

# Performance Analysis of General Selection Combining Diversity Receivers for QAM Signalling

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**Abstract:** In this paper, the Generalized Selection Combining (GSC) diversity scheme over Quadrature Amplitude Modulation (QAM) is presented. The closed-form expressions for Average Symbol Error Rate (ASER) of arbitrary M-ary Cross Quadrature Amplitude Modulation (XQAM) and M-ary rectangular Quadrature Amplitude Modulation is also derived in the paper.

**Keywords:** Average Symbol Error Rate (ASER), Cross-QAM (XQAM), Generalized Selection Combining (GSC), Quadrature Amplitude Modulation (QAM), Rectangular-QAM.

## I. INTRODUCTION

DIVERSITY combining which consists of receiving redundantly the same information-bearing signal over two or more fading channels, then combining these multiple replicas at the receiver in order to increase the overall received Signal-to- Noise Ratio (SNR) is a classical concept [1]-[3], which has been used for the past half century to combat the effects of fading on wireless systems. There are four principal types of combining techniques which depend essentially on the complexity restrictions put on the communication system and the amount of available channel state information at the receiver. The following principal types are: 1) Maximal-Ratio Combining (MRC); 2) Equal-Gain Combining (EGC); 3) Selection Combining (SC); and 4) SWitch Combining (SWC) [4]. In [5], closed-form expressions for ASER for modulation schemes such as BPSK, M-PSK and square M-ary quadrature amplitude modulation (M-QAM) are derived for independent and identically distributed (i.i.d) links. A multimedia transmission over wireless channels demands a large bandwidth. The bandwidth efficient QAM scheme can be a better solution to meet this demand. Rectangular QAM is a generic modulation technique since it includes square QAM, BPSK, Orthogonal Binary Frequency-Shift Keying, Quadrature Phase-Shift Keying and multilevel Amplitude Shift-Keying modulation techniques as special cases [6].

Quadrature Amplitude Modulation (QAM) has been widely used in digital communication systems due to its high bandwidth efficiency. When the number of bits per symbol is even, transmission can be easily implemented by using square QAM. However, if there is a requirement for the transmission of an odd number of bits per symbol, cross-QAM (XQAM) has lower average symbol energy than rectangular-QAM [7]. XQAM has been found to be useful in adaptive modulation schemes, wherein the constellation size is adjusted depending on the channel quality [8]-[13], and in blind equalization [14]-[17]. As a result, XQAMs have been adopted in many practical systems. For example, XQAMs with constellations ranging from 5 to 15 bits have been used in asymmetric digital subscriber lines and very high speed digital subscriber lines [18], [19], and 32- and 128-XQAMs are adopted in digital video broadcasting-cable [20]. Here in the paper, generalized selection combining technique for making X-QAM or Rectangular-QAM is used .

The remainder of this paper is organised as follows. Section II gives the ASER performance. In Section III Simulation performance is discussed. Section IV draws the conclusion.

## II. AVERAGE SYMBOL ERROR RATE PERFORMANCE

### 1. Closed form SEP of XQAM channel

In AWGN channel, the exact SEP for M-ary XQAM (M=32, 128, 512 ...) [6] is given below in (1):

$$P_s(\gamma) = g_1 Q(\sqrt{2A_0\gamma}) + \frac{4}{M} Q(\sqrt{2A_1\gamma}) - g_2 Q^2(\sqrt{2A_0\gamma}) \frac{8}{M} \sum_{k=1}^{M-1} Q_a(\sqrt{2A_0\gamma}, \alpha_k) - \frac{4}{M} \sum_{k=1}^{M-1} Q_a(\sqrt{2A_k\gamma}, \beta_k^+) + \frac{4}{M} \sum_{k=1}^{M-1} Q_a(\sqrt{2A_k\gamma}, \beta_k^-) \quad (1)$$

Here the Q(x) function is expanded in (2)

$$Q_a(x, \theta) = \frac{1}{\pi} \int_0^{\infty} \exp\left(-\frac{x^2}{2\sin^2\theta}\right) d\theta \quad x \geq 0 \quad (2)$$

Switching the order of summation and integration and defining the integral  $I_n(\theta; c_1, c_2)$  in (3) as given in [4]

$$I_n(\theta; c_1, c_2) = \frac{1}{\pi} \int_0^{\theta} \left(\frac{\sin^2\theta}{\sin^2\theta+c_1}\right)^n \left(\frac{\sin^2\theta}{\sin^2\theta+c_2}\right) d\theta \quad (3)$$

Where  $c_1$  and  $c_2$  are constant.

We can rewrite the average symbol error rate in (4) as given in [4]

$$\begin{aligned} P_s(\gamma) = & \mathcal{G}_1 \binom{L}{L_c} \sum_{l=0}^{L-L_c} \frac{(-1)^l \binom{L-L_c}{l}}{1+\frac{l}{L_c}} I_{L_c-1} \left( \frac{\pi}{2}; A_0\gamma, \frac{A_0\gamma}{1+\frac{l}{L_c}} \right) + \frac{4}{M} \binom{L}{L_c} \sum_{l=0}^{L-L_c} \frac{(-1)^l \binom{L-L_c}{l}}{1+\frac{l}{L_c}} I_{L_c-1} \left( \frac{\pi}{2}; A_1\gamma, \frac{A_1\gamma}{1+\frac{l}{L_c}} \right) - \\ & \mathcal{G}_2 \binom{L}{L_c} \sum_{l=0}^{L-L_c} \frac{(-1)^l \binom{L-L_c}{l}}{1+\frac{l}{L_c}} I_{L_c-1} \left( \frac{\pi}{4}; A_0\gamma, \frac{A_0\gamma}{1+\frac{l}{L_c}} \right) - \\ & \frac{8}{M} \sum_{k=1}^{V-1} \left( \binom{L}{L_c} \sum_{l=0}^{L-L_c} \frac{(-1)^l \binom{L-L_c}{l}}{1+\frac{l}{L_c}} I_{L_c-1} \left( \frac{\pi}{2}; A_0\gamma, \frac{A_0\gamma}{1+\frac{l}{L_c}} \right) \right) - \\ & \frac{4}{M} \sum_{k=1}^{V-1} \left( \binom{L}{L_c} \sum_{l=0}^{L-L_c} \frac{(-1)^l \binom{L-L_c}{l}}{1+\frac{l}{L_c}} I_{L_c-1} \left( \frac{\pi}{2}; A_k\gamma, \frac{A_k\gamma}{1+\frac{l}{L_c}} \right) \right) + \\ & \frac{4}{M} \sum_{k=1}^{V-1} \left( \binom{L}{L_c} \sum_{l=0}^{L-L_c} \frac{(-1)^l \binom{L-L_c}{l}}{1+\frac{l}{L_c}} I_{L_c-1} \left( \frac{\pi}{2}; A_k\gamma, \frac{A_k\gamma}{1+\frac{l}{L_c}} \right) \right) \end{aligned} \quad (4)$$

The integral  $I_n(\theta; c_1, c_2)$  is with the help of [4, Appendix III-A]

## 2. Closed form SEP of Rectangular QAM channel

The ASER for the rectangular QAM can be computed as given in [23]

$$P_s(s) = \int_0^{\infty} P_s(e/\gamma_t) f_{\gamma_t}(\gamma_t) d\gamma_t \quad (5)$$

And  $P_s(e/\gamma_t)$  is the conditional symbol error rate for the rectangular  $M = M_1 \times M_2$ -QAM modulation given in [24]

$$P_s(e/\gamma_t) = 2pQ(a\sqrt{\gamma_t}) + 2qQ(b\sqrt{\gamma_t}) - 4pqQ(a\sqrt{\gamma_t})Q(b\sqrt{\gamma_t}) \quad (6)$$

$$\text{Where, } p = 1 - \frac{1}{M_1}, \quad q = 1 - \frac{1}{M_2} \quad \text{and } a = \sqrt{\frac{6}{(M_1^2 - 1) + (M_2^2 - 1)\beta^2}}$$

Here the Q(x) function is expanded with the help of (2).

We can rewrite the symbol error rate just like given in (4)

$$\begin{aligned} P_s(e) = & 2p \binom{L}{L_c} \sum_{l=0}^{L-L_c} \frac{(-1)^l \binom{L-L_c}{l}}{1+\frac{l}{L_c}} I_{L_c-1} \left( \frac{\pi}{2}; g\bar{\gamma}, \frac{g\bar{\gamma}}{1+\frac{l}{L_c}} \right) + 2q \binom{L}{L_c} \sum_{l=0}^{L-L_c} \frac{(-1)^l \binom{L-L_c}{l}}{1+\frac{l}{L_c}} I_{L_c-1} \left( \frac{\pi}{2}; g\bar{\gamma}, \frac{g\bar{\gamma}}{1+\frac{l}{L_c}} \right) \\ & - \\ & 4pq \binom{L}{L_c} \left( \sum_{l=0}^{L-L_c} \frac{(-1)^l \binom{L-L_c}{l}}{1+\frac{l}{L_c}} I_{L_c-1} \left( \frac{\pi}{2} - \arctan\left(\frac{b}{a}\right); \frac{g\bar{\gamma}}{1+\frac{l}{L_c}} \right) \right) + \sum_{l=0}^{L-L_c} \frac{(-1)^l \binom{L-L_c}{l}}{1+\frac{l}{L_c}} I_{L_c-1} \left( \arctan\left(\frac{b}{a}\right); g\bar{\gamma}, \frac{g\bar{\gamma}}{1+\frac{l}{L_c}} \right) \end{aligned} \quad (7)$$

The integral  $I_n(\theta; c_1, c_2)$  is calculated with the help of [4, Appendix III-A]

### III. SIMULATION PERFORMANCE

Various types of Cross-QAM or Rectangular-QAM graph is simulated for three different values of Available path  $L=3,4$  and  $5$  and Diversity path  $L_c = 3$ , with the help of generalized combining technique. This technique discussed in [4] for BPSK, BFSK.

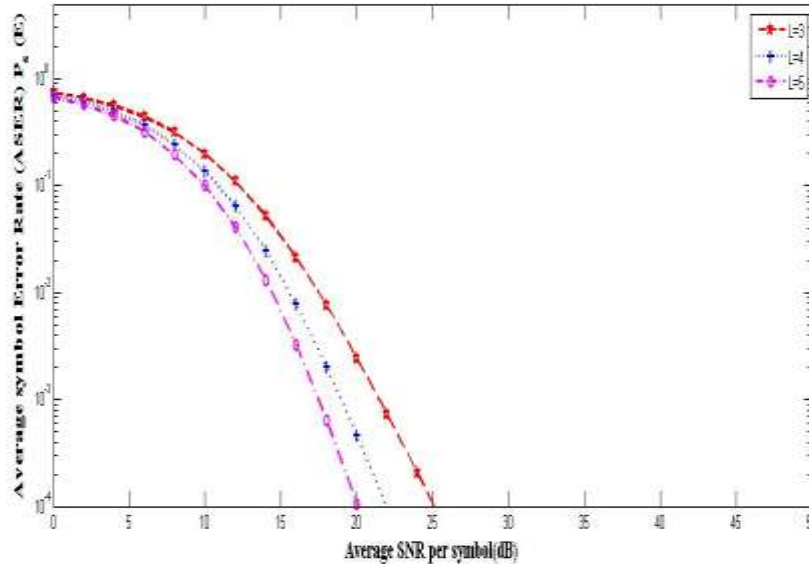


Fig.1: Average symbol error rate of 32- cross QAM

Fig. 1 shows the normalized average combined signal to noise ratio  $\overline{\gamma}_{GSC} / \overline{\gamma}$  as a function of the number of strongest combined path  $L_c$  for various values of the number of the  $L$  available resolvable paths. These curves indicate that for a fixed number of combined paths a better performance improvement can be gained by increasing the number of available diversity paths. The figure shows that minimum ASNR at  $L = 5$  is approximately 20 dB which is 2 dB and 5 dB less than  $L = 4$  and  $L = 3$  respectively at  $ASER = 10^{-4}$ .

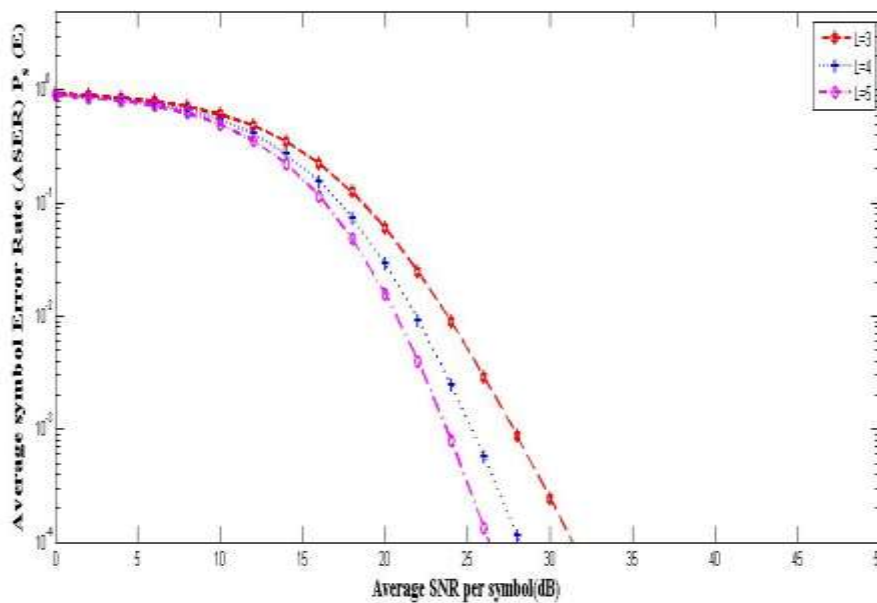


Fig.2 Average symbol error rate of 128- cross QAM

Fig.2 shows that minimum ASNR at  $L = 5$  is approximate 27 dB which is 7 dB higher than 32-cross QAM.

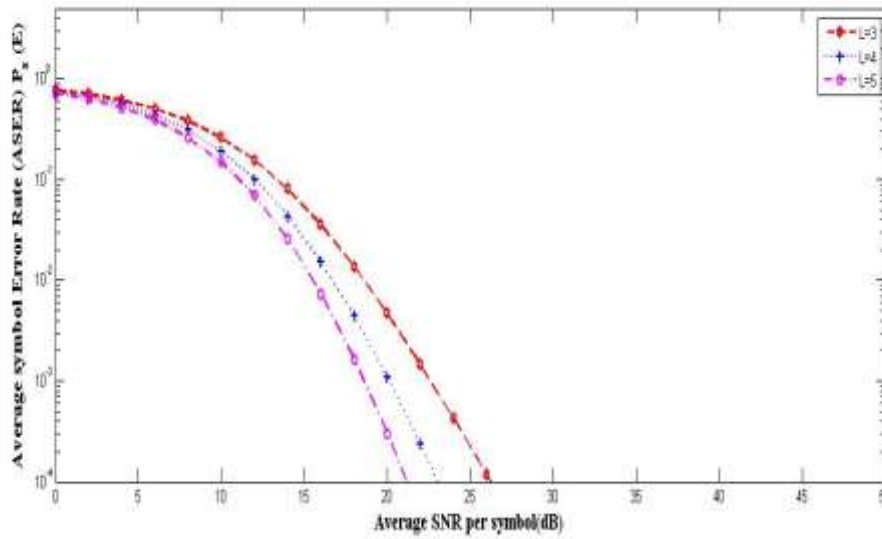


Fig.3 Average symbol error rate of 8X4 Rectangular QAM

Fig.3 shows that minimum ASNR at  $L = 5$  is approximate 26 dB which is 1 dB better than 128- cross QAM and 6 dB higher than 32-cross QAM.

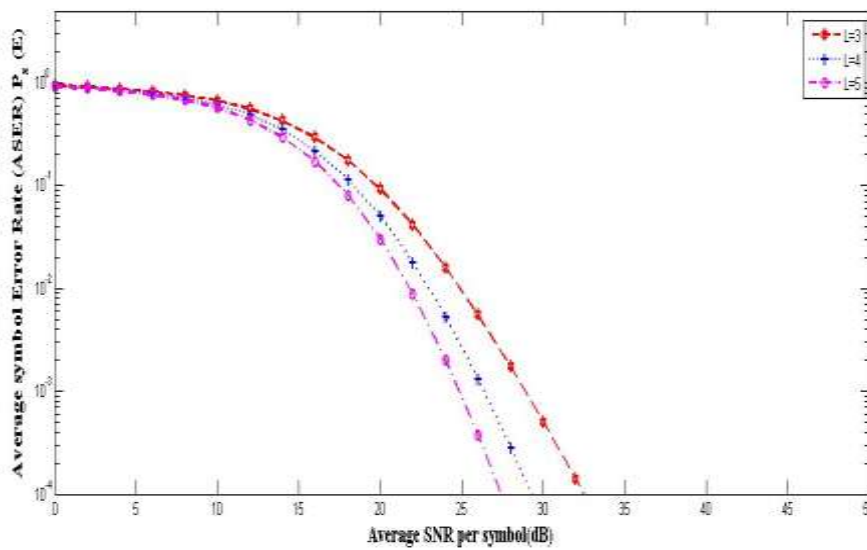


Fig. 4: Average symbol error rate of 16X8 Rectangular QAM

Fig.4 shows that minimum ASNR at  $L = 5$  is approximate 29 dB which is 9 dB higher than 32-cross QAM. From the simulation graphs we can conclude that cross QAM gives better performance at  $L_c = 3$  as compared to rectangular QAM at ASER  $10^{-4}$  as given in TABLE 1.

TABLE 1: DIFFERENT QAM TECHNIQUES

S.NO.	QAM TECHNIQUE	ASNR at ASER $10^{-4}$
1	Cross 32-QAM	20 dB
2	Cross 128-QAM	27 dB
3	Rectangular 8x4 -QAM	26 dB
4	Rectangular 16x8 -QAM	29 dB

#### IV. CONCLUSION

The cross QAM give better symbol error rate .There for over information is not lost .Cross-QAM preferred when the number of odd bit transferred .Both the peak power and average power can be reduced by using a cross-QAM constellation compared to rectangular-QAM and it is also useful in blind equalization and adaptive modulation.

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