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Mathematical Modelling of Dusty Gas Flow through Isotropic Porous Media with Forchheimer Effects

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ABSTRACT

Dusty gas flow through isotropic porous media is considered. The equations governing dusty gas flow through free space are intrinsically averaged in order to derive a comprehensive model that describes flow of a dusty gas through porous media. The developed model is capable of describing flow of mixtures with non-uniform number density through variable porosity media, while taking into account the porous microstructure and both the Darcy resistance and the Forchheimer micro-inertial effects.

Keywords: Gas-particle flow, porous media, Forchheimer effects.

1. INTRODUCTION

Dusty gas flow through porous media has received considerable attention over the second half of the last century due to the various applications of this type of flow in both natural and industrial situations. Concern for environmental pollution over the last fifty years, accompanied with increasing demand for drinking water worldwide made filtration design imperative, [1], [2], [3], [4]. Water shortage for agricultural use, and the need for more agricultural food products, mandated a need for a better understanding of irrigation systems in order to produce more efficient methodologies of distributing plant nutrients into soil layers and into plant roots. Soil pollution and soil contamination by heavy metals necessitate the study of particle-laden fluid flow. In addition, transport of slurries through porous structures, [5], [6], and the study of the movement of oil, water and gas through earth layers have made it imperative to develop transport models involving multi-phase fluid- and gas-particle mixtures, [3], [7], [8].

These and many other applications emphasize the need, and importance of modeling dusty gas flow through porous media, and seeking solutions to initial and boundary-value problems, [5], [9].

In the flow of a dusty gas, the fluid-phase is taken as the carrier fluid while the dust-phase represents the transported particulates. In cases where the porous matrix undergoes fragmentation, porosity of the matrix may change, and more particulates are added to the flow field. In cases where particles settle on the matrix walls (pore walls) a blockage may occur, thus decreasing the porosity. In both cases the porosity must be taken as a non-constant (variable) function of position. In cases where these changes are transient, the porosity is also a function of ,time (*cf.* [8], [10], [11], and the references therein).

Various mathematical models describing the flow of a dusty fluid have been developed based on the continuum approach and involve intrinsic averaging of Saffman's dusty gas model, [12], over an isotropic porous control volume. The available models address the steady-state flow, taking into account the Darcy resistance only and ignoring porosity changes and changes in the particle number density, [13]. The current work will consider development of a more general flow model of a dusty gas through an isotropic porous medium, with variable porosity to allow for possible porous matrix sedimentation and further matrix corrosion studies. The model will provide the flexibility for future studies to describe the fluid-phase and dust-phase velocity fields and to better understand the dependence of the particle distribution on the medium porosity and the changes in particle number distribution. This in turn will offer better understanding of particle settling rates in processes such as filtration.

The current work will also provide partial answers to the existing gap of knowledge with regard to microscopic inertial effects and how they are modeled in a dusty gas flow. This will be accomplished by utilizing porous microstructure descriptions that allow for Forchheimer effects to be taken into account, in addition to Darcy resistance effects. Finally, as the literature on dusty gas flow through porous media enjoys idealizations such as a uniform distribution of particles in the flow field and flow through constant porosity media, the model developed in this work is more general, and offers the flexibility to be sub-classified into simplifying categories that some applications may demand.

2. MODEL DEVELOPMENT

The flow of a viscous, incompressible dusty gas flow through free-space is governed by the following coupled set of field equations, due to Saffman, [12], written here in dyadic form suitable for volume averaging:

Fluid-phase continuity equation

$$\nabla \bullet \vec{U} = 0. \qquad \ldots(1)$$

Fluid-phase momentum equation

$$\rho_f \nabla \bullet \vec{U}\vec{U} = -\nabla P + \mu \nabla^2 \vec{U} + KN(\vec{V} - \vec{U}) - \rho_f \vec{G} \qquad \ldots(2)$$

Dust-phase continuity equation

$$\nabla \bullet N\vec{V} = 0. \qquad \ldots(3)$$

Dust-phase momentum equation

$$\rho_p \nabla \bullet N\vec{V}\vec{V} = KN(\vec{U} - \vec{V}) - \rho_p N\vec{G} + \rho_f N\vec{G} \qquad \ldots(4)$$

where $\vec{U}$ and $\vec{V}$ are the fluid-phase and dust-phase velocity fields, respectively, P is the fluid pressure, ρ_f is the fluid density, ρ_p is the dust particle density, K is the Stokes' coefficient of resistance, N is the particle distribution (or the particle number density, that is, the number of particles per unit volume), $\vec{G}$ is the local gravitational acceleration, and μ is the fluid viscosity coefficient.

Equations (2.1) through (2.4) represent a determinate system of eight scalar equations in the eight unknowns, $\vec{U}, \vec{V}, N,$ and P. Our interest is to develop a continuum model to describe the flow of a particle-fluid mixture through an isotropic porous material, with non-uniform particle distribution (variable number density, N). To accomplish this, the above equations will be averaged over a Representative Elementary Volume, defined as a control volume that contains solids and voids in the same proportion as the whole medium.

The effects of the porous microstructure on the flowing mixture will be accounted for through the concept of a Representative Unit Cell, [14], [15], [16], [17], that provides mathematical idealization to the porous microstructure and facilitates its description. Denoting the pore volume by V_φ, the bulk volume by V, and porosity by $\varphi = \dfrac{V_\varphi}{V}$, then following Bachmat and Bear, [18], and Hamdan, [19], we provide the following notation for volume averaging and its rules. The volumetric phase average of a quantity F (that is, the volumetric volume average of F over the bulk volume, V) is defined as:

$$< F > = \frac{1}{V} \iiint_{V_\phi} F dV \equiv \frac{1}{V} \int_{V_\phi} F dV \qquad \ldots(5)$$

and the intrinsic phase average (that is, the volumetric average of F over the effective pore space, V_ϕ) is defined as:

$$< F >_\varphi = \frac{1}{V_\varphi} \iiint_{V_\phi} F dV \equiv \frac{1}{V_\varphi} \int_{V_\phi} F dV. \qquad \ldots(6)$$

The relationship between the volumetric phase average and the intrinsic phase average is obtained from equations (5), (6) and the definition of porosity, and takes the form:

$$< F >= \varphi < F >_{\varphi} . \qquad \ldots(7)$$

Averaging theorems are written in the following forms, [19]. Let F and H be volumetrically additive scalar quantities, $\vec{F}$ a vector quantity, and c a constant (whose average is itself), then:

$(i)\ldots < cF >= c < F >=c\,\varphi < F >_{\varphi}$.

$$(ii)\ldots < \nabla F >= \varphi\nabla < F >_{\varphi} + \frac{1}{V}\iint_S F^{\circ}\vec{n}dS \equiv \varphi\nabla < F >_{\varphi} + \frac{1}{V}\int_S F^{\circ}\vec{n}dS$$

where S is the surface area of the solid matrix in the REV that is in contact with the fluid, and $\vec{n}$ is the unit normal vector pointing into the solid. The quantity $F^{\circ} = F - < F >$ is the deviation of the averaged quantity from its true (microscopic) value.

$$(iii)\ldots < F \mp H >=< F > \mp < H > = \varphi < F \mp H >_{\varphi} = \varphi < F >_{\varphi} \mp \varphi < H >_{\varphi}$$

$$(iv)\ldots < FH >= \varphi < FH >_{\varphi} = \varphi < F >_{\varphi} < H >_{\varphi} + \varphi < F^{\circ}H^{\circ} >_{\varphi}$$

$$(v) < \nabla \bullet \vec{F} >= \nabla \bullet \varphi < \vec{F} >_{\varphi} + \frac{1}{V}\iint_S \vec{F} \bullet \vec{n}dS \equiv \nabla \bullet \varphi < \vec{F} >_{\varphi} + \frac{1}{V}\int_S \vec{F} \bullet \vec{n}dS.$$

$(vi)\ldots$ Due to the no-slip condition, a surface integral is zero if it contains the fluid velocity vector explicitly.

The above averaging rules are applied to equations (1) through (4), to obtain:

For fluid-phase:

Continuity Equation:

$$\nabla \bullet \varphi < \vec{U} >_{\varphi} + \frac{1}{V}\int_S \vec{U} \bullet \vec{n}dS = 0. \qquad \ldots(8)$$

Momentum Equation:

$$\begin{aligned}
&\rho_f \nabla \bullet \varphi < \vec{U} >_{\varphi} < \vec{U} >_{\varphi} = \\
&-\varphi\nabla < P >_{\varphi} + \mu\nabla^2\varphi < \vec{U} >_{\varphi} + K\varphi < N >_{\varphi} [< \vec{V} >_{\varphi} - < \vec{U} >_{\varphi}] - \rho_f \varphi < \vec{G} >_{\varphi} \\
&+ K\varphi[< N^{\circ}\vec{V}^{\circ} >_{\varphi} - < N^{\circ}\vec{U}^{\circ} >_{\varphi}] - \rho_f \nabla \bullet \varphi < \vec{U}^{\circ}\vec{U}^{\circ} >_{\varphi} - \rho_f \varphi < \vec{G}^{\circ} >_{\varphi} \\
&+ \frac{1}{V}\int_S (\mu\nabla\vec{U} \bullet \vec{n} - \vec{n}P^{\circ})dS + \frac{1}{V}\int_S (\mu\vec{U} \bullet \vec{n} - \rho_f \vec{U}\vec{U} \bullet \vec{n})dS.
\end{aligned} \qquad \ldots(9)$$

For dust-phase:

Continuity Equation:

$$\nabla \bullet \varphi < N >_{\varphi} < \vec{V} >_{\varphi} + \nabla \bullet \varphi < N^{\circ}\vec{V}^{\circ} >_{\varphi} + \frac{1}{V}\int_S N\vec{V} \bullet \vec{n}dS = 0. \qquad \ldots(10)$$

Momentum Equation:

$$\begin{aligned}
&\rho_p \nabla \bullet \varphi < N >_{\varphi} < \vec{V} >_{\varphi} < \vec{V} >_{\varphi} = K\varphi < N >_{\varphi} [< \vec{U} >_{\varphi} - < \vec{V} >_{\varphi}] + \\
&+ (\rho_f - \rho_p)\varphi < N >_{\varphi} < \vec{G} >_{\varphi} + (\rho_f - \rho_p)\varphi < N^{\circ}\vec{G}^{\circ} >_{\varphi} + K\varphi[< N^{\circ}\vec{U}^{\circ} >_{\varphi} - < N^{\circ}\vec{V}^{\circ} >_{\varphi}] \\
&- \rho_p \nabla \bullet \varphi < N^{\circ}\vec{V}^{\circ}\vec{V}^{\circ} >_{\varphi} - \frac{\rho_p}{V}\int_S N\vec{V}\vec{V} \bullet \vec{n}dS .
\end{aligned} \qquad \ldots(11)$$

3. ANALYSIS OF SURFACE INTEGRALS AND DEVIATION TERMS

The deviation terms and surface integrals that appear in equations (8)-(11) contain information on the interactions that take place between the flowing phases and the porous medium. At the outset, Saffman's dusty gas model, [12], in free-space accounts for the effects of the flowing phases on each other by using a friction force proportional to the relative velocity of the flowing phases. In the case of flow of a dusty gas through a porous material, we have assumed that Saffman's relative velocity friction force is still valid.

The flowing phases undergo other forces that arise due to the presence of a solid matrix. In particular, pore walls (solid matrix) present additional solid boundary on which the fluid-phase experiences no-slip on its velocity and the dust-phase experiences additional friction. A greater solid surface area is available for the dust particles to either settle on, or to enhance solid particle reflection back into the flow field. Tortuosity of the flow path in the porous medium and the converging-diverging pore structure may enhance microscopic inertial effects or influence dispersion of the dust particles. In order to shed some light on these processes, we provide analysis of the surface integrals and deviation terms that are involved in the averaged governing equations.

3.1. Surface Integrals Involving the Fluid- and Dust-phase Velocities

When a dusty gas flows in a domain bounded by a solid, the fluid-phase experiences no-slip on the boundary. By invoking Gauss' Divergence Theorem, and making use of the fluid-phase continuity equation (1), we obtain:

$$\int_S \vec{U} \bullet \vec{n} dS = \int_{V_\varphi} \nabla \bullet \vec{U} dV = 0. \quad \ldots(12)$$

Accordingly, the terms $\frac{1}{V}\int_S \vec{U} \bullet \vec{n} dS$ and $\frac{1}{V}\int_S (\mu \vec{U} \bullet \vec{n} - \rho_f \vec{U}\vec{U} \bullet \vec{n}) dS$ vanish equations (8) and (9), which take the following updated forms, respectively:

$$\nabla \bullet \varphi < \vec{U} >_\varphi = 0. \quad \ldots(13)$$

$$\rho_f \nabla \bullet \varphi < \vec{U} >_\varphi < \vec{U} >_\varphi = \frac{1}{V}\int_S (\mu \nabla \vec{U} \bullet \vec{n} - \vec{n} P^\circ) dS$$
$$- \varphi \nabla < P >_\varphi + \mu \nabla^2 \varphi < \vec{U} >_\varphi + K\varphi < N >_\varphi \left[< \vec{V} >_\varphi - < \vec{U} >_\varphi \right] - \rho_f \varphi < \vec{G} >_\varphi$$
$$+ K\varphi [< N^\circ \vec{V}^\circ >_\varphi - < N^\circ \vec{U}^\circ >_\varphi] - \rho_f \nabla \bullet \varphi < \vec{U}^\circ \vec{U}^\circ >_\varphi - \rho_f \varphi < \vec{G}^\circ >_\varphi . \quad \ldots(14)$$

The surface integral $\int_S N\vec{V} \bullet \vec{n} dS$ appearing in the averaged dust-phase continuity equation (10) can be expressed using Gauss' Divergence Theorem and, upon using (3), we have:

$$\int_S N\vec{V} \bullet \vec{n} dS = \int_{V_\varphi} \nabla \bullet N\vec{V} dV = 0 . \quad \ldots(15)$$

The dust-phase continuity equation (10) thus takes the form:

$$\nabla \bullet \varphi < N >_\varphi < \vec{V} >_\varphi + \nabla \bullet \varphi < N^\circ \vec{V}^\circ >_\varphi = 0.$$
…(16)

It is worth noting that Saffman's dusty gas model assumes a small bulk concentration of dust particles (that is, a small volume fraction is occupied by the dust particles). This could justify taking the time rate of change of *N* to be negligibly small. However, the total number of particles within an REV is not constant. In this case, both the number density *N* and the total number of particles within the REV are functions of position, which points to the possibility of particle settling. We note that the total number of particles within the REV is $M = \int_{V_\varphi} N dV$ which can be argued to be approximated by the product $M = V_\varphi < N >_\varphi$. As $< N >_\varphi$ is a function of position, so is *M*.

Now, the surface integral $\frac{1}{V}\int_S N(\vec{V}\vec{V} \bullet \vec{n}) dS$ appearing in equation (11) has been argued to represent a shear force, [13]. Since the dust particle shear is absent, this integral vanishes, and equation (11) is replaced by:

$$\rho_p \nabla \bullet \varphi < N >_\varphi < \vec{V} >_\varphi < \vec{V} >_\varphi = K\varphi < N >_\varphi (< \vec{U} >_\varphi - < \vec{V} >_\varphi) +$$
$$+ (\rho_f - \rho_p) \varphi < N >_\varphi < \vec{G} >_\varphi + (\rho_f - \rho_p) \varphi < N^\circ \vec{G}^\circ >_\varphi + K\varphi (< N^\circ \vec{U}^\circ >_\varphi - < N^\circ \vec{V}^\circ >_\varphi)$$
$$- \rho_p \nabla \bullet \varphi < N^\circ \vec{V}^\circ \vec{V}^\circ >_\varphi . \quad \ldots(17)$$

3.2. Analysis of the Deviation Terms

Averaged terms involving deviations from their microscopic values are present in the fluid-phase momentum equations (14), in the dust-phase continuity equation (16), and in the dust-phase momentum equations (17). These terms are related to the hydrodynamic dispersion of the average phase velocities in the porous medium. Hydrodynamic dispersion through porous media has been argued to be the sum of mechanical dispersion (due to tortuosity of the flow path in the porous microstructure) and molecular diffusion of the fluid-phase vorticity, [19].

The deviation terms in $\nabla \bullet \varphi < \vec{U}^{\circ}\vec{U}^{\circ} >_{\varphi}$ of equation (3.3) and in $\nabla \bullet \varphi < N^{\circ}\vec{V}^{\circ}\vec{V}^{\circ} >_{\varphi}$ of equation (17) involve products of deviations of average phase velocities. They are inertial terms representative of mechanical dispersion due to the porous microstructure. In porous media where velocity and porosity gradients are not high, these terms are small, hence can be neglected. This can be seen from further expansion of the deviation terms, as follows:

$$< \vec{U}^{\circ}\vec{U}^{\circ} >_{\varphi} = < \vec{U}^{\circ} >_{\varphi} < \vec{U}^{\circ} >_{\varphi} + < \vec{U}^{\circ\circ}\vec{U}^{\circ\circ} >_{\varphi} \qquad \text{…(18)}$$

$$< N^{\circ}\vec{V}^{\circ}\vec{V}^{\circ} >_{\varphi} = < N^{\circ} >_{\varphi} < \vec{V}^{\circ} >_{\varphi} < \vec{V}^{\circ} >_{\varphi} + < N^{\circ\circ}\vec{V}^{\circ\circ}\vec{V}^{\circ\circ} >_{\varphi}. \qquad \text{…(19)}$$

In the absence of high porosity and velocity gradients, intrinsic averages of the deviations $< \vec{U}^{\circ} >_{\varphi}$ and $< \vec{V}^{\circ} >_{\varphi}$ are small and their products can be argued to be negligibly small. However, they may be of significance in media with high porosity gradients, and may thus be modeled using dynamic diffusivity, [12].

The term $< \vec{G}^{\circ} >_{\varphi}$ appearing in equation (14) and (17) represents a negligible deviation of the average local gravitational acceleration, hence ignored. The term $< N^{\circ}\vec{G}^{\circ} >_{\varphi}$ of equation (17) represents dispersion of the dust particles due to fluctuations in the local gravitational acceleration. Clearly, this term is negligible due to the small (if any) fluctuations in gravitational acceleration. This can also be seen from the following expansion:

$$< N^{\circ}\vec{G}^{\circ} >_{\varphi} = < N^{\circ} >_{\varphi} < \vec{G}^{\circ} >_{\varphi} + < N^{\circ\circ}\vec{G}^{\circ\circ} >_{\varphi}. \qquad \text{…(20)}$$

Since $< \vec{G}^{\circ} >_{\varphi}$ is negligible, and $< N^{\circ} >_{\varphi}$ is generally small, and the term $< N^{\circ}\vec{G}^{\circ} >_{\varphi}$ is subsequently negligibly small.

The terms $< N^{\circ}\vec{V}^{\circ} >_{\varphi}$ and $< N^{\circ}\vec{U}^{\circ} >_{\varphi}$ represent dispersion of the dust particles due to fluctuations in the dust-phase and fluid-phase average velocity vectors, respectively. The difference between these terms, namely $< N^{\circ}\vec{U}^{\circ} >_{\varphi} - < N^{\circ}\vec{V}^{\circ} >_{\varphi}$, and the negative of this difference, which appear in the fluid-phase and dust-phase momentum equations, represent dispersion of the particles due to fluctuations in the average relative velocity vector. These terms can be expanded as follows:

$$< N^{\circ}\vec{U}^{\circ} >_{\varphi} = < N^{\circ} >_{\varphi} < \vec{U}^{\circ} >_{\varphi} + < N^{\circ\circ}\vec{U}^{\circ\circ} >_{\varphi} \qquad \text{…(21)}$$

$$< N^{\circ}\vec{V}^{\circ} >_{\varphi} = < N^{\circ} >_{\varphi} < \vec{V}^{\circ} >_{\varphi} + < N^{\circ\circ}\vec{V}^{\circ\circ} >_{\varphi} \qquad \text{…(22)}$$

$$< N^{\circ}\vec{U}^{\circ} >_{\varphi} - < N^{\circ}\vec{V}^{\circ} >_{\varphi} = < N^{\circ} >_{\varphi} (< \vec{U}^{\circ} >_{\varphi} - < \vec{V}^{\circ} >_{\varphi}) + (< N^{\circ\circ}\vec{U}^{\circ\circ} >_{\varphi} - < N^{\circ\circ}\vec{V}^{\circ\circ} >_{\varphi}) \qquad \text{…(23)}$$

and analyzed in what follows. In case of a uniform particle distribution, N is constant, and $< N^{\circ} >_{\varphi} = 0$. Thus, the dispersion vectors vanish. For a non-uniform particle distribution, hydrodynamic dispersion may be either modeled as a Fourier diffusion process, or as diffusion expressed as a product of a diffusion coefficient vector, $\vec{\delta}$, and a number density driving differential, $< N >_{\varphi} - N_d$, where N_d is an average reference particle distribution. In this latter case we have:

$$< N^{\circ} >_{\varphi} < \vec{U}^{\circ} >_{\varphi} = \vec{\delta}_1 [< N >_{\varphi} - N_d]. \qquad \text{…(24)}$$

$$< N^{\circ} >_{\varphi} < \vec{V}^{\circ} >_{\varphi} = \vec{\delta}_2 [< N >_{\varphi} - N_d]. \qquad \text{…(25)}$$

$$< N^{\circ} >_{\varphi} [< \vec{U}^{\circ} >_{\varphi} - < \vec{V}^{\circ} >_{\varphi}] = (\vec{\delta}_1 - \vec{\delta}_2)[< N >_{\varphi} - N_d] = \vec{\delta}[< N >_{\varphi} - N_d]. \qquad \text{…(26)}$$

In light of (25), the term $\nabla \bullet \varphi < N^{\circ}V^{\circ} >_{\varphi}$ appearing in the dust-phase continuity equation, is expressed in the form:

$$\nabla \bullet \varphi < N^{\circ}V^{\circ} >_{\varphi} = \nabla \bullet \varphi \vec{\delta}_2 [< N >_{\varphi} - N_d] \qquad \ldots(27)$$

and the dust-phase continuity equation takes the following final form:

$$\nabla \bullet \varphi < N >_{\varphi} < \vec{V} >_{\varphi} + \nabla \bullet \varphi \vec{\delta}_2 [< N >_{\varphi} - N_d] = 0 \qquad \ldots(28)$$

and the fluid-phase momentum equation takes the following form when ignoring averaged deviations in gravitational acceleration:

$$\begin{aligned} &\rho_f \nabla \bullet \varphi < \vec{U} >_{\varphi} < \vec{U} >_{\varphi} = \\ &-\varphi \nabla < P >_{\varphi} + \mu \nabla^2 \varphi < \vec{U} >_{\varphi} + K\varphi < N >_{\varphi} \left[< \vec{V} >_{\varphi} - < \vec{U} >_{\varphi} \right] - \rho_f \varphi < \vec{G} >_{\varphi} \\ &- K\varphi \vec{\delta} [< N >_{\varphi} - N_d] + \frac{1}{V} \int_S (\mu \nabla \vec{U} \bullet \vec{n} - \vec{n} P^{\circ}) dS. \end{aligned} \qquad \ldots(29)$$

The dust-phase momentum equation also reduces to:

$$\begin{aligned} &\rho_p \nabla \bullet \varphi < N >_{\varphi} < \vec{V} >_{\varphi} < \vec{V} >_{\varphi} \\ &= K\varphi \left\{ < N >_{\varphi} (< \vec{U} >_{\varphi} - < \vec{V} >_{\varphi}) + \vec{\delta} [< N >_{\varphi} - N_d] \right\} + (\rho_f - \rho_p) \varphi < N >_{\varphi} < \vec{G} >_{\varphi} \end{aligned} \qquad \ldots(30)$$

3.3. Analysis of the Surface Integral involving Pressure Deviation

The solid porous matrix affects the flowing fluid through the portion of the surface area of the solid that is in contact with the fluid. The surface integral arising in (29), namely, $\frac{1}{V} \int_S (\mu \nabla \vec{U} \bullet \vec{n} - \vec{n} P^{\circ}) dS$, involves the pressure deviation and the fluid-phase velocity gradient. This integral is the same as the integral obtained in Whitaker, [20], [21], and referred to by Whitaker as a *surface filter*. It is also the same integral obtained in Du Plessis and Masliyah, [16], [17], and contains the information necessary to identify and quantify the forces exerted by the porous matrix on the fluid. By comparison with the averaged Navier-Stokes momentum equations, this integral is identified with the force that gives rise to Darcy resistance and the Forchheimer inertial terms. The following cases arise in quantifying this surface integral in the study of dusty gas flow through porous media:

Case 1:
In case of single phase flow through constant porosity porous media, it has been customary to identify this term with the Darcy resistance: $-\frac{\mu}{\eta} < \vec{U} >_{\varphi}$, where η is the permeability, [16], [17]. Since the above surface integral involves the fluid-phase velocity gradient only, the use of Darcy resistance given above is justifiable. However, neither the porous microstructure nor the variable porosity effects are accounted for. In addition, Forchheimer inertial effects cannot be accounted for when the Darcy resistance term alone is used.

Case 2:
For the case of dusty gas flow through constant porosity porous media, Darcy resistance may be expressed in terms of the relative velocity as: $-\frac{\mu}{\eta}(< U >_{\varphi} - < V >_{\varphi})$, [19]. This is justifiable in terms of the two-way interaction that takes place, and the fact that the dust-phase exerts a friction force on the fluid-phase which affects its velocity. However, in this case also there is no account given to the effects of the porous microstructure, the variable porosity and the Forchheimer inertial effects.

Case 3:
Accurate evaluation of the surface integral depends on the knowledge of the porous microstructure and its geometric description. Some important microstructure descriptions have been reported in [14], [15], [16], [17]. For example, in

their description of granular microstructure, Du Plessis and Masliyah, [16], [17], provided the following value for the surface integral that appears in equation (29), above:

$$\frac{1}{V}\int_S (\mu\nabla\vec{U}\bullet\vec{n}-\vec{n}p^{\circ})dS = -[3f(1-\tau)(3\tau-1)l^2/\tau^3]\mu<\vec{U}>_{\varphi} \qquad \ldots(31)$$

where l is a microscopic characteristic length, and

$$\tau = [1-(1-\varphi)^{2/3}]/\varphi. \qquad \ldots(32)$$

The factor f in equation (31) is the product of the Reynolds number and the friction factor associated with the flow of a dusty fluid through porous media; and τ is the tortuosity of the medium. As discussed in [20], [21], [22]. [23], the above surface integral can be decomposed into two parts: one is a shear force integral (which accounts for the viscous drag effects that predominate in the Darcy regime, that is, for small Reynolds number flow), and the other is an inertial force integral (which accounts for inertial drag effects that predominate in the Forchheimer regime, that is, for high Reynolds number flow).

This type of integral has received extensive analysis, and its quantification gives closure to the problem of flow through a porous structure, [15]. Quantification of this surface integral depends on evaluating the surface integral of pressure deviations, namely, $\int_S p^{\circ}\vec{n}dS$, and the surface integral of the directional derivative in the direction of the normal vector, namely, $\int_S \nabla\vec{U}\bullet\vec{n}dS = \int_S \frac{\partial\vec{U}}{\partial\vec{n}}dS$. Clearly, these surface integrals are dependent on the flow velocity but independent of the type of flowing fluid; hence, we will rely on what is already established in the literature to evaluate them.

To accomplish this, we let f_1 be the *velocity-independent* viscous shear geometric factor that depends on the geometry of the porous medium and gives rise to the Darcy resistance, and f_2 the *velocity-dependent* inertial geometric factor that gives rise to the Forchheimer inertial term. Following Du Plessis and Diedericks, [15], the Churchill-Usagi total frictional effects, f, of the porous matrix on the fluid may be expressed as:

$$f^r = f_1^r + f_2^r \qquad \ldots(33)$$

where r is a shifting factor that Du Plessis' results, [14], Du Plessis and Diedericks, [15], have shown to produce reasonable correlation when its value is unity. Furthermore, in terms of the factor f_1, hydrodynamic permeability, η, is given by, [15]:

$$\eta = \frac{\varphi}{f_1}. \qquad \ldots(34)$$

Whitaker, [20], [21], expressed the surface integral in terms of the *superficial velocity average* (namely, $<\vec{U}>=\varphi<\vec{U}>_{\varphi}$) as:

$$-\frac{1}{V}\int_S[-p^{\circ}\vec{n}+\mu\nabla\vec{U}\bullet\vec{n}]dS = -\mu f\varphi<\vec{U}>_{\varphi} = -\mu(f_1+f_2)\varphi<\vec{U}>_{\varphi}\ . \qquad \ldots(35)$$

Expressions for f_1 and f_2 require a mathematical description of the porous matrix and its microstructure. Du Plessis and Diedericks, [15], carried out extensive analysis on evaluating these geometric factors for isotropic porous media, based on Du Plessis and Masliyah's concept of a Representative Unit Cell (RUC), [16], [17], which they defined as the minimal REV in which the average properties of the porous medium are embedded. For granular isotropic porous media, the following expressions, as given in Du Plessis and Diedericks, [15], are adopted in this work for f_1 and f_2, and for the hydrodynamic permeability:

$$f_1 = \frac{36(1-\varphi)^{2/3}}{d^2[1-(1-\varphi)^{1/3}][1-(1-\varphi)^{2/3}]} \qquad \ldots(36)$$

$$f_2 = \frac{\rho d \left|\varphi < \vec{U} >_{\varphi}\right| C_d (1-\varphi)^{2/3}}{d^2 \mu [1-(1-\varphi)^{2/3}]^2} \quad \ldots(37)$$

where d is a microscopic length (such as the mean pore diameter) and C_d is the Forchheimer drag coefficient.

Hydrodynamic permeability for granular isotropic porous media is defined as:

$$\eta = \frac{\varphi}{f_1} = \frac{d^2 \varphi [1-(1-\varphi)^{1/3}][1-(1-\varphi)^{2/3}]}{36(1-\varphi)^{2/3}}. \quad \ldots(38)$$

It is customary to express the Darcy resistance and the Forchheimer term as $-\frac{\mu}{\eta}\varphi < \vec{U} >_{\varphi}$ and $-\frac{\rho C_d}{\sqrt{\eta}} \varphi < \vec{U} >_{\varphi} \left|\varphi < \vec{U} >_{\varphi}\right|$, respectively. We therefore use these expressions to bring the intrinsic averaged fluid-phase linear momentum equations to their final forms. We thus have:

$$\rho_f \nabla \bullet \varphi < \vec{U} >_{\varphi} < \vec{U} >_{\varphi} =$$
$$-\varphi \nabla < P >_{\varphi} + \mu \nabla^2 \varphi < \vec{U} >_{\varphi} + K\varphi < N >_{\varphi} \left[< \vec{V} >_{\varphi} - < \vec{U} >_{\varphi}\right] - \rho_f \varphi < \vec{G} >_{\varphi}$$
$$- K\varphi \vec{\delta}[< N >_{\varphi} - N_d] - \frac{\mu}{\eta} \varphi < \vec{U} >_{\varphi} - \frac{\rho C_d}{\sqrt{\eta}} \varphi < \vec{U} >_{\varphi} \left|\varphi < \vec{U} >_{\varphi}\right|. \quad \ldots(39)$$

3.4. Final Form of Governing Equations

Equations (13) and (39) represent the final form of the intrinsic-averaged fluid-phase continuity and momentum equations, respectively, and equations (28) and (30) are the final form of the intrinsic-averaged dust-phase continuity and momentum equations, respectively. We can write the governing equations in the following final form by letting:

$$< \vec{U} >_{\varphi} \equiv \vec{u}, \ < \vec{V} >_{\varphi} \equiv \vec{v}, \ < N >_{\varphi} \equiv n, \ < \vec{G} >_{\varphi} \equiv \vec{g}, \ < P >_{\varphi} = p \quad \ldots(40)$$

For fluid-phase:

$$\nabla \bullet \varphi \vec{u} = 0. \quad \ldots(41)$$

$$\rho_f \left[\frac{\partial \varphi \vec{u}}{\partial t} + \nabla \bullet \varphi \vec{u}\vec{u}\right] = -\varphi \nabla p + \mu \nabla^2 \varphi \vec{u} + K\varphi n[\vec{v} - \vec{u}] - \rho_f \varphi \vec{g} - K\varphi \vec{\delta}[n - N_d] - \frac{\mu}{\eta}\varphi \vec{u} - \frac{\rho C_d}{\sqrt{\eta}} \varphi \vec{u} |\varphi \vec{u}| \quad \ldots(42)$$

For Dust-phase:

$$\nabla \bullet \varphi n \vec{v} + \nabla \bullet \varphi \vec{\delta}_2 [n - N_d] = 0 \quad \ldots(43)$$

$$\rho_p \nabla \bullet \varphi n \vec{v}\vec{v} = K\varphi n(\vec{u} - \vec{v}) + (\rho_f - \rho_p)\varphi n \vec{g} + K\varphi \vec{\delta}[n - N_d]. \quad \ldots(44)$$

4. CONCLUSION

In this work, we derived a general model that governs the unsteady flow of a dusty gas with non-uniform distribution through an isotropic porous material. Darcian and non-Darcian (Forchheimer) effects have been taken into account. While this work does not provide validation to the developed model, (as it remains a challenge to solve the full model), it points to the need for solving initial and boundary value problems and validating. Some experimental validation may be required, especially for better understanding of the dispersion and settling processes that take place in the porous medium.

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REFERENCES

[1]. C. Choo and C. Tien, "Analysis of transient behavior of deep-bed filtration," Colloid Interface Science, vol. 169, pp. 13-33, 1995.
[2]. T. Iwasaki, "Some notes on sand filtration," American Water Works Association, vol. 29(10), pp. 1591-1602, 1937.
[3]. M. M. Sharma and Y. C. Yortsos "A network model for deep bed filtration processes, " AIChE J., vol. 33(10), pp. 1644-1652, 1987.
[4]. D. Thomas, P. Penicot, P. Contal, D. Leclerc, and J. Vendel, "Clogging of fibrous filters by solid Aerosol particles: experimental and modeling study, " Chemical Engineering Science, vol.56, pp. 3549-3561, 2001.
[5]. M. H. Hamdan and R. M. Barron, "Numerical simulation of inertial dusty gas model of flow through naturally occurring porous media," Developments in Theoretical and Applied Mechanics, vol. XVII, pp. 36-43, 1994.
[6]. M. H. Hamdan and K. D. Sawalha, "Dusty gas flow through porous media, " Applied Mathematics and Computation, vol. 75, pp. 59-73, 1996.
[7]. C. V. Chrysikopoulos, E. A. Voudrias and M. M. Fryillas, "Modeling of contaminant transport resulting from dissolution of Non-aqueous phase liquid pools in saturated porous media, " Transport in Porous Media, vol.16, pp. 125-145, 1994.
[8]. F. Civan, and M. L. Rasmussen, "Analytical models for porous media impairment by particles in rectilinear and radial flows," In: Handbook of Porous Media, 2nd ed., K. Vafai Ed., Taylor and Francis, New York, 2005, pp. 485-542.
[9]. H. Ma and D. W. Ruth, "The microscopic analysis of high Forchheimer Number flow in porous media," Transport in Porous Media, vol. 13, pp.139-160, 1993.
[10]. D. C. Mays and J. R. Hunt, "Hydrodynamic aspects of particle clogging in porous media," Environmental Science and Technology, vol. 39, pp. 557-584, 2005.
[11]. R. C. K. Wong and D. C. A. Mettananda, "Permeability reduction in Qishn sandstone specimens due to particle suspension injection," Transport in Porous Media, vol. 81, pp. 105-122, 2010.
[12]. P. G. Saffman, "On the Stability of Laminar Flow of a Dusty Gas," J. Fluid Mechanics, vol. 13(1), pp. 120-129, 1962.
[13]. F. M. Allan and M. H. Hamdan, "Fluid-particle model of flow through porous media: the case of uniform particle distribution and parallel velocity fields," Applied Mathematics and Computation, vol. 183(2), pp. 1208-1213, 2006.
[14]. J. P. Du Plessis, 1994, "Analytical quantification of coefficients in the Ergun equation for fluid friction in a packed bed," Transport in Porous Media, vol. 16, pp. 189-207, 1994.
[15]. J. P. Du Plessis and G. P. J. Diedericks, "Pore-Scale Modeling of Interstitial Phenomena," In: Fluid Transport in Porous Media, J.P. Du Plessis, ed., Computational Mechanics Publications, Southampton, 1997, pp. 61-104.
[16]. J. P. Du Plessis and J. H. Masliyah, "Mathematical modeling of flow through consolidated isotropic porous media," Transport in Porous Media, vol. 3, pp. 145-161, 1988.
[17]. J. P. Du Plessis and J. H. Masliyah, "Flow through isotropic granular porous media," Transport in Porous Media, vol. 6, pp. 207-221, 1991.
[18]. Y. Bachmat and J. Bear, "Macroscopic modeling of transport phenomena in porous media," I: The Continuum Approach, Transport in Porous Media, vol. 1, pp. 213-240, 1986.
[19]. M. H. Hamdan, 2010, "Mathematical Models of Dusty Gas Flow through Porous Media," Plenary Lecture: 12th WSEAS Conference on Mathematical Methods, Computational Techniques, Intelligent Systems. WSEAS Press, pp. 131-138, 2010.
[20]. S. Whitaker, "Volume Averaging of Transport Equations," In: Fluid Transport in Porous Media, J.P. Du Plessis, ed., Computational Mechanics Publications, Southampton, 1997, pp. 1-60.
[21]. S. Whitaker, "The method of volume averaging," Kluwer Academic Publishers, Dordrecht, 1999.
[22]. D.W. Ruth and H. Ma, "On the derivation of the Forchheimer equation by means of the average theorem," Transport in Porous Media, vol. 7(3), pp. 255-264, 1992.
[23]. D.W. Ruth and H. Ma, "Numerical Analysis of Viscous, Incompressible Flow in a Diverging-Converging RUC," Transport in Porous Media, vol. 13, pp. 161-177, 1993.