}} \quad (11)$$

To find the area at throat of the nozzle

$$A_t = \frac{F}{C_f + P_c} \quad (12)$$

Coefficient of thrust can be calculated with:

$$C_f = \sqrt{\frac{2\gamma^2}{\gamma-1}(\frac{2}{\gamma+1})^{\frac{\gamma+1}{\gamma-1}}[1 - \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}] + \frac{P_e - P_a}{P_c}(\frac{A_e}{A_t})} \quad (13)$$

The mass flow rate of the nozzle can be found from the basic geometry and the fluid properties that can be given as

$$\dot{m} = \rho_t v_t A_t = \frac{p_0 A_t \sqrt{\gamma}}{\sqrt{R T_0}} * (\frac{2}{\gamma+1})^{\frac{\gamma+1}{2(\gamma-1)}} \quad (14)$$

The exit velocity of the rocket is given by:

$$v_e = M_e \sqrt{\gamma R T_e} \quad (15)$$

Radius of throat:

$$r_t = \sqrt{\frac{A_t}{\pi}} \quad (16)$$

Radius of chamber:

$$r_c = \sqrt{\frac{A_c}{\pi}} \quad (17)$$

Length of divergence:

$$L_{divergence} = \frac{r_e}{tan\, tan\ \theta} \quad (18)$$

Length of convergence:

$$L_{convergence} = \frac{r_c}{tan\, tan\ \beta} \quad (19)$$

Theoretical calculated values for lowernozzle

Assumptions

Pc= 1000 psi, Tc = 3200K, γ= 1.18, molecular weight= 24.63 mol, R= 337.55 (J/kg-k), F=10,000N

Using equation (09) & (10) we can find pressure and temperature at throat

Pt= 284.2099 psi
Tt= 2935.779 K
Using equation (04) we can find the exit Mach number
Me= 3.16
Using equation (13) coefficient of thrust is calculated
C_f= 1.6
Using equation (12) area at exit of nozzle can be calculated
At = 0.000906 m^2
Using equation (10) ratio of exit to throat area is calculated

$$\frac{A_e}{A_t} = 10$$

And value of exit area is
Ae= 0.00825 m^2
Exit temperature and pressure is calculated using equation (05) & (06)
Te= 1684.21K, Pe=14.94 psi
Mass flow rate is calculated using equation (14)
ṁ = 3.874 kg/s
Exit velocity is calculated using equation (15)
Ve= 2544.184 m/s
Specific impulse is calculated using equation (02)
Isp= 263.1306 sec
Radius of throat and chamber is calculated using equation (16) & (17)

$$r_c = 0.135m, r_t = 0.0169m$$

Length of divergence and convergence are calculated using equation (18) & (19)
Taking $\beta = 45^0, \theta = 15^0$
$L_{convergence} = 0.13m, L_{divergence} = 0.18m$

Theoretical calculated values for upper nozzle
Assumptions
Pc=80 psi, Tc = 3200K, γ= 1.2, molecular weight= 19.84 mol, R= 419 (J/kg-k), F=1,000N
Using equation (09) & (10) we can find pressure and temperature at throat
Pt= 45.157 psi
Tt= 2909.09 K
Using equation (04) we can find the exit Mach number
Me= 4.2
Using equation (13) coefficient of thrust is calculated
C_f= 2.245
Using equation (12) area at exit of nozzle can be calculated
At = 0.0008 m^2
Using equation (10) ratio of exit to throat area is calculated

$$\frac{A_e}{A_t} = 37.8$$

And value of exit area is
Ae= 0.0305 m^2
Exit temperature and pressure is calculated using equation (05) & (06)
Te= 1157.742K, Pe=0.1794 psi
Mass flow rate is calculated using equation (14)
ṁ = 0.246 kg/s
Exit velocity is calculated using equation (15)
Ve= 3204.44 m/s
Specific impulse is calculated using equation (02)
Isp= 326 sec
Radius of throat and chamber is calculated using equation (16) & (17)
$r_c = 0.07m, \quad r_t = 0.01595m$
Length of divergence and convergence are calculated using equation (18) & (19)
Taking $\beta = 45^0, \theta = 15^0$
$L_{convergence} = 0.367m, L_{divergence} = 0.069m$

Maximum height
The calculations are done in drag free conditions.
As per Newton's law of motion,

$\frac{W}{a} = \frac{M}{g}$ (20)

$T = ma$ (21)

For multi stage rockets we will use subscripts 1, 2……. And n refers to conditions of the first and second stages during thrusting flight.

$a_n = \left(\frac{T_n}{M_n} - g\right)$ (22)

$v_n = \left(\frac{T_n}{M_n} - g\right) t_n$ (23)

$$v = v_1 + v_2 + v_3 \ldots\ldots\ldots\ldots + v_n$$

$t_n = \frac{M_{propellant}}{\dot{m}}$ (24)

Where, t_n = burn time of propellant

$v_n = \left(\frac{T_1}{M_1} - g\right) t_1 + \left(\frac{T_2}{M_2} - g\right) t_2 + \cdots \ldots\ldots\ldots \left(\frac{T_n}{M_n} - g\right) t_n$ (25)

$H_n = \left(\frac{T_n}{M_n} - g\right) \frac{t_n^2}{2} + t_n [\left(\frac{T_1}{M_1} - g\right) t_1 + \left(\frac{T_2}{M_2} - g\right) t_2 + \cdots \ldots\ldots\ldots \left(\frac{T_n}{M_n} - g\right)] t_{n-1}$ (26)

The total altitude will then be,

$H = H_1 + H_2 + \cdots \ldots\ldots + H_n + H_c$ (27)

(Or)

$H = \left(\frac{T_1}{M_1} - g\right) \frac{t_1^2}{2} + \left[\left(\frac{T_2}{M_2} - g\right) \frac{t_2^2}{2} + \left(\frac{T_1}{M_1} - g\right) t_1 t_2\right] + \left[\left(\frac{T_3}{M_3} - g\right) \frac{t_3^2}{2} \left(\frac{T_2}{M_2} - g\right) t_2 t_3 + \left(\frac{T_1}{M_1} - g\right) t_1 t_3\right] + \cdots \ldots\ldots\ldots\ldots\ldots \frac{v^2}{2g}$

(28)

$T_n = \frac{I_n g}{t_n}$ Where, I= total impulse (29)

$v_n = \left(\frac{I_n}{M_n} - t_n\right) g$ Where, v is velocity (30)

$v_n = \left(\frac{I_1}{M_1} - t_1\right) g + \left(\frac{I_2}{M_2} - t_2\right) g + \cdots \ldots\ldots\ldots \left(\frac{I_n}{M_n} - t_n\right) g$ (31)

$H_n = \left(\frac{I_n}{M_n} - t_n\right) \frac{g t_n}{2} + t_n g [\left(\frac{I_1}{M_1} - t_1\right) + \left(\frac{I_2}{M_2} - t_2\right) \ldots\ldots\ldots + (\frac{I_{n-1}}{M_{n-1}} - t_{n-1})]$ (32)

(Or)

$H_n = \frac{v_n t_n}{2} + t_n (v_1 + v_2 + v_3 .. + v_{n-1})$ (33)

$$H = H_1 + H_2 + \cdots \ldots\ldots\ldots + H_n + \frac{v^2}{2g}$$

(Or)

$H = \left(\frac{I_1}{M_1} - t_1\right) \frac{g t_1}{2} + \left[\left(\frac{I_2}{M_2} - t_2\right) \frac{g t_2}{2} + \left(\frac{I_1}{M_1} - t_1\right) g t_2\right] + \left[\left(\frac{I_3}{M_3} - t_3\right) \frac{g t_3}{2} + \left(\frac{I_2}{M_2} - t_2\right) g t_3 + \left(\frac{I_1}{M_1} - t_1\right) g t_3\right] + \cdots \ldots\ldots + \frac{v^2}{2g}$ (34)

(Or)

$H = \frac{v_1 t_1}{2} + \left(\frac{v_2}{2} + v_1\right) t_2 + \left(\frac{v_3}{2} + v_2 + v_1\right) t_3 + \cdots \ldots\ldots\ldots + \frac{v^2}{2g}$ (35)

Maximum height calculation

Thrust taken as

T1=10000N, T2=1000N

Using equation (29) we get I (total impulse) of both the stages

$I_1 = 52625.89$ Ns ,$I_2 = 8664.62$ Ns

Burn time is calculated using equation (24)

t1 =51.626 sec, t2=85 sec

Using equation (31) we get the total velocity as

V= 1101.835+1055.03716 =2156.87216 m/s

Using equation (33) / (39) to get the maximum height achieved by the rocket

H= 167,046.706m = 167.046 km

RESULTS

After calculating nozzle dimensions and its performance theoretically using above equations, following data is obtained:

Table-1 Nozzles dimensions of both stages

DIMENSIONS	UPPER NOZZLE(m)	LOWER NOZZLE(m)
CHAMBER DIAMETER(Dc)	0.14	0.27
EXPANSION RATIO(Ae/At)	37.8	10
DIVERGENCE LENGTH(Ldiv)	0.367	0.188
THROAT DIAMETER(Dt)	0.0319	0.0338
EXIT DIAMETER(De)	0.4925	0.1024
CONVERGENCE LENGTH(Lcon)	0.069	0.13

Figure. 3 upper nozzle 3D design

Figure. 4 lower nozzle 3D design

Table-2 Nozzle performance parameters

PARAMETERS	SUSTAINER STAGE	BOOSTER STAGE
CHAMBER PRESSURE(PSI)	80	1000
THRUST(N)	1000	10000
EXIT PRESSURE(PSI)	0.1794	14.94
PROPELLANT	LIQUID OXYGEN+LIQUID METHANE	AP/AL/HTPB
EXPANSION RATIO(Ae/At)	37.8	10
GAMMA	1.18	1.2
CHAMBER TEMPRETURE(K)	3200	3200
MACH NUMBER	4.2	3.16
ISP(sec)	326	263.13
EXIT TEMPRETURE(k)	1157.742	1684.21
EXIT VELOCITY(m/s)	3204.444	2588.184
MASS FLOW RATE(Kg/s)	0.246	3.874
coefficient of thrust(Cf)	2.245	1.6

After calculating maximum height of the rocket achieved with respect to propellants used in different stages, following data is obtained:

Table-3 Maximum height calculations

parameters	values
velocity (v1)[m/s]	1101.835
velocity (v2)[m/s]	1055.03716
total velocity(v)[m/s]	2156.87216
total impulse(I1)[Ns]	52625.89
total impulse (I2)[Ns]	8664.62
total height (H)[Km]	167.046

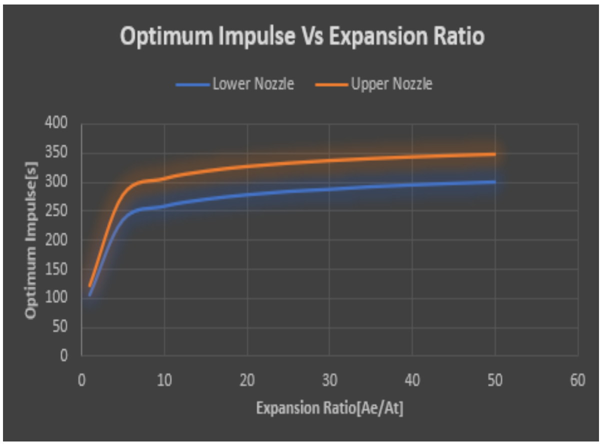

Figure. 5 Optimum Impulse vs. Expansion Ratio graph

In this figure we can see the changes in the optimum impulse with respect to the expansion ratio, the impulse keeps on increasing as we increase the expansion ratio.

CONCLUSION

Liquid oxygen as an oxidizer and liquid methane as a fuel is used as an upper stage rocket propellant, HTPB + aluminium as an oxidizer and ammonium perchlorate as a fuel is used as a lower stage rocket propellant. Secondary fuel ejector is being used as a thrust vector control system because of its various advantages over other systems.A sustainer and booster stage nozzles of two stage sounding rocket are designed with mathematical calculations to compare it with analytical calculations using rocket nozzle and isentropic gas equations, providing following data, exit area of $A_e = 0.305m^2$, expansion area ratio of 37.8:1, Mach number of 4.2, exit pressure value as 0.1794psi, exit velocity of 3204.444m/s & exit area of$A_e = 0.00825m^2$, expansion area ratio of 10:1, Mach number of 3.16, exit pressure of 14.9psi, exit velocity of 2588.184m/s respectively.

Declaration of Conflict of interests
The author(s) declared no potential conflicts of interest with respectto the research, authorship, and/or publication of this article.

ACKNOWLEDGMENT

The authors are very thankful to the ASTROEX RESEARCH ASSOCIATION, DEORIA for providing excellent professional support to complete this research work successfully.

REFERENCES

[1]. Marco Pallone, Mauro Pontani, Paolo Teofilatte and Angelo Minotti, “Design Methodology and performance evaluation of new generation sounding rocket”, International journal of Aerospace engineering, 2018.
[2]. Sidhant Singh, “Solid Rocket Motor for Experimental Sounding Rockets”, Advances in Aerospace Science and Applications, Vol.3 (3), pg. no-199-208, 2013.
[3]. Thada Suskila, “Experimental investigation of the solid rocket motors for small sounding rockets”, International conference on mechanical engineering, TSME-ICoME 2017.
[4]. Karl Weiland naumann, Helmut k.ciezki, ralfstierle, Konrad Schmid, jurgenramsel, “Rocket propulsion with gelled propellants for sounding rockets” ,ESA Symposium on European rocket and balloon programs related research, ESA SP-700 ,Pg. no -143-148,2011.
[5]. Ankit Kumar Mishra, Rishi Suhas Karandikar, Janani Kavipriya. S,Ashutoshkumar, “Study of nozzle flow separations in divergent section of nozzle”, International Journal of Universal Science and Engineering, Vol. No. 7, Jan-Dec,2021.
[6]. Ankit Kumar Mishra, Janani Kavipriya.V.S, “A Technical review on effect of spray angles and characteristics for a pintle injector”, Journal of Advanced Engineering Research, vol.7, No.2, pp. 30-36, 2020.
[7]. Ankit Kumar Mishra, Madhumitha M, Vaishnavi Shenoy “Numerical and Computational Analysis on Two Stage Sounding Rocket”, International Journal of Science, Engineering and Technology, vol. 9(4), pp. 01-06, 2021.
[8]. Ankit Kumar Mishra, Kshitij Gandhi, Kuldeep Sharma, Neerumalla Sumanth, “Conceptual design and

analysis of two stage sounding rocket", International Journal of Universal Science and Engineering, vol. 7(1), pp. 52-72, 2021.

[9]. Ankit Kumar Mishra, Amandeep Singh, "Computational fluid dynamics analysis on the convergent nozzle design", International Journal of Advances in Engineering Research, vol. 20(3), pp. 01-06, 2020.

[10]. Ankit Kumar Mishra, Madhumitha M, Rama Devi, Prathiksha G Shetty, "Comparative study on propellant characteristics for reusable launch vehicles", International Journal of Universal Science and Engineering, vol.7(1), pp.40-47, 2021.

[11]. Ankit Kumar Mishra, Vinit Goswami, "Aerodynamic Thrust Characteristics and Performance Valuations for an aerospike nozzle: review", International Journal of Engineering Applied Sciences and Technology, Vol.5, No.11, pp. 128-133, 2021, https://doi.org/10.33564/IJEAST.2021.v05i11.018.

[12]. Ankit Kumar Mishra, Shruti Jadhav, Masetty Akshay, "Theoretical Aspects on Design and Performance Characteristics of solid rocket motor", International Journal of All Research Education and Scientific Methods, Vol.10, No.2, pp. 894-898, 2022.

[13]. Ankit Kumar Mishra, MS Ragul, "Theoretical Model Study on Chemical Compositions Affecting the Space Launch Vehicles", REST Journal on Emerging Trends in Modelling and Manufacturing, Vol. 8(1), 2022.https://doi.org/10.46632/jemm/8/1/6.