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The Modified Method for solving non-linear programming problems

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ABSTRACT

In this research, a new algorithm of conjugation algorithms was developed to solve nonlinear optimization problems. This algorithm depend on a new search direction($d_i = -H_i^{-1}g_i + pg_i$). Numerical results proved the efficiency of the proposed algorithm compared to other algorithms

1. INTRODUCTION

Let us focus on solving the nonlinear unconstrained optimization problem by conjugate gradient method

$$\min f(x) \qquad x \in R^n \quad \text{……………..(1)}$$

Starting with an initial point $x_0 of \ f(x)$ [6]

$$x_{i+1} = x_i + \alpha_i d_i \quad \text{……………………. (2)}$$

Where α_i is a step size, which α_i >0 and satisfied Wolfe conditions

$$\left.\begin{aligned} f(x_i + \alpha_i d_i) &\le f(x_i) + \delta_1 \alpha_i d_i^T g_i \\ g(x_i + \alpha_i d_i)^T d_i &\ge \delta_2 d_i^T g_i \end{aligned}\right\} \quad \text{…….(3)}$$

With $\delta_1 < 1/2 \ and \ \delta_1 < \delta_2 < 1$ [1]

And d_i is the search direction calculated by

$$d_i = \begin{cases} -g_i & for\ i = 1 \\ -g_i + \beta_i d_{i-1} & for\ i \ge 2 \end{cases} \quad \dots\dots\dots\dots\dots (4)$$

Where g_i *denotes* $\nabla f(x)$and β_i is known as the conjugate gradient parameter there are many ways to calculate β_i .[2]

Ibrahim,M.A.H. , Mamat,M. , Liza, P., and Salleh,Z.[5] suggested a search direction with fiexd parameter which is stated as follows :

$$\begin{aligned} d_i &= -H_i^{-1} g_i + p g_i \\ (H_i - pI) d_i &= -g_i \end{aligned} \quad \text{………….(5)}$$

Where I is the identity matrix and p< 0 .the matrix H is updated by the BFGS formula as[7]

$$H_{i+1} = H_i - \frac{H_i v_i v_i^T H_i}{v_i^T H_i v_i} + \frac{y_i y_i^T}{v_i^T y_i} \text{………………. (6)}$$

Which satisfied the QN condition :

$$H_{i+1} v_i = y_i$$

Where :

$$\begin{aligned} y_i &= g_{i+1} - g_i \\ v_i &= x_{i+1} - x_i \end{aligned}$$

2. NEW ALGORITHM FOR CONJUGATE GRADIENT METHOD

In this paper we present a modified of a beta in CG method by using new direction of equ.(5)

$$d_i = -H_i^{-1} g_i + \rho g_i \qquad \ldots\ldots\ldots(7)$$

And the CG direction

$$d_i = -g_i + \beta_i d_{i-1} \qquad \ldots\ldots\ldots.(8)$$

By equality 7 and 8

$$-H_i^{-1} g_i + \rho g_i = -g_i + \beta_i d_{i-1} \qquad \ldots\ldots\ldots.(9)$$

$$[\beta_i d_{i-1} = g_i (I + \rho I - H_i^{-1})] \qquad * y_i$$

$$\beta_i d_{i-1}^T y_i = g_i^T (I + \rho I - H_i^{-1}) y_i$$

$$\beta_i = \frac{g_i^T (I + \rho I - H_i^{-1}) y_i}{d_{i-1}^T y_i} \qquad \ldots\ldots(10)$$

3. ALGORITHEM

Step (1): put a starting point X1 and $H_1 = I_n$ and compute $f(x_1), g(x_1)$, $d_1 = -g_1$ and k=1
Step (2): test convergent
If $\|g_i\| \le \epsilon \quad , \epsilon = 10^{-7}$ stop x_i is the optimal solution else go to step (3)
Step (3): computation α_i *by equ.*(3) and update variable $x_{i+1} = x_i + \alpha_i d_i$
Step (4): update (H_i) by equ.(6)
Step (5): computation direction d_i by equ.(5).
Step (6): computation β by equ.(10).
Step (7): calculate f_{i+1} , g_{i+1} , y_i , v_i
Step (8): set i=i+1 and go to step (2)

4. CONVERGENCE

We suppose that the sequence $\{H_i\}$, $\{y_i\}$,$\{X_i\}$ all created by algorithm BFGS-SD as given in apepare are positive definite . We as well suppose that every search direction d_i satisfied the desent condition . for all $i \ge 0$
There exists a constant $C_1 > 0$ such that

$$\frac{g_i^T d_i}{\|g_i\|^2} \le C_i \; for \; all \; i \ge 0 \;\ldots\ldots.. \quad (11)$$

Then the search directions satisfy the sufficient descent condition we can prove in theorem4.1 hence. We need to make a few assumption based on the objective function .

Assumption 4.1 :[4]

A- The level set L is convex moreover , positive constants C_1
$C_1 \|Z\|^2 \le Z^T F(x) Z \le C_2 \|Z\|^2$ for all $Z \in R^n$ and $x \in L$, where F(x) is the Hessian matrix for f.
$B -$ The Hessian matrix is lipschitz continuous at the point X^*, that is there exists the positive constant C_3 satisfying
$\|g(x) - g(x^*)\| \le c_3 \|x - x^*\|$ *for all x in aneighborhood of* x^*

Theorem 4.1:

Assume $\{H_i\}$ is bounded then the condition equ.(11) is holeds for all $i \ge 0$.

Proof :
From $d_i = -g_i + \beta_i d_{i-1}$

$$g_i^T d_i = -g_i^T g_i + g_i^T \beta_i d_{i-1}$$

$$= -\|g_i\|^2 + g_i^T \frac{g_i (I + \rho I - H_i^{-1}) y_i}{d_{i-1}^T y_i} d_{i-1}$$

$$\le -\|g_i\|^2 + \|g_i\|^2 \frac{(I + \rho I - H_i^{-1}) y_i^T d_{i-1}}{d_{i-1}^T y_i}$$

$$\le -\|g_i\|^2 + ((1+\rho) I - H_i^{-1}) \|g_i\|^2$$

$$\le -\|g_i\|^2 + \lambda \|g_i\|^2$$

$$\le -(1 - \lambda) \|g_i\|^2$$

$$\leq -c\|g_i\|^2$$

Where $c = (1-\lambda)$ which bounded away from zero . and $0 < \lambda < 1$.

Theorem 4:2 (Global Convergence)

Suppose that theorem 4.1 and assumption 4.1 hold then

$$\lim_{i\to\infty}\|g_i\|^2 = 0$$

Proof :

$$d_i = -g_i + B_i d_{i-1}$$

$$\|d_i\| \leq \|g_i\| + \|B_i d_{i-1}\|$$
$$\leq \|g_i\| + \left\|\frac{g_i^T(I+\rho I - H_i^{-1})y_i}{d_{i-1}^T y_i} d_{i-1}\right\|$$
$$\leq \|g_i\| + \|(I+PI-H_i^{-1})g_i\|$$
Let $C_1 = (I+PI-H_i^{-1})$
$$\leq \|g_i\| + |C_1| * \|g_i\|$$
$$[\|d_i\| \leq |(1+C_1)| * \|g_i\|]^2$$
Let $C = (1+C_1)^2$
$$\|d_i\|^2 \leq C\|g_i\|^2 < \emptyset^2$$
$$\sum_{i\to1}^{\infty}\frac{1}{\|d_i\|^2} \geq \frac{1}{\emptyset^2}\sum 1 = \infty$$
$$\therefore \lim_{i\to\infty}\|g_i\|^2 = 0$$

5. NUMERICAL RESULT

In the search, we show some numerical result, so that we use the test functions are the standerd unconstrained optimization problem from andrei [3], we analysis the performance of the Hs method with the new algorithem, the dimensions of the tests range between 1000 and 10000, the comparative performance of the algorithm is evaluated by considering both the (NOF) which is number of function evaluations and the (NOI) which is number of iterations and all these methods terminate when the following stooping criterion i,

$$\|g_i\| \leq \epsilon \qquad ,\epsilon = 10^{-7}$$

Table 1: Numerical Comarisons between the Hessen method and New method.

N	Test fu.	Dim	Hs	New
1	Powell	1000 10000	37(75) 52(157)	26(63) 32(132)
2	Beale	1000 10000	11(32) 23(64)	8(25) 11(43)
3	gencralzed wood	1000 10000	34(84) 56(94)	27(65) 43(83)
4	Cosine cute	1000 10000	6(13) 6(13)	6(13) 6(13)
5	Raydon2	1000 10000	5(10) 5(10)	5(10) 5(10)
6	Dixmaan A cute	1000 10000	8(16) 8(16)	8(16) 8(16)
7	Dixmaan B cute	1000 10000	13(22) 14(23)	11(20) 12(20)
8	Dixmaan C cute	1000 10000	16(28) 17(29)	13(26) 15(27)
9	Fletcher cute	1000 10000	8(19) 8(21)	7(17) 7(19)
10	Diagonal 4	1000 10000	5(8) 5(8)	6(9) 6(9)

11	Exteaded Himmeblau	1000 10000	12(21) 22(36)	10(19) 20(32)
12	Extended Psci	1000 10000	8(15) 9(16)	7(14) 7(15)
13	Nondia cute	1000 10000	15(28) 8(17)	13(26) 7(13)
14	Dixmaan j cute	1000 10000	18(30) 22(42)	16(28) 20(39)
15	Vardim	1000 10000	16(43) 18(54)	18(45) 20(56)

CONCOLUSION

In this research, we obtained a new algorithm, we proved the condition of generate sufficient descent directions as well as the achievement of globally convergent. It showed the numerical results that the new algorithm at the level of efficiency to solve the general functions and also proved to be efficient when it relied on NOF and NOI.

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APPENDIX

The specifics of the test functions , Which use in search , can be Look in [3]

1- Generalization Powell Function:

$$f = \sum_{i=1}^{n/4} [x_{4i-3} + 10\, x_{4i-2})^2 + 5\,(x_{4i-1} - x_{4i})^2 + (x_{4i-2} - 2\,x_{4i-1})^4$$

$$+ 10\,(x_{4i-3} - x_{4i})^4] \qquad ; \qquad x_o = (3,\ -1,\ 0,\ 1,\ ...)^T$$

2- Beale Function,

$$x_0 = (0,0)^T \quad F(x) = (1.5 - x_1(1 - x_2))^2 + (2.25 - x_1(1 - x_2^2))^2 + (2.625 - x_1(1 - x_2^3))^2,$$

3- Generalized Wood Function

$$f = \sum_{i=1}^{n/4} [100\ (x_{4i-1} - x_{4i-9}^2)^2 + (1 - x_{4i-9})^2 + 90\,(x_{4i} - x_{4i-1}^2)^2 + (1 - x_{4i-1})^2 +$$

$$10.1((x_{4i-2} - 1)^2 + x_{4i-1}^2) + 19.8(x_{4i-2} - 1)(x_{4i} - 1)]$$

$$x_0 = (-3.0, -1.0, -3.0, -1.0; ....)^T$$

4- Cosin Function (Cute)

$$f(x) = \sum_{i=1}^{n-1} \cos(x_i^2 - \frac{x_{i+1}}{2})$$

$$x_0 = [1, 1, 1, \ldots\ldots..1]$$

5- Raydan2 Function

$$f(x) = \sum_{i=1}^{n} (\exp(x_i) - x_i),$$

$$x_0 = [1,1,..., 1].$$

6- Dixmaana Function (Cute):

$$f(x) = 1 + \sum_{i=1}^{n} \alpha\, x_i^2 \left(\frac{i}{n}\right)^{k1} + \sum_{i=1}^{n-1} \beta\, x_i^2 \left(x_{i+1} + x_{i+1}^2\right)^2 \left(\frac{i}{n}\right)^{k2} + \sum_{i=1}^{2m} \gamma\, x_i^2 x_{i+m}^4 \left(\frac{i}{n}\right)^{k3}$$

$$+ \sum_{i=1}^{m} \delta\, x_i\, x_{i+2m} \left(\frac{i}{n}\right)^{k4},$$

$$m = n/3 \;;\; \alpha = 1;\beta = 0;\gamma = 0.125 \;;\delta = 0.125 \;;k1 = 0, k2 = 0, k3 = 0, k4 = 0$$

$$x_0 = [2., 2.,..., 2.]^T$$

7- Dixmaanb Function (Cute):

$$f(x) = 1 + \sum_{i=1}^{n} \alpha\, x_i^2 \left(\frac{i}{n}\right)^{k1} + \sum_{i=1}^{n-1} \beta\, x_i^2 \left(x_{i+1} + x_{i+1}^2\right)^2 \left(\frac{i}{n}\right)^{k2} + \sum_{i=1}^{2m} \gamma\, x_i^2 x_{i+m}^4 \left(\frac{i}{n}\right)^{k3}$$

$$+ \sum_{i=1}^{m} \delta\, x_i\, x_{i+2m} \left(\frac{i}{n}\right)^{k4},$$

$$m = n/3 \;;\; \alpha = 1;\beta = 0.0625 \;;\gamma = 0.0625 \;;\delta = 0.0625 \;;k1 = 0, k2 = 0, k3 = 0, k4 = 1$$

$$x_0 = [2., 2.,..., 2.]^T$$

8- Dixmaanc Function (Cute):

$$f(x) = 1 + \sum_{i=1}^{n} \alpha\, x_i^2 \left(\frac{i}{n}\right)^{k1} + \sum_{i=1}^{n-1} \beta\, x_i^2 \left(x_{i+1} + x_{i+1}^2\right)^2 \left(\frac{i}{n}\right)^{k2} + \sum_{i=1}^{2m} \gamma\, x_i^2 x_{i+m}^4 \left(\frac{i}{n}\right)^{k3}$$

$$+ \sum_{i=1}^{m} \delta\, x_i\, x_{i+2m} \left(\frac{i}{n}\right)^{k4},$$

$$m = n/3 \;;\; \alpha = 1;\beta = 0.125 \;;\gamma = 0.125 \;;\delta = 0.125 \;;k1 = 0, k2 = 0, k3 = 0, k4 = 0$$

$$x_0 = [2., 2.,..., 2.]^T$$

9- Fletchcr Function

$$f(x) = +\sum_{i=1}^{n-1} c(x_{i+1} - x_i + 1 - x_i^2)^2$$

$$x_0 = [0,0,0 \ldots\ldots,0] c = 100$$

10- Diagonal-4 Function:

$$f(x) = \sum_{i=1}^{n/2} \frac{1}{2}(x_{2i-1}^2 + c\, x_{2i}^2),$$

$$x_0 = [1,1,..., 1]^T \quad , c = 100$$

11- Extended Himmelblau Function:

$$f(x) = \sum_{i=1}^{n/2} \left(x_{2i-1}^2 + x_{2i} - 11\right)^2 + \left(x_{2i-1} + x_{2i}^2 - 7\right)^2,$$

$$x_0 = [1, 1, \ldots, 1]^T$$

12- Extended Psc1 Function:

$$f(x)=\sum_{i=1}^{n/2}\left(x_{2i-1}^{2}+x_{2i}^{2}+x_{2i-1}x_{2i}\right)^{2}+\sin^{2}(x_{2i-1})+\cos^{2}(x_{2i}),$$

$$x_0=[3,0.1,\ldots,3,0.1]^T$$

13- Nondia Fuction

$$f(x)=(x_1-1)^2+\sum_{i=2}^{n}100(x_i-x_{i-1}^2)^2$$

$$x_0=[-1,-1,\ldots\ldots..-1]$$

14- Dixmaan j cute

$$f(x)=1+\sum_{i=1}^{n}\alpha x_i^2\left(\frac{i}{n}\right)^{k1}+\sum_{i=1}^{n-1}\beta\, x_i^2(x_i+x_{i+1}^2)^2\left(\frac{i}{n}\right)^{k2}+\sum_{i=1}^{2m}\gamma x_i^2x_{i+m}^4\left(\frac{i}{n}\right)^{k3}+\sum_{i=1}^{m}\delta x_i x_{i+2m}\left(\frac{i}{n}\right)^{k4}$$

$$m=\frac{n}{3},\alpha=1,\ \beta=0,\ \gamma=0.125,\delta=0.125,\qquad k_1=1,k_2=0,k_3=0,\ k_4=1$$

15- Vardim function(cute)

$$f(x)=\sum_{i=1}^{n}(x_i-1)^2+\left(\sum_{i=1}^{n}ix_i-\frac{n(n+1)}{2}\right)^2+\left(\sum_{i=1}^{n}ix_i-\frac{n(n+1)}{2}\right)^4$$

$$x_0=[1-\frac{1}{n},1-\frac{2}{n},\ldots\ldots..1-\frac{n}{2}]$$