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The Geometric Approach to Existence Linear $[n,k,d]_{13}$ Codes

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ABSTRACT

Let [n,k,d]q codes be Linear codes of Length n ,dimension k and minimum Hamming distance d over GF(q) . In this paper we give geometric proofs for several results on existence Linear $[n,k,d]_{13}$ Codes that arise from the geometric approach,as described in the paragraph the geometrical contraction method in PG(2,13),and as shown in the method of obtaining new examples (1, 2, ... and 11).

Keywords: linear Codes, Arc, Double Blocking set, geometric approach.

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INTRODUCTION

A projective plane PG(2,q) over Galois field GF(q) of q elements, are a two-dimensional projective space, which consists of points and lines with incidence relation between them. In PG(2,q) there are $q^2 + q + 1$ points, and $q^2 + q + 1$ lines, every line contains 1 + q points and every point is on 1 + q lines, all these points in PG(2,q) have the form of a triple (a_1,a_2,a_3) where a1, a2, a3 ∈ GF(q); such that $(a_1,a_2,a_3) \neq (0,0,0)$. Two points (a_1,a_2,a_3) and (b_1,b_2,b_3) represent the same point if there exists $\gamma \in GF(q)\backslash\{0\}$, such that $(b_1,b_2,b_3)= \gamma\,(a_1,a_2,a_3)$
There exists one point of the form (1,0,0). There exists q points of the form (x,1,0) There exists q2 points of the form (x,y,1), similarly for the lines.
A point $p(x_1,x_2,x_3)$ is incident with the line $L[a_1,a_2,a_3]$ if f
$a_1x_1 + a_2x_2 + a_3x_3 = 0$.[1],[2]

Definition (1) " Double Blocking set "
A double blocking set in a projective plane PG(2,q) is a set S of points with the property that every line contains at least two points of S .[5]

Definition (2) " A (k,r) –arc "
A (k,r) –arc K in PG(2,q) is a set of k points with condition no line of the plane contains more than k points and there exist at least one line of the plane which contains k points .
A (k,,r) –arc is called complete arc if is not contained in a (k+1,r)- arc . [1]

Definition (3) " The Linear [n,k,d]q Codes "
The Linear [n,k,d]q Codes in PG(2,q) where n is the length of codes and k is the dimension of codes , and minimum Hamming distance between the codes is called d over the Galois field GF(q) . [3],[6]

Definition (4) " i-secant "
A line L in PG(2,q) is an i-secant of a (k , r)-arc if $|L \cap K|=i$. [1]

Theorem 1 : there exists linear [n,3,d] q codes if and only if there exists an (n ,n-d) - arc in PG(2,q) . [4] , [5].

The geometrical contraction method in PG (2, 13)

Let A=(1,2,15,29) be the set of reference unit and reference points in PG(2,13) where: 1=(1,0,0) , 2=(0,1,0),15=(0,0,1) ,29=(1,1,1)
A is(4,2)-arc, since no three points of A are collinear,
[1,2]=[1,2,3,4,5,6,7,8,9,10,11,12,13,14]
[1,15]=[1, 15 ,16,17,18,19,20,21,22,23,24,25,26,27]
[1,29]=[1,28,29,30,31,32,33,34,35,36,37,38,39,40]
[2,15]=[2,15,28,41,54,67,80,93,106,119,132,145,158,171]
[2,29]=[2,16,29,42,55,68,81,94,,107,120,133,146,159,172]
[15.29]=[3,15,29,43,57,71,85,99,113,127,141,155,169,183]
The diagonal points of A are the points {3,16,28} where,
$L_1 \cap L_6 = 3$; $L_2 \cap L_5 = 16$; $L_3 \cap L_4 = 28$.

There are One hundred points of index zero for A , which are:
44,45,46,47,48,49,50,51,52,53,56,58,59,60,61,62,63,64,65,66,69,70,72,73,74,75,76,77,78,79,82,83,84,86,87,88,89,90,91,92,95,96,97,98,100,101,102,103,104,105,108,109,110,111,112,114,115,116,117,118,121,122,123,124,125,126,128,129,130,131,134,135,136,137,138,139,140,142,143,144,147,148,149,150,151,152,153,154,156,157,160,161,162,163,164,165,166,167,168,170172,174,175,176,177,178,179,180,181,182.
Hence ,A is incomplete (4,2)_arc .

The Conics in PG(2,13) Through the Reference and Unit Points (1)

The general equation of the conic is:

$$a_1x^2{}_1 + a_2x^2{}_2 + a_3x^2{}_3 + a_4x_1x_2 + a_5x_1x_3 + a_6x_2x_3 = 0 \quad \ldots (1)$$

By substituting the points of the arc A in [1], then:
1 = (1,0,0) implies that $a_1 = 0$, 2 = (0,1,0), then $a_2 = 0$, 15 = (0,0,1), then $a_3 = 0$, 29 = (1,1,1), then

$$a_1 = a_2 = a_3 = 0$$

$a_4 + a_5 + a_6 = 0$.
Hence, from equation (1)
$a_4 x_1 x_2 + a_5 x_1 x_3 + a_6 x_2 x_3 = 0$ … (2)
If $a_4 = 0$, then the conic is degenerated, therefore for $a_4 \neq 0$, similarly $a_5 \neq 0$ and $a_6 \neq 0$,
Dividing equation [2] by a4, one can get:
$x_1 x_2 + \alpha x_1 x_3 + \beta x_2 x_3 = 0$
where $\alpha = a_5/a_4$, $\beta = a_6/a_4$
then $\beta = -(1+\alpha)$, since $1+\alpha+\beta=0$ (mod 13).
where $\alpha \neq 0$ and $\alpha \neq 12$, for if $\alpha = 0$ or $\alpha = 12$, then degenerated conics, thus α = 1, 2, 3, 4, 5, 6, 7, 8, 9,10,11 and can be written (2) as :

$$x_1x_2 + \alpha x_1x_3 - (1+\alpha)x_2x_3 = 0 \quad \ldots (3)$$

The Equation and the Points of the Conics of PG(2,13) Through the Reference and Unit Points (1)

1. If α= 1, then the equation of the conic
$C_1 = x_1x_2 + x_1x_3 + 11x_2x_3 = 0$,
the points of C_1: {1,2,15,29,51,62,79,86,104,111, 128,139,148,162} which is a complete (14,2)-arc, since there are no points of index zero for C_1.

2. If α= 2, then the equation of the conic
$C_2 = x_1x_2 + 2x_1x_3 + 10x_2x_3 = 0$,
the points of C_2 : {1,2,15,29,49,61,69,84,105,117,124,138,154,181 }, which is a complete (14,2)-arc, since there are no points of index zero for C_2.

3. If α= 3, then the equation of the conic
$C_3 = x_1x_2 + 3x_1x_3 + 9x_2x_3 = 0$,
the points of C_3 : {1,2,15,29,53,56,73,89,100,114,129,135,163,182 }, which is a complete (14,2)-arc, since there are no points of index zero for C_3.

4. If α= 4, then the equation of the conic
$C_4 = x_1x_2 + 4x_1x_3 + 8x_2x_3 = 0$,
the points of C_4: {1,2,15,29,47,58,76,90,96,108,131,156,166,178 }, which is a complete (14,2)-arc, since there are no points of index zero for C_4.

5. If α= 5, then the equation of the conic
$C_5 = x_1x_2 + 5x_1x_3 + 7x_2x_3 = 0$,
the points of C_5: {1,2,15,29,52,66,74,83,101,116,134,149,167,176 }, which is a complete (14,2)-arc, since there are no points of index zero for C_5.

6. If α= 6, then the equation of the conic
$C_6 = x_1x_2 + 6x_1x_3 + 6x_2x_3 = 0$,
the points of C_6: {1,2,15,29,46,65,75,82,103 ,123,144,151,161,180 }, which is a complete (14,2)-arc, since there are no points of index zero for C_6.

7. If α= 7, then the equation of the conic
$C_7 = x_1x_2 + 7x_1x_3 + 5x_2x_3 = 0$,
the points of C_7: {1,2,15,29,50,59,77,92,110,125,143,152,160,174 }, which is a complete (14,2)-arc, since there are no points of index zero for C_7.

8. If α= 8, then the equation of the conic
$C_8 = x_1x_2 + 8x_1x_3 + 4x_2x_3 = 0$,
the points of C_8: {1,2,15,29,48,60,70,95,118,136,150,130,168,179}, which is a complete (14,2)-arc, since there are no points of index zero for C_8.

9. If α= 9, then the equation of the conic
$C_9 = x_1x_2 + 9x_1x_3 + 3x_2x_3 = 0$,
the points of C_9: {1,2,15,29,44,63,91,97,112,126,137,153,170,173}, which is a complete (14,2)-arc, since there are no points of index zero for C_9.

10. If α= 10, then the equation of the conic
$C_{10} = x_1x_2 + 10x_1x_3 + 2x_2x_3 = 0$,
the points of C_{10}: {1,2,15,29,45,72,88,102,109,121,142,165,157,177}, which is a complete (14,2)-arc, since there are no points of index zero for C_{10}.

11. If α= 11, then the equation of the conic
$C_{11} = x_1x_2 + 11x_1x_3 + x_2x_3 = 0$,
the points of C_{11}: {1,2,15,29,64,78,87,98,115,122,140,147,164,175}, which is a complete (14,2)-arc, since there are no points of index zero for C_{11}.

Example(1):Existence of $[155,3,142]_{13}$ codes

We take one conic, and take π= PG(2,q) over Galois filed GF(q) is contains 183 points and line, every line is contains 14 points and every point there are 14 line, say C_1 ,and let
$K=\pi - C_1$

{3,4,5,6,7,8,9,10,11,12,13,14,16,17,18,19,20,21,22,23,24,25,26,27,28,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,47,48,49,50,52,53,54,55,56,57,58,59,60,61,63,64,65,66,67,68,69,70,71,72,73,74,75,76,77,78,80,81,82,83,84,85,87,88,89,90,91,92,93,94,95,96,97,98,99,100,101,102,103,105,106,107,108,109,110,112,113,114,115,116,117,118,119,120,121,122,123,124,125,126,127,129,130,,131,132,133,134,135,136,137,138,140,141,142,143,144,145,147,149,150,151,152,153,154,155,156,157,158,159,160,161,161,163,164,165,166,167,168,169,170,171,172,173,174,175,176,177,178,179,180,181,
182,183}.
The geometrical Construction method must satisfies the following:

i. K intersects any line of π in at most 13 points .
ii. Every point not in K is on at least one 13-secant of K .

The point : 41,42,54,67,75,80,63,91,101,119,88,93,171,158,132,106,145
Are eliminated from K to satisfy (1) . The points of index zero for 1,51,62 are added to K to satisfy (2) , then
K_{13} =K∪ [1,51,62] / [41,42,54,67,75,80,63,91,101,119,88,93,171,158,132,106,145]
K_{13}=[1,3,4,5,6,7,8,9,10,11,12,13,14,16,17,18,19,20,21,22,23,24,25,26,27,28,30,31,32,33,34,35,36,37,38,39,40,43,44,45,46,47,48,49,50,51,52,53,55,56,
57,58,59,60,61,62,64,65,66,68,69,70,71,72,73,74,76,77,78,81,82,83,84,85,
87,89,90,92,94,95,96,97,98,99,100,102,103,105,107,108,109,110,112,113,114,115,116,117,118,120,121,122,123,124,125,126,127,129,130,133,134,135,136,137,138,140,141,142,143,144,145,147,149,150,151,152,153,154,155,156,157,159,160,161,161,163,164,165,166,167,168,169,170,172,173,174,175,176,177,178,180,181,182,183].

Is a complete (155,13) –arc as shown in table (1) .
Let $\beta_1 = \pi - k_{13}$ ={2,15,29,41,42,54,63,67,75,79,80,86,91,93,101,104,106,111,119,128,132,139,145,158,162,141} is (28,1)-blocking set as shown in (Table1). $\beta 1$ is of Redei -type contains the line L1 ={2,15,28,41,54,67,80,93,106,119,132,145,158,171}/{28} and one point on each line through the point 28 which are non-collinear points 1,27,48,50,61,78,85,103,110,95,162,122,137 by theorem (1) ,there exists a projective $[155,3,142]_{13}$ code which is equivalent to the complete (155,13)-arc$k_{13.}$

Table(1)

I	$K_{13}\cap$ Li	B1 ∩ Li
1	28	2,15,41,54,67,80,93,106, 119,132,145,158,171
2	1,16,17,18,19,20,21,22,23,24,25,26,27	15
. . .		
182	8,22,28,47,66,72,97,116,122,141,147,166, 172	91
183	14,16,28,53,65,77,89,113,125,137,149,161,173	101

Example(2):Existence of $[130,3,118]_{13}$ codes

We take two conic, say C_1 and C_2 ,and let
K=π - $C_1 \cup C_2$
{3,4,5,6,7,8,9,10,11,12,13,14,16,17,18,19,20,21,22,23,24,25,26,27,28,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,47,48,50,52,53,54,55,56,57,58,59,60,63,64,65,66,67,68,70,71,72,73,74,75,76,77,78,80,81,82,83,85,87,88,89,90,91,92,93,94,95,96,97,98,99,100,101,102,103,106,107,108,109,110,112,113,114,115,116,118,119,120,121,122,123,125,126,127,129,130,131,132,133,134,135,136,137,140,141,142,143,144,145,147,149,150,151,152,153,155,156,157,158,159,160,161,161,163,164,165,166,167,168,169,170,171,172,173,174,175,176,177,178,179,180,182,183}.
The geometrical Construction method must satisfies the following:

i. K intersects any line ofπ in at most 12 points .
ii. Every point not in K is on at least one 12-secant of K .

The point:

28,65,54,67,80,93,77,96,81,158,119,145,132,106,171,167,60,45,73,125,121,100,109,134,131,176,155,107,110,30,166]
Are eliminated from K to satisfy (1) . The points of index zero for 29,79 are added to K to satisfy (2) , then K_{12} =K∪ [29,79] / 28,65,54,67,80,93,77,96,81,158,119,145,132,106,171,167,60,45,73,125,121,100,109,134,131,176,155,107,110,30,166]
K_{12}={3,4,5,6,7,8,9,10,11,12,13,14,16,17,18,19,20,21,22,23,24,25,26,27,29,31,32,33,34,35,36,37,38,39,40,41,42,43,44,46,47,48,50,52,53,55,56,57,58,59,63,64,66,68,70,71,72,74,75,76,78,79,82,83,85,87,88,89,90,91,92,94,95,97,98,99,101,102,103,108,112,113,114,115,116,118,120,122,123,126,127,129,130,133,135,136,137,140,141,142,143,144,147,149,150,151,152,153,156,157,159,160,161,161,163,164,165,168,169,170,172,173,174,175,177,178,179,180,182,183}.
Is a complete (130,12) –arc as shown in (Table2).
Let $\beta_2 = \pi - k_{12}$ ={1,2,15,28,30,45,49,51,54,60,61,62,65,67,69,73,77,80,81,84,86,93,96,100,104,105,106,107,109,110,111,117,119,121,124,125,128,131,132,134,138,139,145,148,154,155,158,162,166,167,171,176,181}
(53,2)-blocking set as shown in table , by theorem (1) ,there exists projective $[130,3,118]_{13}$ code which is equivalent to the complete (130,12)-arc k_{12}.

Table(2)

I	$K_{12}\cap$ Li	B2 ∩ Li
1	41	2,15,28,54,67,80,93,106,119,132,145,158,171
2	16,17,18,19,20,21,22,23,24,25,26,27	1,15
. . .		
182	8,22,47,66,72,91,97,116,122,141,147,166,172	28
183	14,16,53,89,101,113,137,149,161,173	65,28,77,125

Example(3):Existence of $[110,3,99]_{13}$ codes

We take two conic , say C_1 and C_2 , C_3 and let K=π - C_1∪ C_2∪ C_3
{3,4,5,6,7,8,9,10,11,12,13,14,16,17,18,19,20,21,22,23,24,25,26,27,28,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,47,48,50,52,54,55,57,58,59,60, 63,64,65,66,67,68, 70,71,72,74,75,76,77,78,80,81,82,83,85,87,88,90,91 92,93,94,95,96,97,98,99, 101,102,103,106,107,108,109,110,112,113,115, 116, 118,119,120,121,122,123, 125,126,127,130,131,132,133,134,136,137140,141,142,143,144,146,147149,150,151,152,153,155,156,157,158,159,160,161,164,165,166,167,168,169,170,171,172,173,174,175,176,177,178,179,180, 183}.

The geometrical Construction method must satisfies the following:

i. K intersects any line ofπ in at most 11 points .
ii. Every point not in K is on at least one 11-secant of K .

The point:
171,93,67,76,80,41,107,158,119,145,132,106,28,25,85,10,55,65,90,160,159,109,165,115,121,112,183,110,125,133,166,113,168,108,107,170,32,161,118,78,131,22,172]

Are eliminated from K to satisfy (1) . The points of index zero for 104,111 are added to K to satisfy (2) , then K_{11} =K∪ [104,111] /
171,93,67,76,80,41,107,158,119,145,132,106,28,25,85,10,55,65,90,160,159,109,165,115,121,112,183,110,125,133,166,113,168,108,107,170,32,161,118,78,131,22,172]
K_{11}= [3,4,5,6,7,8,9,11,12,13,14,16,17,18,19,20,21,23,24,26,27,30,31,33,34,

35,36,37,38,39,40, 42,43,44,45,46,47,48 ,50,52,54,57,58,59,60, 63,64,66,68, 70,71,72,74,75,77,81,82,83,87,88,91,92,94,95,96,97,98,99,101,102,103,104,111,116,120,122,123,126,127,130,134,136,137,140,141,142,143,144,147,149,150,151,152,153,155,156,157,164,167,169,173,174,175,176,177,178,179,180}.
Is a complete (110,11) –arc as shown in table (3) .Let $\beta_3 = \pi - k_{11}$ ={1,2,10,15,22,28,29,32,41,49,51,53,55,56,61,62,65,67,69,73,76,79,80,84,85,86,89,90,93,100,105,106,107,108,109,110,112,113,114,115,117,118,119,121,124,125,128,129,131,132,133,135,138,139,145,148,170,171,172,181,182183,168,166,165,163,162,161,160,154,158,159,25}.(73,3)-blocking set as shown in (Table3) , by theorem (1) ,there exists a projective $[110,3,99]_{13}$ code which is equivalent to the complete (110,11)-arc k_{11}.

Table(3)

I	K_{11}∩ Li	B3 ∩ Li
1	54	2,15,28,41,67,80,93,106,119,132,145,158,171
2	16,17,18,19,20,21,23,24,26,27	1,15,22,25
. . .		
182	8,47,66,72,91,97,116,122,141, 147	22,28,166,172
183	14,16,77,101,137,149,173	28,53,65,89,113,125,161

Example(4):Existence of $[99,3,89]_{13}$ codes

We take two conic , say C_1 and C_2 ,C_3 ,C_4 and let K=π - C_1∪ C_2∪ C_3∪C_4
{3,4,5,6,7,8,9,10,11,12,13,14,16,17,18,19,20,21,22,23,24,25,26,27,28,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,48 ,50,52,54,55,57, 59,60, 63,
64,65,66,67,68, 70,71,72,74,75, 77,78,80,81,82,83, 85,87,88,91,92,93,94,95
97,98,99, 101,102,103, 106,107,109,110,112,113,115,116,118,119,120,121
122,123, 125,126,127,130,132,133,134, 136,137,140,141,142,143,144,145
147,149,150,151,152,153,155,157,158,159,160,161,164,165,167,168,169,
170,171,172,173,174,175,176,177,179,180, 183}.

The geometrical Construction method must satisfies the following :

i. K intersects any line ofπ in at most 10 points .
ii. Every point not in K is on at least one 10-secant of K .

The point :

67,54,80,41,119,145,132,106,22,88,24,161,8,18,25,19,85,113,14,10,159,171115,160,116,50,112,161,179,165,109,183,77,110,155,95,118,122,180,35,38,65,17]

Are eliminated from K to satisfy (1). The points of index zero for 79,128 are added to K to satisfy (2) , then K_{10} =K∪ [79,128] / 67,54,80,41,119,145,132,106,22,88,24,161,8,18,25,19,85,113,14,10,159,171115,160,116,50,112,179,165,109,183,77,110,155,95,118,122,180,35,38,65,17]
K_{10}={3,4,5,6,7,9,11,12,13,16,20,21,23,26,27,28,30,31,32,33,34,36,37,39,40, 42,43,44,45,46,48 ,52, 55,57, 59,60, 63,64,66,68,70,71,72,74,75,78,79,81 82,83, 85,87,91,92,93,94,97,98,99, 101,102,103, 107,120,121,123,125,126 127,128,130, 133,134, 136,137,140,141,142,143,144,147,149,150,151,152 153, 157,158,164,167,168,169,170,172,173,174,175,176,177}.

Is a complete (99,10) –arc as shown in (Table4).Let $\beta_4 = \pi - k_{10}$ ={1,2,8,10,14,15,17,18,19,22,24,25,29,35,38,41,47,49,50,51,53,54,56,58,61,62,65,67,69,73,76,77,80,84,85,86,88,89,90,95,96,100,104,105,106,108,109,110,111,112,113,114,115,116,117,118,119,122,124,129,131,132,135,138,139,145,148,154,155,156,159,160,161,162,163,165,166,171,178,179,180,181,182,183}
(84,4)-blocking set as shown in (Table4)., by theorem (1) ,there exists a projective $[99,3,89]_{13}$ code which is equivalent to the complete (99,10)-arc k_{10}.

Table (4)

I	K_{10}∩ Li	B4 ∩ Li
1	28,158	2,15,41,54,67,80,93,106,119,132,145,171
2	16,20,21,23,26,27	1,15,17,18,19,24,25,22
. . .		
182	28,66,72,91,97,141,147,172	166,116,122,47,22,8
183	16,28,101,125,137,149,173	161,113,89,77,65,53,14

Example(5):Existence of $[76,3,67]_{13}$ codes

We take two conic , say C_1 and C_2 ,C_3,C_4,C_5 let K=π - C_1∪ C_2∪ C_3∪C_4∪ C_5
{3,4,5,6,7,8,9,10,11,12,13,14,16,17,18,19,20,21,22,23,24,25,26,27,28,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,48 ,50,54,55,57, 59,60,63,6465,67,68,70,71,72,75,77,78,80,81,82,85,87,88,91,92,93,94,95,97,98,99,102,103,106,107,109,110,112,113,115,118,119,120,121,122,123,125,126,127,130,132,133,136,137,140,141,142,143,144,145,147,150,151,152,153,155,157 158,159,160,161,164,165,168,169,170,171,172,173,174,175,177,179,180, 183}.

The geometrical Construction method must satisfies the following :

i. K intersects any line ofπ in at most 9 points .
ii. Every point not in K is on at least one 9-secant of K.

The point:

106,160,119,45,65,28,41,54,16,20,25,93,113,141,155,10,12,14,158,132,107,133,171,165,115,145,35,122,85,159,112,50,125,60,95,3,169,130,150,44,17,78,110,18,183,80,180,63,103,22,43,126,161,88,30]

Are eliminated from K to satisfy (1). The points of index zero for 105,181 are added to K to satisfy (2), then K_9 =K∪ [105,181] / 106,160,119,45,65,28,41,54,16,20,25,93,113,141,155,10,12,14,158,132,107,133,171,165,115,145,35,122,85,159,112,50,125,60,95,3,169,130,150,44,17,78,110,18,183,80,180,63,103,22,43,126,161,88,30]
K_9 ={4,5,6,7,8,9, 11, 13,19,21,23,24,26,27,31,32,33,34, 36,37,38,39,40,42, 46,48 , 55,57, 59,64,67,68, 70,71,72, 75, 77, 81,82, 87,91,92,94,97,98,99, 102, 105 ,109,118,120,121,123,127,136,137,140,142,143,144,146,147,151 152,153,157,164, 168, 170, 172,173,174,175,177,179,181 }.
Is a complete (76,9) –arc as shown in In (Table5).
Let $\beta_5 = \pi - k_9$ ={1,2,3,10,12,14,15,16,17,18,20,22,25,28,29,30,35,41,43,44,45,47,49,50,51,52,53,54,56,58,60,61,62,63,65,66,69,73,74,76,78,79,80,83,84,85,86,88,89,90,93,95,96,100,101,103,104,106,107,108,110,111,112,113,114,115,116,117,119,122,1

24,125,126,128,129,130,131,132,133,134,135,138,139,141,145,148,149,150,154,155,156,158,159,160,161,162,163,165,166,167,169,171,176,178,180,182,183}
(107,5)-blocking set as shown in table, by theorem (1),there existsa projective $[76,3,67]_{13}$ code which is equivalent to the complete (76,9)-arc k_9.

Table (5)

I	$K_9 \cap$ Li	B5 $\cap$ Li
1	67	2,15,28,41,54,80,93,106,119,132,145,158,171
2	19,21,23,24,26,27	1,15,16,17,18,22,25,20
. . .		
182	8,72,91,91,147,172	22,28,47,66,116,122,166,141
183	77,137,173	14,16,28,65,53,89,101,113,125,149,161

Example(6):Existence of $[64,3,56]_{13}$ codes

We take two conic , say C_1 and C_2 ,C_3,C_4,C_5,C_6 let
$K=\pi - C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6$
{3,4,5,6,7,8,9,10,11,12,13,14,16,17,18,19,20,21,22,23,24,25,26,27,28,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45, 48 ,50,54,55,57, 59,60, 63,64,

67,68,70,71,72,77,78,80,81,85,87,88,91,92,93,94,95,97,98,99,102,106,107,109,110,112,113,115,118,119,120,121,122,125,126,127,130,132,133,136,137,140,141,142,143,145,147,150,152,153,155,157,158,159,160,164,165,168,169,170,171,172,173,174,175,177,179,183}.

The geometrical Construction method must satisfies the following :

i. K intersects any line of π in at most 8 points .
ii. Every point not in K is on at least one 8-secant of K .

The point :

54,80,119,28,41,16,22,25,113,141,163,10,14,6,107,159,115,85,152,160,81,60,93,121,78,67,173,50,35,85,13,118,77,183,150,88,63,95,110,18,170,158,143,40,145,5,132,91,71,125,102,112,106,99,146,168,177,39]

Are eliminated from K to satisfy (1) . The points of index zero for 65,82 are added to K to satisfy (2) , thenK_8 =K∪ [65,82] / 54,80,119,28,41,16,22,25,113,141,163,10,14,6,107,159,115,85,152,160,81,60,93,121,78,67,173,50,35,13,118,77,183,150,88,63,95,110,18,170,158,143,40,145,5,132,91,71,125,102,112,106,99,146,168,177,39]
K_8={3,4,7,8,9,11,12,17,19,20,21,23,24,26,27,30,31,32,33,34,36,37,38,42,43,44,45,48,55,57,59,64,65,68,70,72,77,82,92,94,97,98,109,120,122,126,127,130,133,136,137,140,142,147,153,155,157,164,165,171,172,174,175,179}
Is a complete (64,8) –arc as shown in (Table 6).Let $\beta_6 = \pi - k_8$
={1,2,5,6,10,13,14,15,16,18,22,25,28,29,35,39,40,41,46,47,49,50,51,52,53,54,56,58,60,61,62,63,66,67,69,71,73,74,75,76,78,79,80,81,83,84,85,86,87,88,89,90,91,93,95,96,99,100,101,102,103,104,105,106,107,108,110,111,112,113,114,115,116,117,118,119,121,123,124,125,128,129,131,132,134,135,138,139,141,143,144,145,146,148,149,150,151,152,154,156,158,159,160,161,162,163,166,167,168,169,170,173,176,177,178,180,181,182,183}
(119,6)-blocking set as shown in(Table6) , by theorem (1) ,there exists a projective $[64,3,56]_{13}$ code which is equivalent to the complete (64,8)-arc k_8.

Table (6)

I	$K_8 \cap$ Li	B6 $\cap$ Li
1	171	2,15,28,41,54,67,80,93,106,119,132,145,158
2	17,19,20,21,23,24,26,27	1,15,16,18,22,25
. . .		
182	8,72,91,97,122,147,172	22,28,47,66,116,141,166
183	16,77,137,65	14,28,53,89,101,113,125,149,161,173

Example(7):Existence of [54,3,47]$_{13}$codes

We take two conic, say C_1 and C_2 ,C_3,C_4,C_5,C_6 , C_7 let K=π - $C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6 \cup C_7$
{3,4,5,6,7,8,9,10,11,12,13,14,16,17,18,19,20,21,22,23,24,25,26,27,28,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45, 48,54,55,57, 60, 63,64, 67,68, 70,71,72, 78,80,81, 85,87,88,91,93,94,95,97,98,99,102, 106,107,109, 112,113,115,118,119,120,121,122,126,127,130,132,133,136,137,140,141,142, 145,147,150, 153,155, 157,158,159,164,165,168,169,170,171,172,173, 175,177,179,183}.

The geometrical Construction methodmust satisfies the following :

i. K intersects any line ofπ in at most 7 points .
ii. Every point not in K is on at least one 7-secant of K .

The point :

28,41,54,67,80,16,18,19,22,25,121,85,141,169,113,5,6,10,12,14,81,107,159,133,171,115,40,60,93,63,170,95,78,118,183,150,88,109,30,165,173,158,155,4,72,145,39,8,132,179,106,91,17,11,35,70,98]
Are eliminated from K to satisfy (1) . The points of index zero for 117,131 are added to K to satisfy (2) , then
K_7 =K∪ [117,131]
/
28,41,54,67,80,16,18,19,22,25,121,85,141,169,113,5,6,10,12,14,81,107,159,133,171,115,40,60,93,63,170,95,78,118,183,150,88,109,30,165,173,158,155,4,72,145,39,8,132,179,106,91,17,11,35,70,98]
K_7={3,4,9,13,20,21,23,24,26,27,31,32,33,34,36,37,38,42,43,44,45,48,55,57,64,68,71,87,94,97,99,102,112,117,119,120,122,126,127,130,131,136,137,140,142,146,147,153,157,164,168,172,175,177}Is a complete (54,7) –arc as shown in(Table 7) .Let $\beta_7 = \pi - k_7$
={1,2,4,5,6,8,10,11,12,14,15,16,17,18,19,22,25,28,29,30,35,39,40,41,46,47,49,50,51,52,53,54,56,58,59,60,61,62,63,65,66,67,69,70,72,73,74,75,76,77,78,79,80,81,82,83,84,85,86,88,89,90,91,92,93,95,96,98,100,101,103,104,105,106,107,108,109,110,111,113,114,115,116,118,121,123,124,125,128,129,132,133,134,135,138,139,141,143,144,145,148,149,150,151,152,154,155,156,158,159,160,161,162,163,165,166,167,169,170,171,173,174,176,178,179,180,181,182,183}
(129,7)-blocking set as shown in table , by theorem (1) ,there exists a projective [54,3,47]$_{13}$ code which is equivalent to the complete (54,7)-arc k_7.

Table (7)

I	$K_7 \cap$ Li	B7 ∩ Li
1	119	2,15,28,41,54,67,80,93,106,132,145,158,171
2	20,21,23,24,26,27	1,15,16,17,18,19,22,25
. . .		
182	97,112,147,172	8,22,28,47,66,72,91,116,141,166
183	173	14,16,28,53,65,77,89,101,113,125,149,161,137

Example(8):Existence of [45,3,39]$_{13}$codes

We take two conic , say C_1 and C_2 ,C_3,C_4,C_5,C_6 , C_7 ,C_8 let
K=π - $C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6 \cup C_7 \cup C_8$
{3,4,5,6,7,8,9,10,11,12,13,14,16,17,18,19,20,21,22,23,24,25,26,27,28,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45, 54,55,57, 63,64, 67,68, 71,72, 78,80,81, 85,87,88,91,93,94,97,98,99,102, 106,107,109, 112,113,115, 119,120,121,122, 126,127, 132,133,137,140,141,142, 145,146,147, 153,155, 157,158,159,164,165,169,170,171,172,173, 175,177,183}.

The geometrical Construction method must satisfies the following:

i. K intersects any line ofπ in at most 6 points .
ii. Every point not in K is on at least one 6-secant of K .

The point:
28,41,54,67,93,106,16,18,19,22,25,17,121,141,169,183,113,85,155,3,5,12,8,10,14,133,81,107,159,94,115,171,146,122,112,4,102,13,165,109,88,175,78,170,44,6,40,145,173,153,177,11,30,32,34,36]
Are eliminated from K to satisfy (1) . The points of index zero for 61,62 are added to K to satisfy (2) , thenK_6 =K∪ [61,62] /

28,41,54,67,93,106,16,18,19,22,25,17,121,141,169,183,113,85,155,3,5,12,8,10,14,133,81,107,159,94,115,171,146,122, 112,4,102,13,165,109,88,175,78,170,44,6,40,145,173,153,177,11,30,32,34,36]
K_6={7,9,20,21,23,24,26,27,31,33,35,37,38,39,42,43,45,119,55,57,63,64,68,71,72,80,87,91,97,98,99,120,127,132,137,1 40,142,126,147,157,158,164,61,62,172}
Is a complete (45,6) –arc as shown in (Table8).
Let $\beta_8 = \pi - k_6$
={1,2,3,4,5,6,8,10,11,12,13,14,15,17,18,19,22,25,28,29,30,32,34,36,40,41,44,46,47,48,49,50,51,52,53,54,56,58,59,60,6 5,66,67,69,70,73,74,75,76,77,78,79,81,82,83,84,85,86,88,89,90,92,93,94,95,96,100,101,102,103,104,105,106,107,108, 109,110,111,112,113,114,115,116,117,118,121,122,123,124,125,128,129,130,131,133,134,135,136,138,139,141,143,1 44,145,146,148,149,150,151,152,153,154,155,156,159,160,161,162,163,165,166,167,168,169,170,171,173,174,175,17 6,177,178,179,180,181,182,183}
(138,8)-blocking set as shown in table , by theorem (1) ,there exists a projective $[45,3,39]_{13}$code which is equivalent to the complete (45,6)-arc k_6.

Table (8)

I	$K_6 \cap$ Li	B8 ∩ Li
1	80,119,158	2,15,28,41,54,67,93,106,132,145,171
2	16,20,23,24,26,27	1,15,17,18,19,21,22,25
. . .		
182	72,91,97,147,172	8,22,28,47,66,116,122,141,166
183	16,137	14,28,53,65,77,89,101,113,125,149,161,173

Example(9):Existence of $[34,3,29]_{13}$ codes

We take two conic , say C_1 and C_2 ,C_3,C_4,C_5,C_6 , C_7 ,C_8 , C_9 let
$K=\pi - C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6 \cup C_7 \cup C_8 \cup C_9$
={3,4,5,6,7,8,9,10,11,12,13,14,16,17,18,19,20,21,22,23,24,25,26,27,28,30,31,32,33,34,35,36,37,38,39,40,41,42,43,45,5 4,55,57,64,67,68,71,72,78,80,81,85,87,88,93,94,98,99,102,106,107,109,113,115,119,120,121,122,127,132,133140,141, 142,145,146,147,155,157,158,159,164,165,169,171,172,175,177,183}.
The geometrical Construction method must satisfies the following :

i. K intersects any line ofπ in at most 5 points .
ii. Every point not in K is on at least one 5-secant of K .

The point :
41,54,67,80,93,106,119,16,17,18,19,22,24,25,121,183,169,155,141,113,85,3,4,5,8,12,10,14,107,81,55,133,159,171,40, 115,140,88,78,94,102,165,9,13,109,146,35,122,158,145,45,30,132,31, 32,33,34]
Are eliminated from K to satisfy (1). The points of index zero for 92,179 are added to K to satisfy (2) , then
K_6 =K ∪ [92,179] / 41,54,67,80,93,106,119,16,17,18,19,22,24,25,121,183,169,155,141,113,85,3,4,5,8,12,10,14,107,81,55,133,159,171,40, 115,140,88,78,94,102,165,9,13,109,146,35,122,158,145,45,30,132,31, 32,33,34]
K_5={6,7,20,21,23,26,27,28,36,37,38,39,42,43,57,64,68,71,72,87,98,99,120,127,142,147,157,164,172,175,179,177,92,1 1
Is a complete (34,5) –arc as shown in(Table 9) .
Let $\beta_9 = \pi - k_5$
={1,2,3,4,5,8,9,10,12,14,15,16,17,18,19,22,24,25,29,30,31,32,34,33,35,40,41,44,45,46,47,48,49,50,51,52,53,54,55,56,5 8,59,60,61,62,63,65,66,67,69,70,73,74,75,76,77,78,79,80,81,82,83,84,85,86,88,89,90,91,93,95,94,96,97,100,101,102,1 03,104,105,106,107,108,109,110,111,112,113,114,115,116,117,118,119,121,122,123,124,125,126,128,129,130,131,13 2,133,134,135,136,137,138,139,140,,141,143,144,145,146,148,149,150,151,152,153,154, 155,156,158,159,160,161,162,163,165,166,167,168,169,170,171,173,174,176,178,180,181182,183}
(149,9)-blocking set as shown in(Table9) , by theorem (1) ,there exists a projective $[34,3,29]_{13}$code which is equivalent to the complete (34,5)-arc k_5.

Table (9)

I	$K_5 \cap$ Li	B9 ∩ Li
1	28	2,15,41,54,67,80,93,106,119,132,145,158,171
2	21,20,23,26,27	1,15,16,18,18,19,22,24,25
. . .		
182	28,72,147,172	8,22,47,66,91,97,116,122,141,166
183	28	14,16,53,65,77,89,101,113,125,137,149,161,173

Example(10):Existence of $[20,3,16]_{13}$codes

We take two conic , say C_1 and C_2 ,C_3,C_4,C_5,C_6 , C_7 ,C_8 , C_9,C_{10} let
$K=\pi$ - $C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6 \cup C_7 \cup C_8 \cup C_9 \cup C_{10}$
{3,4,5,6,7,8,9,10,11,12,13,14,16,17,18,19,20,21,22,23,24,25,26,27,28,30,31,32,33,34,35,36,37,38,39,40,41,42,43,54,55,57,64,67,68,71,78,80,81,85,87,93,94,98,99,106,107,113,115,119,120,122,127,132,133,140,141,145,146,147,155,158,159,164, 169,171,172,175,183}.
The geometrical Construction method must satisfies the following :

i. K intersects any line ofπ in at most 4 points .
ii. Every point not in K is on at least one 4-secant of K .

The point :
28,54,67,80,93,106,119,41,3,85,113,141,155,169,183,71,5,6,16,17,18,19,20,25,22,24,39,8,10,12,13,14,55,81,94,107,133,159,68,171,115,122,140,9,43,64,7,158,175,40,4,30,87,78,11,132,31,32,33,34,36]
Are eliminated from K to satisfy (1). The points of index zero for 52,62 are added to K to satisfy (2) , then
K_4 =K∪ [52,62] /
28,54,67,80,93,106,119,41,3,85,113,141,155,169,183,71,5,6,16,17,18,19,20,25,22,24,39,8,10,12,13,14,55,81,94,107,133,159,68,171,115,122,140,9,43,64,7,158,175,40,4,30,87,78,11,132,31,32,33,34,36]
K_4={21,23,26,27,35,37,38,42,57,98,99,120,127,145,164,147,172,146,62,52}
Is a complete (20,4) –arc as shown in table (10) .
Let $\beta_{10} = \pi - k_4$
={1,2,3,4,5,8,9,10,11,12,13,14,15,16,17,18,19,20,22,24,25,28,29,30,31,32,33,34,36,39,40,41,43,44,45,46,47,48,49,50,51,53,54,55,56,58,59,60,61,63,64,65,66,67,68,69,70,71,72,73,74,75,76,77,78,79,80,81,82,83,84,85,86,87,88,89,90,91,92,93,95,94,96,97,100,101,102,103,104,105,106,107,108,109,110,111,112,113,114,115,116,117,118,119,121,122,123,124,125,126,128,129,130,131,132,133,134,135,136,137,138,139,140,,141,142,143,144,148,149,150,151,152,153,154,155,156,157,158,159,160,161,162,163,165,166,167,168,169,170,171,173,174,176,177,178,179,180,181,182,183}
(163,10)-blocking set as shown in(Table10) , by theorem (1) ,there exists a projective $[20,3,16]_{13}$code which is equivalent to the complete (20,4)-arc k_4.

Table (10)

I	$K_4 \cap$ Li	B10 ∩ Li
1	145	2,15,28,41,54,80,93,106,119,132,67,171,158
2	21,23,27	1,15,16,17,18,19,20,22,24,25,26
. . .		
182	Ø	14,28,16,53,65,77,89,101,113,125,137,149,161,173
183	23	12,33,159,43,66,76,86,98,106,129,139,149,182

Example(11):Existence of $[13,3,10]_{13}$codes

We take two conic , say C_1,C_2,C_3,C_4,C_5,C_6 , C_7 ,C_8 , C_9 ,$C_{10}$and C_{11} let $K=\pi$ - $C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6 \cup C_7 \cup C_8 \cup C_9 \cup C_{10} \cup C_{11}$
{3,4,5,6,7,8,9,10,11,12,13,14,16,17,18,19,20,21,22,23,24,25,26,27,28,30,31,32,33,34,35,36,37,38,39,40,41,42,43,54,55,57, 67,68, 71,80,81, 85,93,94, 99, 106,107,113,119,120,127, 132,133,141, 145,146, 155,158,159,164, 169,171,172,183}.
The geometrical Construction method must satisfies the following :

i. K intersects any line ofπ in at most 3 points .
ii. Every point not in K is on at least one 3-secant of K .

The point :
28,54,67,80,93,106,119,132,145,16,17,18,19,20,24,22,25,21,39,144,3,71,85,113,141,155,169,183,4,5,8,9,10,12,13,14,55,81,94,107,133,159,171,120,27,30,43,146,11,33,172,32,34,35,36,23,6,31]
Are eliminated from K to satisfy (1) . The points of index zero for 87,173 are added to K to satisfy (2) , then K_3 =K∪ [87,173] /
28,54,67,80,93,106,119,132,145,16,17,18,19,20,24,22,25,21,39,144,3,71,85,113,141,155,169,183,4,5,8,9,10,12,13,14,55,81,94,107,133,159,171,120,27,30,43,146,11,33,172,32,34,35,36,23,6,31]
K_3={7,26,37,38,40,41,42,47,99,127,158,173,87}
Is a complete (13,3) –arc as shown in (Table 11) .

Let $\beta_{11} = \pi - k_3$
={1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,24,25,27,28,29,30,31,32,33,34,
35,36,39,43,44,45,46,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,65,66,67,68,69,70,71,72,73,74,75,76,77,78,79,80,81,82,83,84,85,86,88,89,90,91,92,93,95,94,96,97,98,100,101,102,103,104,105,106,107,108,109,110,111,112,113,
114,115,116,117,118,119,120,121,122,123,124,125,126,128,129,130,131,132,133,134,135,136,137,138,139,140,,141,142,143,144,145,146,147,148,149,150,151,152,153,154,155,156,157,159,160,161,162,163,164,165,166,167,168,169,170,171,174,175,176,177,178,179,180,181,182,183}
(170,11)-blocking set as shown in(Table11) , by theorem (1) ,there exists a projective
$[13,3,10]_{13}$code which is equivalent to the complete (13,3)-arc k_3.

Table (11)

I	$K_3 \cap$ Li	B11 $\cap$ Li
1	41,158	2,15,28,54,67,80,93,106,119,132,145,171
2	19,26	1,15,16,17,18,20,21,22,23,24,25,27
. . .		
182	Ø	8,22,141,28,47,66,72,91,97,116,122,147,166,172
183	173	16,14,28,53,65,77,89,101,113,125,137,149,161

REFERENCES

[1]. J .W.P.Hirsehfeld , projective Geometries over finite fields , oxford university, press oxfored(1979) .
[2]. J .W.P.Hirsehfeld , L.Stome ,The Paching Problem in statistics, coding theory and finite projective spaes: update 2001 ,School of Mathematical Sciences University of Sussex Brighton BN1 9QH United Kingdom,(2001) .
[3]. J.H.Vanlint ,An Introduction to Coding Theory , second Edition , Springer ,Belin , (1992) .
[4]. R .Hill , optimal linear codes , Cryptography and codind II, oxford university ,1992, 41-70
[5]. S. BALL,On the Size of a Double Blocking Set in PG(2, q)University of Sussex, Falmer, East Sussex, United Kingdom, 1994 ,125-133
[6]. T. Kløve,On the existence of proper codes for error detection,University of Bergen, Norway.Singapore, (2009)