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Analysis of Composite Beam under Multiple Boundary Conditions

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Abstract: A composite beam consists of laminate consisting of more than one lamina bonded together through their thickness. Thicknesses of Lamina are of order 0.005 inch (0.125mm), implying that to take realistic loads several lamina will be required. For a typical unidirectional lamina the mechanical properties are severely limited in the transverse direction. Stacking several unidirectional layers, may lead to an optimum laminate for unidirectional loads. However, this would not be desirable for complex loading and stiffness requirements,. One can overcome this problem by making a laminate with layers stacked at different angles to withstand different loading and stiffness requirements. Usually more than one lamina is bonded together through their thickness to get real structure. Each layer can be identified, its material, its angle of orientation with respect to a reference axis and by location in the laminate. Reduced stiffness matrix was obtained by using properties of composite material. These properties are longitudinal elastic modulus, Transverse elastic modulus, Major poisons ratio and Shear modulus. Using these properties composite compliance matrix was obtained. Inverse of compliance matrix was taken and reduced stiffness matrix was obtained. Then reduced stiffness matrix for each and every layer was calculated. Reduced stiffness matrix for each and every layer was calculated taking in consideration angles of the fiber in lamina. Mid plane symmetry was taken and position of each layer was calculated with respect to mid plane .D11 matrix was determine by formula using relative position of layer from mid plane and reduced stiffness matrix of all lamina (effect of angle of fiber was included) . Density of composite material was obtained by using densities of each material and there volume composition. Value of natural frequency in rad/sec and per sec was obtained by using formulation for finding frequency of composite material. Frequency was obtained for all the supports i.e. simple –simple, clamped –clamped and clamped –free and for first five mode of vibration. Glass/epoxy, graphite /epoxy composites were used to obtain tabulation for natural frequency in hertz. Comparison of frequency for these composite, frequency of composite under different mode condition were done for these composites. Taking beam as Euler beam , equation of Euler beam was considered and was solved for simple –simple clamped –clamped and clamped –free case taking in consideration boundary condition of simple – simple support condition i.e. displacement at support and bending moment at support is equal to zero .

Keywords: Composite Beam, Stiffness matrix, Ansys, Mid plane Symmetry, Fiber orientation, Modes of Vibration, Lamina.

Hooke's Law for composite laminates

Figure 1 shows a schematic representation of a composite lamina. The direction along the fiber axis is designated 1 (x axis). The direction transverse to the fiber axis but in the plane of the lamina is designated 2 (y axis). The direction transverse to both the fiber axis and the plane of the lamina (out of page) is designated 3 (z axis). This direction is not shown in the figure as it only becomes necessary in three-dimensional cases.
The 1-2 co-ordinate system can be considered to be local co-ordinates based on the fiber direction. However this system is inadequate as fibers can be placed at various angles with respect to each other and the structure. Therefore a new co-ordinate system needs to be defined that takes into account the angle the fiber makes with its surroundings as shown in fig 3.1. This new system is referred to as global co-ordinates (x-y system) and is related to the local co-ordinates (1-2 system) by the angle θ.

Fig : 1 Global co-ordinate system in relation to local co-ordinate system.
1-2 : Local Coordinates
x-y : Global Coordinates
A composite material is not isotropic and therefore its stresses and strains cannot be related by the simple Hooke's Law ($\sigma = \varepsilon E$). This law has to be extended to two-dimensions and redefined for the local and global co-ordinate systems [Fig: 4]. The result is Equations (1) and (2).

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$$\begin{bmatrix} \sigma 1 \\ \sigma 2 \\ \tau 12 \end{bmatrix} = \begin{bmatrix} Q11 & Q12 & 0 \\ Q21 & Q22 & 0 \\ 0 & 0 & Q33 \end{bmatrix} \begin{bmatrix} \in 1 \\ \in 2 \\ \gamma 12 \end{bmatrix} \text{....(1)}$$

Where,
σ1,2 are the normal stresses in directions 1 and 2 .
τ12 is the shear stress in the 1-2 plane;
ε1,2 are the normal strains in directions 1 and 2;
γ12 is the shear strain in the 1-2 plane;
[Q] is the reduced stiffness matrix;

$$\begin{bmatrix} \sigma x \\ \sigma y \\ \tau xy \end{bmatrix} = \begin{bmatrix} Q'11 & Q'12 & Q'16 \\ Q'21 & Q'22 & Q'26 \\ Q'16 & Q'26 & Q'66 \end{bmatrix} \begin{bmatrix} \in x \\ \in y \\ \gamma xy \end{bmatrix}$$

....(2)

Where,
σxy are the normal stresses in directions x and y;
τxy is the shear stress in the x-y plane;
εx,y are the normal strains in directions x and y;
γxy is the shear strain in the x-y plane;
[Q'] is the transformed reduced stiffness matrix.

The elements of Q matrix in equation are dependent on material constants and may be calculated using equation (3).

$$Q11 = \frac{E1}{1-\nu 12 \nu 21} \qquad \text{..........(3)}$$

$$Q_{12} = \frac{\nu 12 E2}{1-\nu 12 \nu 21}$$

$$Q_{22} = \frac{E2}{1-\nu 12 \nu 21}$$

where
E1,2 are Young's modulus in directions 1 and 2;
G12 is the shear modulus in the 1-2 plane;
ν12, are Poisson's ratios in the 1-2 and 2-1 planes.

Since normal stresses applied in the 1–2 direction do not result in any shearing strains in the 1–2 plane because Q16 = Q26 = 0 therefore unidirectional lamina is an especially orthotropic lamina. Also, the shearing stresses applied in the 1–2 plane do not result in any normal strains in the 1 and 2 directions because Q16 = Q26 = 0.

The [Q'] matrix in Equation (2) may be determined by Equation (4).

$[Q'] = [T]^{-1}[Q][R][T][R]^{-1}$

.....(34)
Where
[T] is the transformation matrix;
[R] is the Reuter matrix.
These matrices are given by:

$$T = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \text{....(3.5)}$$

$$T^{-1} = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Here
c = cosθ and s=sinθ

The local stresses and strains in Equation (1) are related to the global stresses and strains in Equation (2) by Eq. (6).

$$\begin{bmatrix} \sigma x \\ \sigma y \\ \tau xy \end{bmatrix} = [\mathrm{T}]^{-1} \begin{bmatrix} \sigma 1 \\ \sigma 2 \\ \tau 12 \end{bmatrix}$$

$$\begin{bmatrix} \epsilon 1 \\ \epsilon 2 \\ \gamma 12 \end{bmatrix} = [\mathrm{R}][\mathrm{T}][\mathrm{R}]^{-1} \begin{bmatrix} \epsilon x \\ \epsilon y \\ \gamma xy \end{bmatrix}$$

Equations (1) to (6) are used to determine the stresses and strains for a single composite layer. Since composites are multi-layered entities, equations for this case must also be set up. The result is equation (7).

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \epsilon^{\theta} \\ \kappa \end{bmatrix} \ldots..(7)$$

where
N is the vector of resultant forces;
M is the vector of resultant moments;
ϵ^{θ} is the vector of the mid-plane strains;
κ is the vector of mid-plane curvatures.
Vectors ε and κ are related to the global co-ordinates by Equation (7).

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A11 & A12 & A16 \\ A21 & A22 & A26 \\ A16 & A26 & A66 \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B11 & B12 & B16 \\ B21 & B22 & B26 \\ B16 & B26 & B66 \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B11 & B12 & B16 \\ B21 & B22 & B26 \\ B16 & B26 & B66 \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} D11 & D12 & D16 \\ D21 & D22 & D26 \\ D16 & D26 & D66 \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

……(8)

The [A], [B], and [D] matrices in Equation are known as the extensional, coupling, and bending stiffness matrices, respectively. The elements of these matrices may be determined from Equations (9) to (11).

$$\mathrm{A_{ij}} = \sum_{k=0}^{N} \left[\overline{\mathrm{Q_{ij}}}\right]_K (\mathrm{h_k} - \mathrm{h_{k-1}}) \quad \text{i=1,2,6 \& j=1,2,6}$$

…(9)

$$\mathrm{B_{ij}} = \frac{1}{2} \sum_{k=0}^{N} \left[\overline{\mathrm{Q_{ij}}}\right]_K (\mathrm{h_k^2} - \mathrm{h_{k-1}^2}) \quad \text{i=1,2,6 \& j=1,2,6}$$

…..(10)

$$D_{ij} = \frac{1}{3}\sum_{k=0}^{N}\left[\overline{Q_{ij}}\right]_K (h_k^3 - h_{k-1}^3) \quad i=1,2,6 \;\&\; j=1,2,6$$

……(11)

Where, n is the number of layers; is the i-th, j-th element of the [Q] matrix of the k-th layer; $h_k$is the distance of the top or bottom of the k-th layer from the mid-plane of the composite. Figure 1 illustrates how to determine the distance hk from the mid-plane.

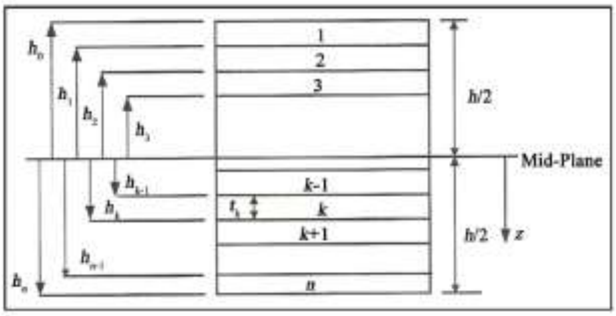

Fig 1: Locations of layers in a composite structure [21]

Vibration of Composite Beam by Classical Beam theory

On a structure dynamic loading can vary from recurring cyclic loading of the same repeated magnitude, such as a unbalanced motor which is turning at a specified number of revolutions per minute on a structure (for example), to the other extreme of a short time, intense, nonrecurring load, termed shock or impact loading, such as a bird striking an aircraft component during flight. A continuous infinity of dynamic loads exists between these extremes of harmonic oscillation and impact Associated mode shapes. Mathematically, there are infinity of natural frequencies and mode shapes in a continuous structure[26]. Dynamic loading can vary from intense, nonrecurring load known as shock load such as bird striking aero plane to recurring cyclic loading of magnitude which repeats itself such as unbalanced motors rotating at particular R.P.M. Any structures amplitude may rapidly grows with time if its frequency of oscillation matches its natural frequency. Structure can be overstressed which leads to its failure or due to large oscillations amplitude may be limited at large value which further leads to fatigue damages. Time dependent loading should be compared with natural frequency to ensure structural integrity of any structure. These two frequencies should be considerably different. While designing structure over deflecting and overstressing should be taken care of and resonances should be avoided.

ω_n is the natural circular frequency in radians per unit time for the nth vibrational mode.
Note that in this case there is one natural frequency for each natural mode shape, for n = 1, 2, 3,..., etc.
ω_n can be expressed as

$$\omega_n = \frac{\pi^2 n^2}{L^2}\sqrt{\frac{b\,D_{ij}}{\rho_m A}} \qquad …..(12)$$

Where b= Breadth of beam
A= Area of cross section
L= Length of beam
ρ_m = Density of composite material .
Transverse-shear effect is not taken into consideration in this equation .
For each n there would be different natural frequency .Frequency in hertz can be determined by
$f_n = \frac{\omega_n}{2\pi}$ in hertz.

…..(13)
The natural frequencies of a free-free supported beam are equal to natural frequency of clamped-clamped supported beam. Natural frequencies would be lower if transverse shear deformation effects were included.

First Order Shear Deformation Theory (FOSDT)

A generally laminated composite beam, is made of many piles of orthotropic materials, principal material axis of a ply may be oriented at an angle with respect to the x axis. Consider the origin of the beam is on mid-plane of the beam and x-axis coincident with the beam axis[27].

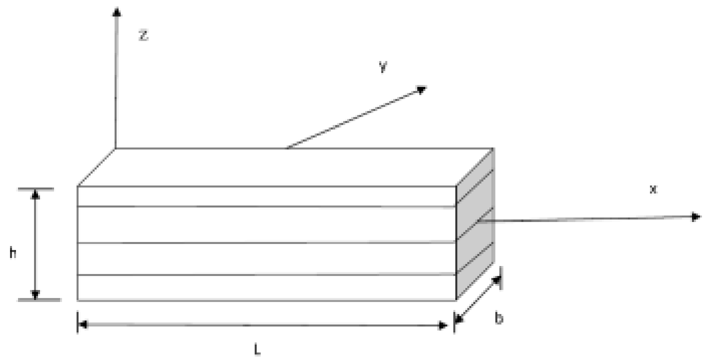

Fig 2: Geometry of a laminated composite beam[27]

Based on first- order shear deformation theory, assumed displacement field for the laminated composite beam can be written as

$$u(x,z,t) = u_0(x,t) + z\theta(x,t)$$
$$v(x,z,t) = z\theta_y(x,t)$$
$$w(x,z,t) = w_0(x,t)$$

………(14)
where, u =axial displacements of a point on the mid plane in the x-directions,
w = axial displacements of a point on the mid plane in the z-directions
θ= rotation of the normal to the mid-plane about the y axis,
θ_y= rotation of the normal to the mid-plane about the y axis,
t= time.
The strain-displacement relations are given by- (by theory of elasticity)—

$$\varepsilon_x = \partial u_0 / \partial x + z\partial\theta / \partial x$$
$$\gamma_{xz} = \partial w_0 / \partial x$$
$$\gamma_{xy} = \partial \psi / \partial x$$
$$k_x = \partial\theta / \partial x$$
$$k_{xy} = \partial\theta / \partial x$$

……(15)
By the classical lamination theory, the constitutive equations of the laminate can be obtained as-

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{16} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{16} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{11} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad \text{…….(16)}$$

Where,(i,j=1,2,6)
Nx, Ny,and N_{xy} are in plane forces
M_x ,M_y and M_{xy} are the bending and twisting moments
ε_x, ε_y and γ_{xy} are mid plane strain
κ_x , κ_y and κ_{xy} are bending and twisting curvatures

for the case of laminated composite beam,

Ny and N_{xy} , the in plane forces and bending moment $M_y = 0$
ε_y, γ_{xy} and κ_y the curvature assumed to be non-zero

Then equation can be rewritten as

$$\begin{Bmatrix} N_x \\ M_x \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{A}_{11} & \bar{B}_{11} & \bar{B}_{16} \\ \bar{B}_{11} & \bar{D}_{11} & \bar{D}_{16} \\ \bar{B}_{16} & \bar{D}_{16} & \bar{D}_{66} \end{bmatrix} \begin{Bmatrix} \partial u_0 / \partial x \\ \partial \theta / \partial x \\ \partial \psi / \partial x \end{Bmatrix}$$

…….(17)
Now considering the effect of transverse shear deformation then

$$Q_{xz} = A_{55}\gamma_{xz} = A_{55}\left(\partial \omega_0 / \partial x + \theta\right)$$

……..(18)
Where
Q_{xz} is the transverse shear force per unit length and

$$\begin{bmatrix} \overline{A_{11}} & \overline{B_{11}} & \overline{B_{16}} \\ \overline{B_{11}} & \overline{D_{11}} & \overline{D_{16}} \\ \overline{B_{16}} & \overline{D_{16}} & \overline{D_{66}} \end{bmatrix} = \begin{bmatrix} A_{11} & B_{11} & B_{16} \\ B_{11} & D_{11} & D_{16} \\ B_{16} & D_{16} & D_{66} \end{bmatrix} - \begin{bmatrix} A_{12} & A_{16} & B_{12} \\ B_{12} & B_{16} & D_{12} \\ B_{26} & B_{66} & D_{26} \end{bmatrix} * \begin{bmatrix} A_{22} & A_{26} & B_{22} \\ A_{26} & A_{66} & B_{26} \\ B_{22} & B_{26} & D_{22} \end{bmatrix}^{-1} * \begin{bmatrix} A_{12} & A_{16} & B_{12} \\ B_{12} & B_{16} & D_{12} \\ B_{26} & B_{66} & D_{26} \end{bmatrix}^{T}$$

The laminate stiffness coefficients Aij, Bij , Dij (i,j= 1,2,6) which are functions of laminate ply orientation , material properties and stack sequences, are given as-

$$A_{ij} = \int_{-h/2}^{h/2} \bar{Q}_{ij}\,dz$$

$$B_{ij} = \int_{-h/2}^{h/2} \bar{Q}_{ij}\,z\,dz$$

$$D_{ij} = \int_{-h/2}^{h/2} \bar{Q}_{ij}\,z^2\,dz$$

$$A_{55} = k\int_{-h/2}^{h/2} \bar{Q}_{55}\,dz$$

……..(19)
The transformed reduced stiffness constants Qij (i,j=1,2,6) are given as-

$$\overline{Q_{11}} = \left(Q_{11}\cos^4\phi + 2(Q_{12}+2Q_{66})\sin^2\phi\cos^2\phi + Q_{22}\cos^2\phi\right)$$

$$\overline{Q_{12}} = (Q_{11}+Q_{22}-4Q_{66})\sin^2\phi\cos^2\phi + Q_{12}\left(\sin^4\phi+\cos^4\phi\right)$$

$$\overline{Q_{22}} = \left(Q_{11}\sin^4\phi + 2(Q_{12}+2Q_{66})\sin^2\phi\cos^2\phi + Q_{22}\cos^4\phi\right)$$

$$\overline{Q_{16}} = (Q_{11}-Q_{22}-2Q_{66})\sin\phi\cos^3\phi + (Q_{12}-Q_{22}+2Q_{66})\cos\phi\sin^3\phi$$

$$\overline{Q_{26}} = (Q_{11}-Q_{22}-2Q_{66})\sin^3\phi\cos\phi + (Q_{12}-Q_{22}+2Q_{66})\cos^3\phi\sin\phi$$

$$\overline{Q_{66}} = (Q_{11}-Q_{22}-2Q_{66}-2Q_{12})\sin^2\phi\cos^2\phi + Q_{66}\left(\sin^4\phi+\cos^4\phi\right)$$

……(20)

Where,-
φ is the angle between the fiber direction and longitudinal axis of the beam.
The reduced stiffness constants Q11 , Q12 , Q22 and Q66 can be obtained in terms of the engineering constants

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}$$
$$Q_{12} = \frac{E_2\nu_{12}}{1-\nu_{12}\nu_{21}},$$
$$Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}}$$
$$Q_{66} = G_{66}$$

…..(21)

The total strain energy V of the laminated composite beam given as—

$$V = \frac{1}{2}\int_0^L \left[N_x \varepsilon_x^0 + M_x k_x + M_{xy} k_{xy} + Q_{xz}\gamma_{xz} \right] b.dx$$

……(22)
Substituting ε_x , κ_x , κ_{xy} and γ_{xy} values from equation (3.15) into equation (3.22) then

$$V = \frac{1}{2}\int_0^L [N_x \partial u_0 / \partial x + M_x \partial\theta / \partial x + M_{xy}\partial\psi / \partial x + Q_{xz}\left(\partial w_0 / \partial x + \theta\right)] b.dx$$

……(23)
Total kinetic energy T of the laminated composite beam is given as –

$$T = \frac{1}{2}\int_0^L \int_{-h/2}^{h/2} \rho\left[\left(\partial u / \partial t\right)^2 + \left(\partial v / \partial t\right)^2 + \left(\partial w / \partial t\right)^2\right] b.dx \quad …….(24)$$

Where,-
ρ is the mass density per unit volume.
Now substituting u, ν and ω from equation (14) into equation (23) and after integration with respect to z we get—

$$T = \frac{1}{2}\int_0^l I_1\left(\partial u_o / \partial t\right)^2 + I_3\left(\partial\theta / \partial t\right)^2 + 2I_2\left(\partial u_o / \partial t\right)\left(\partial\theta / \partial t\right) \qquad + I_3\left(\partial\psi / \partial t\right)^2 + I_1\left(\partial\omega_o / \partial t\right)^2$$

…..(25)
Where

$$I_1 = \int_{-h/2}^{h/2} \rho.dz$$

$$I_2 = \int_{-h/2}^{h/2} \rho.zdz$$

$$I_3 = \int_{-h/2}^{h/2} \rho.z^2 dz$$

By the use of Hamilton's principle, the governing equations of motion of the laminated composite beam can be expressed in the form-

$$\int_{t_1}^{t_2} \left(\delta T - \delta V\right) dt = 0$$

………(26)

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At $t = t_1$ and t_2
$\delta\psi = \delta\theta = \delta u_o = \delta w_o = 0$

After substitution the variational operations yields the following governing equation of motion

$$-I_1\left(\frac{\partial^2 u_0}{\partial t^2}\right) - I_2\left(\frac{\partial^2\theta}{\partial t^2}\right) + \overline{A_{11}}\left(\frac{\partial^2 u_0}{\partial x^2}\right) + \overline{B_{11}}\left(\frac{\partial^2\theta}{\partial x^2}\right) + \overline{B_{16}}\left(\frac{\partial^2\psi}{\partial x^2}\right) = 0$$

$$-I_1\left(\frac{\partial^2 w_0}{\partial t^2}\right) + A_{55}\left(\frac{\partial^2 w_0}{\partial x^2}\right) + A_{55}\left(\frac{\partial\theta}{\partial x}\right) = 0$$

$$-I_3\left(\frac{\partial^2\theta}{\partial t^2}\right) - I_2\left(\frac{\partial^2 u_0}{\partial t^2}\right) + \overline{B_{11}}\left(\frac{\partial^2 u_0}{\partial x^2}\right) + \overline{D_{11}}\left(\frac{\partial^2\theta}{\partial x^2}\right) + \overline{D_{16}}\left(\frac{\partial^2\psi}{\partial x^2}\right) - A_{55}\left(\frac{\partial w_0}{\partial x}\right) - A_{55}\theta = 0$$

$$-I_3\left(\frac{\partial^2\psi}{\partial t^2}\right) + \overline{B_{16}}\left(\frac{\partial^2 u_0}{\partial x^2}\right) + \overline{D_{16}}\left(\frac{\partial^2\theta}{\partial x^2}\right) + \overline{D_{66}}\left(\frac{\partial^2\psi}{\partial x^2}\right) = 0$$

Model Development

Problem Statement: Vibrational Analysis
Vibration analysis of a beam can be done on Ansys by providing structural data and load conditions on different supports. Structural data which are required for simple vibration analysis of beam on Ansys are

- Young's modulus
- Poisson's ratio
- Density
- Length , Breadth and Height .

For the sake of simplicity laminate is considered to be of unidirectional lamina

Problem 1
Vibrational analysis of graphite/ epoxy composite beam for different boundary conditions :
E1= 221 GPa , E2= 6.9 GPa , E3= 8.59 GPa
G12=4.8 GPa , G13= 4.14 GPa ,
G23=3.45 GPa , ν12= 0.3 , ρ= 1550.1 kg/m³,
L= length of the composite laminated beam =0.381 m,
b= width of the laminated composite beam= 25.4 mm,
h= thickness of the each ply = 25.4 mm.

Problem 2
Vibrational analysis of Glass / epoxy composite beam for different boundary conditions :
E1= 144.80 GPa , E2= 9.65 GPa , E3= 7.72 GPa
G12=4.14 GPa , G13= 4.14 GPa ,
G23=3.45 GPa , ν12= 0.3 , ρ= 1389.2 kg/m³,
L= length of the composite laminated beam =0.381 m,
b= width of the laminated composite beam= 25.4 mm,
h= thickness of the each ply = 25.4 mm.

RESULTS

Natural frequencies obtained from ANSYS are listed in tables and those results comparing with the available results of references for the composite laminated beam with different boundary conditions.
Here different numerical example taken for the analysis of natural frequencies and mode shapes of the composite laminated beam, where the numerical example contained the following data-

1. Material properties of the laminated composite beam.
2. Length of the laminated composite beam.
3. Width of the laminated composite beam.
4. Thickness of the laminated composite beam.
5. Lay-up of layers.(angles of fiber)
6. Density of the material.
7. Boundary conditions.

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The results obtained for Ansys and Euler's Beam Theory and comparing with the results of First order shears deformation theory which is available in literature.

5.1. Frequency Results from Numerical Analysis on Ansys

Table5.1:

Result for Graphite Epoxy Composite for Clamped-Clamped End Conditions

***** INDEX OF DATA SETS ON RESULTS FILE *****

Set	Time/Freq	Load Step	Substep	Cumulative
1	720	1	1	1
2	1815	1	2	2
3	4055	1	3	3

Table5.2:

Result for Graphite Epoxy Composite for Clamped-Free End Conditions

***** INDEX OF DATA SETS ON RESULTS FILE *****

Set	Time/Freq	Load Step	Substep	Cumulative
1	178	1	1	1
2	854	1	2	2
3	1982	1	3	3

Table5.3:

Result for Graphite Epoxy Composite for Simply Supported –Simply Supported End Conditions

***** INDEX OF DATA SETS ON RESULTS FILE *****

Set	Time/Freq	Load Step	Substep	Cumulative
1	387	1	1	1
2	1243	1	2	2
3	3108	1	3	3

Table5.4:

Result for Glass Epoxy Composite for Clamped-Clamped End Conditions

***** INDEX OF DATA SETS ON RESULTS FILE *****

Set	Time/Freq	Load Step	Substep	Cumulative
1	480	1	1	1
2	1298	1	2	2
3	3100	1	3	3

Table5.5:

Result for Glass Epoxy Composite for Clamped-Free End Conditions

***** INDEX OF DATA SETS ON RESULTS FILE *****

Set	Time/Freq	Load Step	Substep	Cumulative
1	230	1	1	1
2	860	1	2	2
3	1250	1	3	3

Table5.6:

Result for Glass Epoxy Composite for Simply Supported –Simply Supported End Conditions

***** INDEX OF DATA SETS ON RESULTS FILE *****

Set	Time/Freq	Load Step	Substep	Cumulative
1	512	1	1	1
2	912	1	2	2

3	1680	1	3	3

CONCLUSION

The natural frequencies of different boundary conditions of laminated composite beam have been reported. The program result shows in general a good agreement with the existing literature. Natural frequencies and mode shapes are obtained using first order shear deformation theory for different types of laminated composites. It was found that natural frequency increases with increase in mode of vibration as shown in above diagrams. Different examples were taken in analysis and it is found that natural frequencies increase with the value of E1 increases. Mode shape was plotted for differently supported laminated beam with the help of ANSYS to get exact idea of mode shape. Vibration analysis of laminated composite beam was also done on ANSYS to get natural frequency and same trend of natural frequency was found to be repeated.

It is found that natural frequency is minimum for clamped –free supported beam and maximum for clamped-clamped supported beam .In between these two, natural frequencies of simple-simple supported beam lies. An analytical formulation can be derived for modeling the behavior of laminated composite beams with integrated piezoelectric sensor and actuator. Analytical solution for active vibration control and suppression of smart laminated composite beams can be found. The governing equation should be based on the first-order shear deformation theory An algorithm based on the finite element method (FEM) can be developed to study the dynamic response of composite laminated beams subjected to the moving oscillator. The first order shear deformation theory (FSDT) should be assumed for the beam model.

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