

Design of Smart Structures with Controllers For Active Vibration Control

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ABSTRACT

This paper deals with the active vibration control of beam like structures with distributed piezoelectric sensor and actuator layers bonded on top and bottom surfaces of the beam. The contribution of the piezoelectric sensor and actuator layers on the mass and stiffness of the beam is considered. The patches are located at the three region i.e. fixed end & middle, middle & free end, free & fixed end. The study is demonstrated through simulation in MATLAB for various controllers like proportional controller by output feedback and linear quadratic regulator (LQR) by state feedback. The entire structure is modeled in state space form using the concept of piezoelectric theory, Euler-Bernoulli beam theory, Finite Element Method (FEM) and the state space techniques. The numerical simulation shows that the sufficient vibration attenuation can be achieved by the proposed method.

Keywords: Smart structure, finite element model, state space model, proportional output feedback, LQR, vibration control.

1. INTRODUCTION

The developments of high strength to weight ratio mechanical structures are attracting engineers for the applications in light weight aerospace structures. However, vibration problems of structures have been more complicated with increase of strength to weight ratio. Till the last decade, passive techniques are among the most widely used structures. Passive vibration reduction can be achieved by adding mass damping and stiffness at appropriate locations. However Major drawback of Passive techniques is low response with increase in weight of structure. Hence, vibration control of high strength to weight ratio mechanical structures can be achieved using smart structures. The smart structures can be defined as:

” The structure that can sense external disturbance and respond to that with active control in real time to maintain the mission requirements.”

The present work considers the application of piezoelectric patches to smart beam-like structures for the purpose of active vibration control. The finite element method is powerful tool for designing and analyzing smart structures. Both structural dynamics and control engineering need to be dealt to demonstrate smart structures. A design method is proposed by incorporating control laws such as Proportional Output Feedback (POF) and Linear Quadratic Regulator (LQR) to suppress the vibration. Brij N Agrawal and Kirk E Treanor [1] presented the analytical and experimental results on optimal placement of Piezoceramics actuators for shape control of beam structures. Halim & Moheimani [2] aimed to develop a feedback controller that suppresses vibration of flexible structures. The controller is applied to a simple-supported PZT laminate beam and it is validated experimentally. S. Narayanan. Balamurugan [3] presented finite element modeling of laminated structures with distributed piezoelectric sensor and actuator layers. Beam, plate and shell type elements have been developed incorporating the stiffness, mass and electromechanical coupling effects of the piezoelectric laminates.

T. C. Manju Nath, B. Bandyopadhyay [4] presented the modeling and design of a multiple output feedback based discrete sliding mode control scheme application for the vibration control of a smart flexible. N.S. Villianil, S.M.R. Khalili [5] Presented the active buckling control of smart functionally graded (FG) plates using piezoelectric sensor/actuator patches is studied. The sensor output is used to determine the input to the actuator using the feedback control algorithm. Y Yu, X N

Zhang and S L Xie, 2009[6] presented this deal with the shape control of a cantilever beam structure by using laminated piezoelectric actuators (LPAs) with a low control voltage. The shape control equation of the cantilever beam partially covered with LPAs is derived based on the constitutive relations of the material and piezoelectric material and shear deformation theory. K. B. Waghulde, Bimleshkumar Sinha [7] presented vibration of a smart beam is being controlled. This smart beam setup is comprised of actuators and sensors placed at the root of the cantilever beam. Levent Malgaca [8] presented vibration control problem can be directly and systematically solved in a single analysis stage using commercial finite element problems. Pagano, S., Russo, R., Strano, S., and Terzo, M.[9] presented Non-linear modeling and optimal control of hydraulically actuated seismic isolator test rig. Khan, S., Suresh, A., and Seetharamaiah, N.[10] presented application of Magneto Rheological fluid Damper in flow shear mode for the optimal solution. Hernandez, A., Marichal, G., poncela, A., and Padron, I.[11] presented design of intelligent control strategies using a magnatorheological damper for span structure.

In most of present researches, FEM formulation of smart cantilever beam usually done in ANSYS and design of control laws are carried out in Mat LAB control system toolbox. Hence, for designing piezoelectric smart structures with control laws, it is necessary to develop a general design scheme of actively controlled piezoelectric smart structures. The objective of this work is to address a general design and analysis scheme of piezoelectric smart structures with control laws. The LQR optimal control approach using state feedback and proportional value of gain by output feedback has analyzed to achieve the desired control. Numerical examples are presented to demonstrate the validity of the proposed design scheme. This paper has organized in to three parts, FEM formulation of piezoelectric smart structure with designing control laws, Numerical simulation and Conclusion.

2. MODELING OF SMART CANTILEVER BEAM WITH CONTROL LAWS

2.1 Finite Element Formulation of beam element

A beam element is considered with two nodes at its end. Each node is having two degree of freedom (DOF). The shape functions of the element are derived by considering an approximate solution and by applying boundary conditions. The mass and stiffness matrix is derived using shape functions for the beam element. Mass and stiffness matrix of piezoelectric (sensor/actuator) element are similar to the beam element. To obtain the mass and stiffness matrix of smart beam element which consists of two piezoelectric materials and a beam element, all the three matrices added. The cantilever beam is modeled by FEM assembly of beam element and smart beam element. The last two row's two elements of first matrix are added with first two row's two element of next matrix. The global mass and stiffness matrix is formed. The boundary conditions are applied on the global matrices for the cantilever beam. The first two rows and two columns should be deleted as one end of the cantilever beam is fixed. The actual response of the system, i.e., the tip displacement $u(x, t)$ is obtained for all the various models of the cantilever beam with and without the controllers by considering the first two dominant vibratory modes.

A beam element of length l_b with two DOFs at each node i.e. translation and rotation is considered.

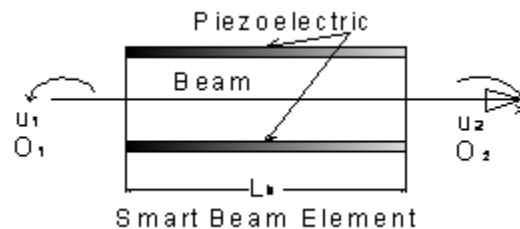


Fig 1. Smart structure.

The displacement u is given by

$$u(x)=[N]^T[p] \quad (1)$$

$$= [N_1(x) \quad N_2(x) \quad N_3(x) \quad N_4(x)] \begin{bmatrix} u_1 \\ \theta_1 \\ u_2 \\ \theta_2 \end{bmatrix} \quad (2)$$

$N_1(x), N_2(x), N_3(x), N_4(x)$ are the shape functions and u_1, θ_1 and u_2, θ_2 are the DOF's at the node1 and node2 respectively

$$\text{Where } N_1(x) = 1 - \frac{3x^2}{l_b^2} + \frac{2x^3}{l_b^3} \quad (3)$$

$$N_2(x) = x - \frac{2x^2}{l_b} + \frac{x^3}{l_b^2} \quad (4)$$

$$N_3(x) = \frac{3x^2}{l_b^2} - \frac{2x^3}{l_b^3} \quad (5)$$

$$N_4(x) = \frac{-x^2}{l_b} + \frac{x^3}{l_b^2} \quad (6)$$

The kinetic energy and bending strain energy of the element can be expressed as

$$T = \frac{1}{2} \int_0^{l_b} \rho_b A_b \left[\frac{\partial u(x,t)}{\partial t} \right]^2 dx = \frac{1}{2} \dot{u}^T [m] \dot{u} \quad (7)$$

$$V = \frac{1}{2} \int_0^{l_b} E_b I_b \left[\frac{\partial^2 u(x,t)}{\partial t^2} \right]^2 dx = \frac{1}{2} \ddot{u}^T [k] \ddot{u} \quad (8)$$

Where, ρ_b is the density of beam, E_b is the Young's modulus, I_b is the moment of inertia of cross-section, A_b is the area of cross-section

The governing differential equation of motion for the beam element can be represented as

$$M^b \ddot{p} + C\dot{p} + K^b p = q \quad (9)$$

where M^b , C , K^b , q are the mass, damping, stiffness, and the force co-efficient vectors of beam element.

The consistent mass matrix and stiffness matrices are obtained as

$$[M^b] = \rho_b A_b \int_0^{l_b} [N]^T [N] dx \quad (10)$$

$$[K^b] = E_b I_b \int_0^{l_b} [\ddot{N}]^T [\ddot{N}] dx \quad (11)$$

$$[M^b] = \rho_b A_b \int_0^{l_b} \begin{bmatrix} 1 - \frac{3x^2}{l_b^2} + \frac{2x^3}{l_b^3} \\ x - \frac{2x^2}{l_b} + \frac{x^3}{l_b^2} \\ \frac{3x^2}{l_b^2} - \frac{2x^3}{l_b^3} \\ -\frac{x^2}{l_b} + \frac{x^3}{l_b^2} \end{bmatrix} \begin{bmatrix} 1 - \frac{3x^2}{l_b^2} + \frac{2x^3}{l_b^3} & x - \frac{2x^2}{l_b} + \frac{x^3}{l_b^2} & \frac{3x^2}{l_b^2} - \frac{2x^3}{l_b^3} & -\frac{x^2}{l_b} + \frac{x^3}{l_b^2} \end{bmatrix} dx \quad (12)$$

$$[M^b] = \frac{\rho_b l_b A_b}{420} \begin{bmatrix} 156 & 22l_b & 54 & -13l_b \\ 22l_b & 4l_b^2 & 13l_b & -3l_b^2 \\ 54 & 13l_b & 156 & -22l_b \\ -13l_b & -3l_b^2 & -22l_b & 4l_b^2 \end{bmatrix} \quad (13)$$

$$[K^b] = E_b I_b \int_0^{l_b} \begin{bmatrix} \frac{6}{l_b^2} + \frac{12x}{l_b^3} \\ -\frac{4}{l_b} + \frac{6x}{l_b^2} \\ \frac{6}{l_b^2} - \frac{12x}{l_b^3} \\ -\frac{2}{l_b} + \frac{6x}{l_b^2} \end{bmatrix} \begin{bmatrix} \frac{6}{l_b^2} + \frac{12x}{l_b^3} & -\frac{4}{l_b} + \frac{6x}{l_b^2} & \frac{6}{l_b^2} - \frac{12x}{l_b^3} & -\frac{2}{l_b} + \frac{6x}{l_b^2} \end{bmatrix} dx \quad (14)$$

$$[K^b] = \frac{E_b I_b}{l_b^3} \begin{bmatrix} 12 & 6l_b & -12 & 6l_b \\ 6l_b & 4l_b^2 & -6l_b & 2l_b^2 \\ -12 & -6l_b & 12 & -6l_b \\ 6l_b & 2l_b^2 & -6l_b & 4l_b^2 \end{bmatrix} \quad (15)$$

2.2 Finite Element Formulation of Smart Beam Element

The mass and stiffness matrix for the smart beam element with piezoelectric patches placed at the top and bottom surfaces as a collocated pair is given by

$$M^p \ddot{p} + C^p \dot{p} + K^p p = \{f_a\} \{\phi_a(t)\} \quad (16)$$

Where M^p , C^p , K^p are the mass, damping, stiffness, and $\{f_a\}$ is the force co-efficient vectors which maps the applied actuator voltage to the induced displacements of smart beam element, $\phi_a(t)$ the voltage applied to the actuator, develops effective control forces and moments.

The mass matrix of smart beam element is given by

$$[M^p] = \frac{\rho_b I_b A_b}{420} \begin{bmatrix} 156 & 22l_b & 54 & -13l_b \\ 22l_b & 4l_b^2 & 13l_b & -3l_b^2 \\ 54 & 13l_b & 156 & -22l_b \\ -13l_b & -3l_b^2 & -22l_b & 4l_b^2 \end{bmatrix} + 2 \frac{\rho_p I_p A_p}{420} \begin{bmatrix} 156 & 22l_p & 54 & -13l_p \\ 22l_p & 4l_p^2 & 13l_p & -3l_p^2 \\ 54 & 13l_p & 156 & -22l_p \\ -13l_p & -3l_p^2 & -22l_p & 4l_p^2 \end{bmatrix} \quad (17)$$

$$[K^p] = \frac{E I_{eq}}{l_p^3} \begin{bmatrix} 12 & 6l_p & -12 & 6l_p \\ 6l_p & 4l_p^2 & -6l_p & 2l_p^2 \\ -12 & -6l_p & 12 & -6l_p \\ 6l_p & 2l_p^2 & -6l_p & 4l_p^2 \end{bmatrix} \quad (18)$$

$$E I_{eq} = E_b I_b + 2 E_p I_p \quad (19)$$

$$I_p = \frac{1}{12} b t_a^3 + b t_a \left(\frac{t_a + t_b}{2} \right)^2 \quad (20)$$

2.3 Control Laws

The various control laws such as one control law, which is based on output feedback by assuming arbitrary value and one optimal control law Linear quadratic regulator (LQR) based on state feedback and one control law, which is based on pole placement by state feedback has been explained as:-

2.3.1 LQR optimal control by state feedback

LQR optimal control theory is used to determine the active control gain. The following quadratic cost function is minimized

$$J = \frac{1}{2} \int_0^\infty (x^T Q x + u^T R u) dt \quad (21)$$

Q and R represent weights on the different states and control channels and their elements are selected to provide suitable performance. They are the main design parameters. J represents the weighted sum of energy of the state and control. Assuming full state feedback, the control law is given by

$$u = -Kx \quad (22)$$

With constant control gain

$$K = R^{-1}B^T S \quad (23)$$

Matrix S can be obtained by the solution of the Riccati equation, given by

$$A^T S + SA + Q - SBR^{-1}B^T S = 0 \quad (24)$$

The closed loop system dynamics with state feedback control is given by

$$\dot{x} = (A - BK)x + E_r(t) \quad (25)$$

2.3.2 Control by output feedback

Output feedback control provides a more meaningful design approach in practice. Measured outputs (ϵ) from sensors are directly feed back to actuators through

$$u = -K\epsilon \quad (26)$$

The closed loop system dynamics with output feedback control is given by

$$\dot{x} = (A - BKC)x + E_r(t) \quad (27)$$

$$A_c = (A - BKC)$$

2.4 Laminar Sensor Equation

The sensor voltage of piezoelectric element is given by, (Manjunath T.C, Bandyopadhyay B., 2009)

$$V^s(t) = K_c G_c d_{31} E_p \left(\frac{t_b}{2} + t_a \right) b [0 \ -1 \ 0 \ 1 \ 0 \ 1] [\dot{p}] \quad (28)$$

Where G_c is gain, t_b , t_a are the thickness of beam and actuator, K_c is the controller gain
 $V^s(t) = g^T [\dot{p}]$

2.5 Controlling Force from Actuator

Similar to the sensor, the piezoelectric layer which acts as actuator bonded to the structure. The geometrical arrangement is such that the useful direction of expansion is normal to that of the electric field. Thus, the activation capability is governed by piezoelectric constant d_{31} . With standard engineering notation, the equation of stress for piezoelectric material given by Premont is

$$\sigma_{11} = E_p \epsilon_{11} - e_{31} \frac{V}{t_p} \quad (29)$$

where, V is the voltage applied to the piezoelectric material. The controlling force equation given by

$$f_a = E_p d_{31} b \{-1 \ 0 \ r_a \ 1 \ 0 \ -r_a\}^T w(t) \quad (30)$$

where, r_a is the distance measured from the neutral axis of the beam to the mid plane of actuator layer, E_p Young's modulus of piezoelectric material, d_{31} is Piezo strain constant, b is the width of material

$$f_a = h w(t) \quad (31)$$

$$\text{where, } h = E_p d_{31} b \{-1 \ 0 \ r_a \ 1 \ 0 \ -r_a\}^T$$

2.6 Model Reduction

After assembly of each element of beam, the final equation for the smart cantilever beam with piezoelectric patches placed at the top and bottom surfaces as a collocated pair is given by

$$M \ddot{p} + C \dot{p} + K p = q + \{f_a\} \{\phi_a(t)\} \quad (32)$$

The external force is taken as unit impulse force.

where M , C , K , are the global mass, Rayleigh damping, stiffness, and further Rayleigh damping co-efficient,

$$C = \alpha[M] + \beta[K] \quad (33)$$

where, α and β are the damping constants

In active vibration control of flexible structures, the use of smaller order model has computational advantages. Therefore, it is necessary to apply a model reduction technique to the state space representation. The reduced order system model extraction techniques solve the problem by keeping the vital properties of the full model only. The frequency range is selected to span first two frequencies of the smart beam in order to find the reduced order model of the system.

Consider a generalized co-ordinate for reduction as

$$p = Vz \quad (34)$$

where V is the modal vectors corresponding to the first two eigen values. After reduction eqn (38) becomes

$$M_{red} \ddot{z} + C_{red} \dot{z} + K_{red} z = f_{ext} + f_{red} \quad (35)$$

2.7 State space formulation

In state space formulation, the second order differential equations are converted to first order differential equations.

First order dynamical system is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & I \\ -M_{red}^{-1} K_{red} & -M_{red}^{-1} C_{red} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ M_{red}^{-1} V^T h \end{bmatrix} w(t) + \begin{bmatrix} 0 \\ M_{red}^{-1} V^T f \end{bmatrix} r(t) \quad (36)$$

$$\dot{X} = A x(t) + B w(t) + E r(t) \quad (37)$$

where, A is known as the system matrix, $x(t)$ is the state vector, matrix B is input matrix, $w(t)$ is a column vector formed by the voltages applied to the actuators and acting as a control force, E is the external force acting on the beam

$$\& \quad Y = C x(t) + D w(t) \quad (38)$$

Where C is the output matrix, and D is the direct transmission matrix

$$Y = V^s(t) = g^T [\dot{p}] = g^T V \dot{z} = g^T V \begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} \quad (39)$$

$$= [0 \quad g^T V] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad (40)$$

Here

$$A = \begin{bmatrix} 0 & I \\ -M_{red}^{-1} K_{red} & -M_{red}^{-1} C_{red} \end{bmatrix} \quad (41)$$

$$B = \begin{bmatrix} 0 \\ M_{red}^{-1} V^T \{f_a\} \end{bmatrix} \quad (42)$$

$$C = [0 \quad g^T V] \quad (43)$$

$$D = \text{null matrix} \quad (44)$$

$$E = \begin{bmatrix} 0 \\ M_{red}^{-1} V^T f \end{bmatrix} \quad (45)$$

3. NUMERICAL SIMULATION

Table 1 Material properties and dimensions of Smart beam

Physical Parameters	Beam Element	Piezoelectric sensor/actuator
Length(m)	$l_b=0.226$	$l_b=0.075$
Breath(m)	$b=0.025$	$b=0.025$
Thickness(m)	$t_b=0.965e-3$	$t_a=0.75e-3$
Elastic Modulus(GPa)	$E_b=68$	$E_p=61$
Density(Kg/m ³)	$\rho_b=2800$	$\rho_p=7500$
Piezo strain constant(m/V)		$d_{31}=274e-12$
Piezo stress constant(Vm/N)		$g_{31}=10.5e-3$
Damping constants	$\alpha=0.001, \beta=0.0001$	

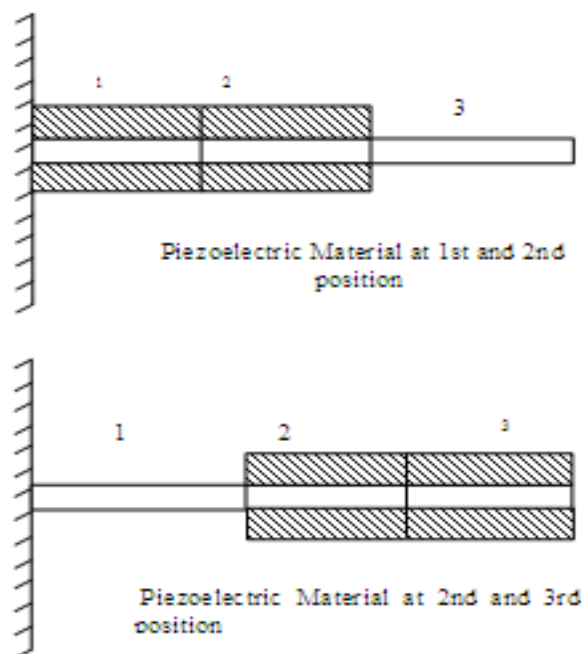
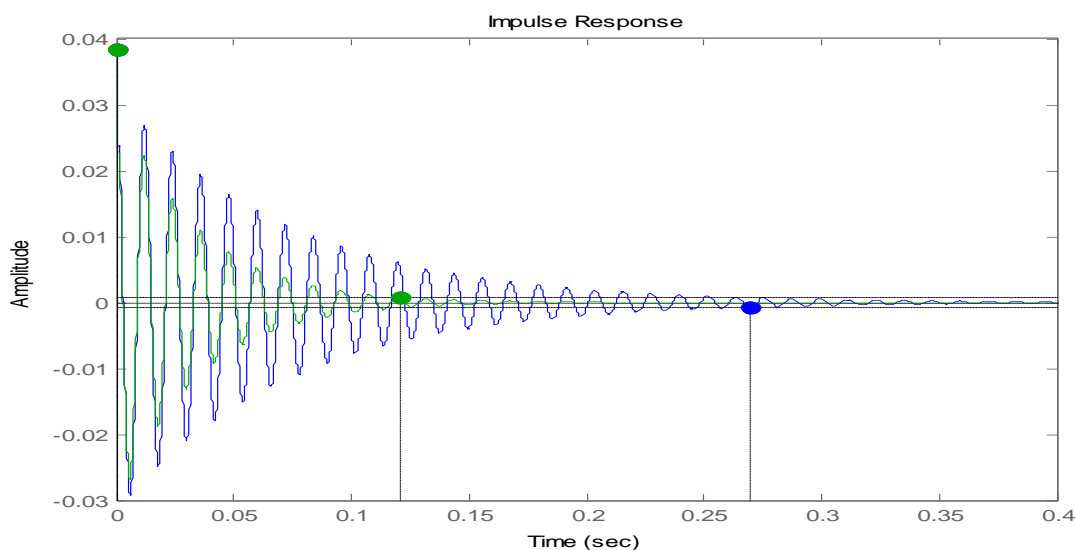


Fig 2. Position of sensor/actuator on Cantilever Beam.

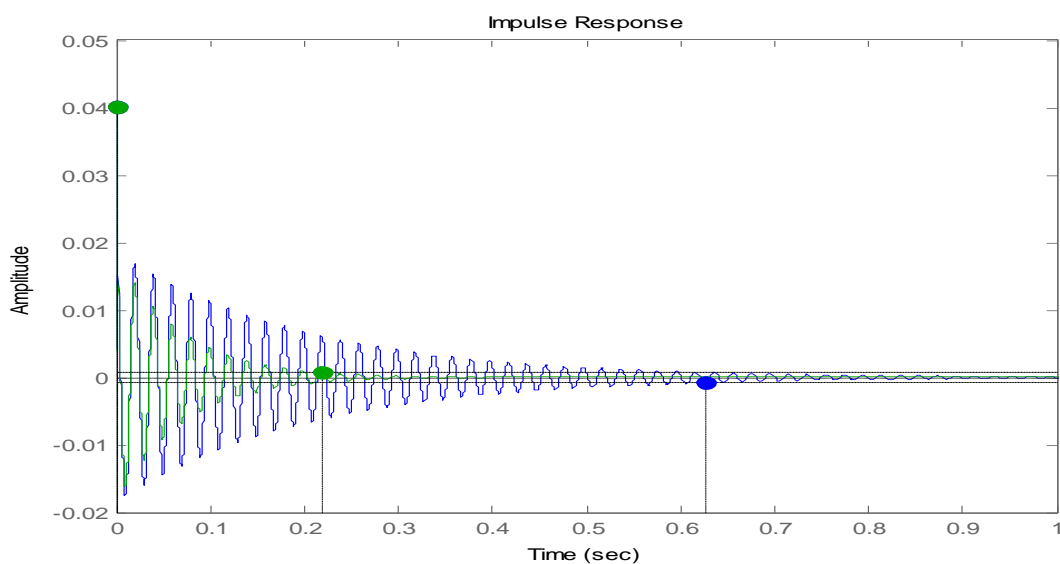
A cantilever beam with three elements of equal length is considered here. The piezoelectric sensor and actuator are placed at three different positions i.e. at fixed end & middle, middle & free end, free & fixed end. The structure consists of an aluminum beam with PZT-5H sensor and actuators patches. The material properties and dimensions of the beam and piezo patches are similar to the experiment performed by Xu and Koko(2004). For analysis, only collocated positions are considered. The physical properties of sensor and actuator have been given in table 1.

Case1.

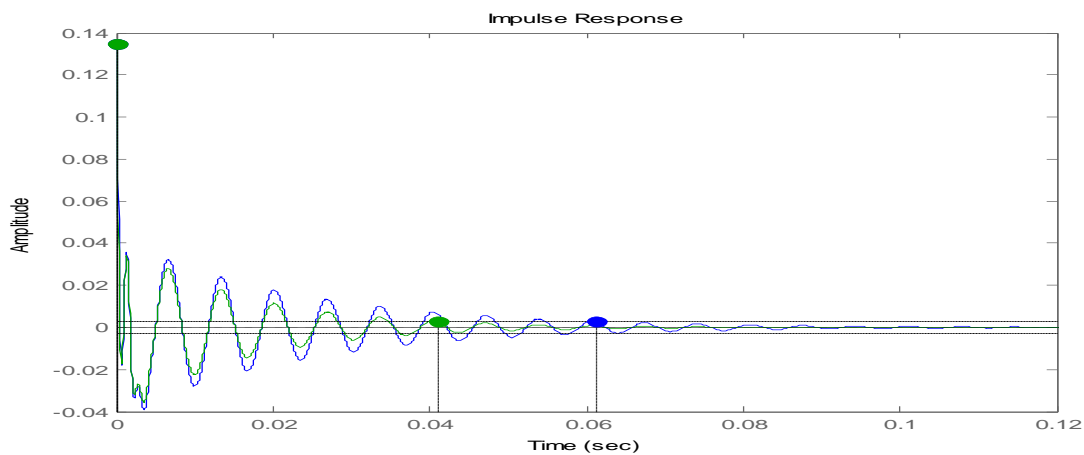
In the first case, the responses are taken by giving impulse input. The output feedback controller are designed by taking the arbitrary value of gain. In practical designs problems all the states are always, not known for feedback. On the other hand, output feedback control provides a more consequential design. The responses are also plotted by changing the position of sensor and actuator on the beam i.e. fixed end & middle, middle & free end, free & fixed end.



Graph 1 Tip displacement of cantilever beam when piezoelectric materials at 1st and 3rd position (POF Controller).



Graph 2 Tip displacement of cantilever beam when piezoelectric materials at 2nd and 3rd position (POF Controller).



Graph 3 Tip displacement of cantilever beam when piezoelectric materials at 1st and 2nd position (POF Controller).

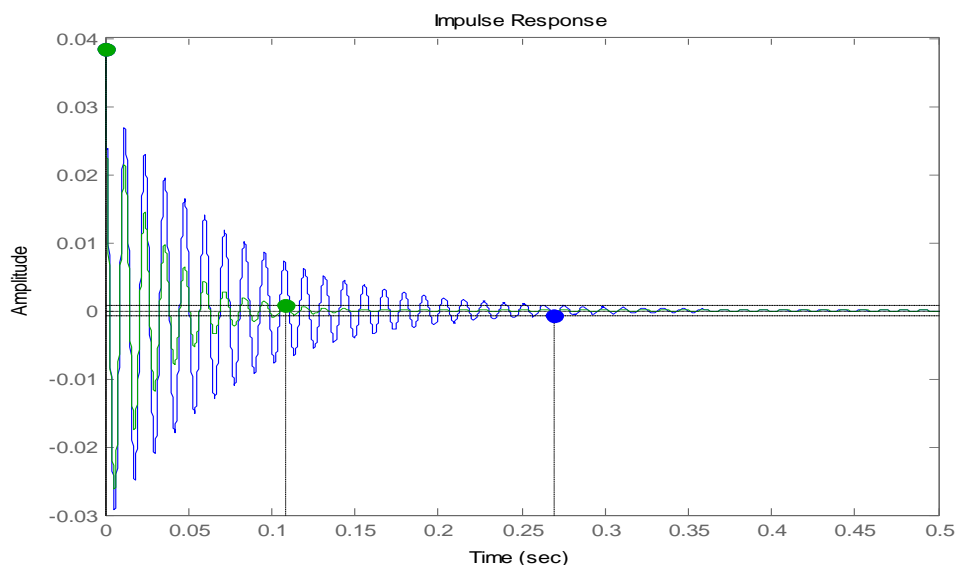
Case 2.

In second case, an optimal control is designed to minimize the cost function j

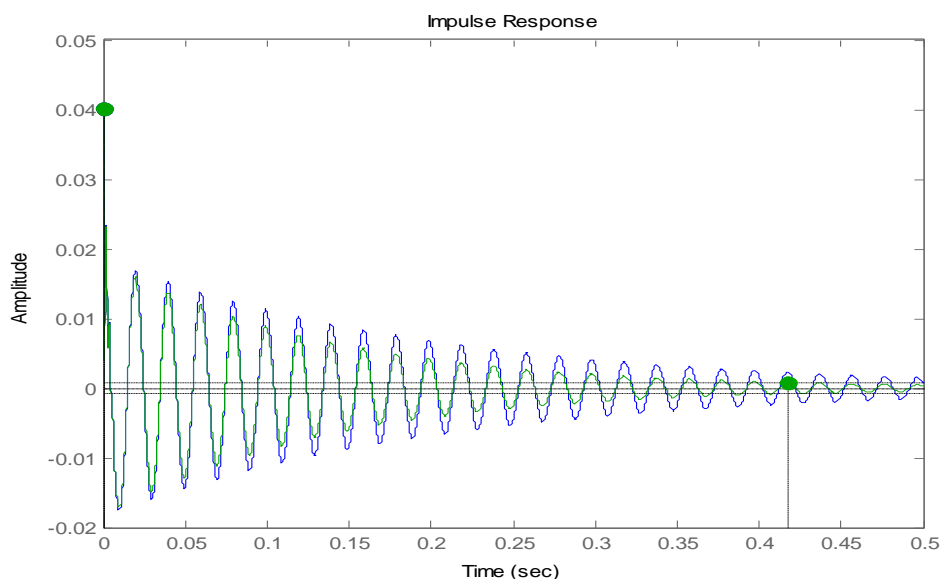
$$j = \frac{1}{2} \int_0^{\infty} (x^T Q x + u^T R u) dt$$

$$Q = 1e8 * \begin{bmatrix} 20 & 10 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 20 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix} \quad R = 100$$

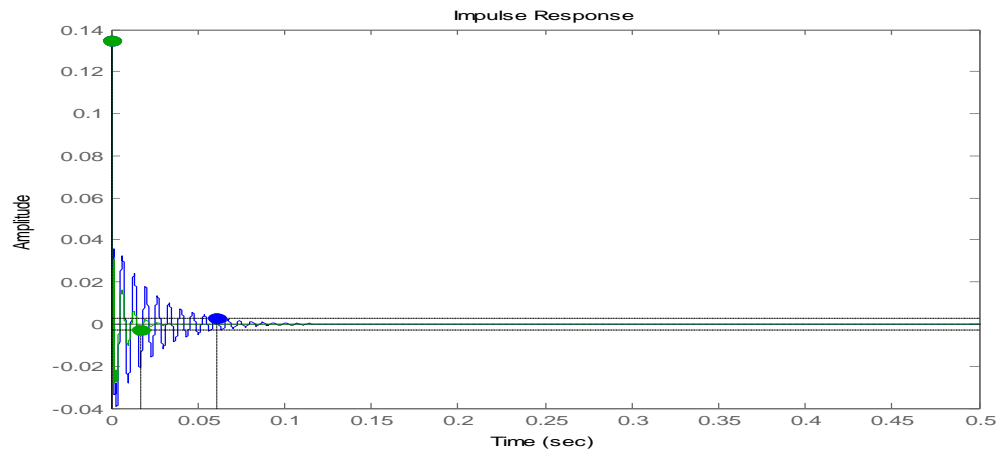
For this an optimal value of gain is find out by solving Riccati equation.



Graph 4 Tip displacement of cantilever beam with or without LQR controller when Piezoelectric materials at 1st and 3rd position.



Graph 5 Tip displacement of cantilever beam with or without LQR controller when Piezoelectric materials at 2nd and 3rd position.



Graph 6 Tip displacement of cantilever beam with or without LQR controller when Piezoelectric material 1st and 2nd position.

CONCLUSION

Present work deals with the mathematical formulation and the computational model for the active vibration control of a piezoelectric smart structure. A general scheme of analyzing and designing piezoelectric smart structures with control laws is successfully developed in this study. The present scheme has the flexibility of designing the system as collocated and non-collocated and user-selected a feedback control law. The active vibration control performance of piezoelectric cantilever structure is studied by taking arbitrary value of gain with output feedback and, the linear quadratic regulator (LQR) scheme, which is an optimal control theory based on full state feedback. It has been observed that without control the transient response is predominant and with control laws, sufficient vibrations attenuation can be achieved. The study revealed that the LQR control scheme is very effective in controlling the vibration as the optimal gain is obtained by minimizing the cost function. Numerical simulation showed that modeling a smart structure by including the sensor / actuator mass and stiffness and by varying its location on the beam from the free end to the fixed end introduced a considerable change in the system's structural vibration characteristics. From the responses of the various locations of sensor/actuator on beam, it has been observed that best performance of control is obtained, when the piezoelectric element is placed at 1st and 2nd position.

Table2. Responses of controlled and uncontrolled loop

Different type of controller	1 st and 3 rd position		1 st and 2 nd position		2 nd and 3 rd position	
	Settling time(in sec.)	peak response	Settling time(in sec.)	Peak response	Settling time(in sec.)	Peak response
POF for controlled loop	0.12	0.039	0.042	0.135	0.22	0.04
Uncontrolled loop	0.27	0.039	0.07	0.135	0.62	0.04
LQR for controlled loop	0.11	0.039	0.02	0.13	0.42	0.04
Uncontrolled loop	0.27	0.039	0.07	0.13	0.6	0.03

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