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Emergent phenomena in complex systems and their detection

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ABSTRACT

The paper turns attention to opinion of Robert Laughlin (Nobel Prize laureate) saying that "Emergence is an organization principle". Emergence is a basic principle not only in physics. There are recapitulated our recent results in the field of emergence phenomena in Complex systems and especially the detection of emergent situation indicating violations of structural invariants. The paper moves in the border between physics and cybernetics with rich references to special fields of mathematics. A lot of concepts and approaches are original ones and open the space for novel ways of basic research.

Keywords: Emergent phenomena, complex systems, detection of emergent situation, symmetries, uncertainties, violation of structural invariants.

1. INTRODUCTION

This paper has been inspired by the book of Robert B. Laughlin "A Different Universe (Reinventing Physics from the bottom down.)" [1]. Laughlin understands and introduces "emergence as an organization principle" and from this point of view explores a wide field of physics. He does not write directly "a principle of organization of matter" – may be because, he got Nobel Prize for quantum liquids. This principle appears (according to Laughlin) not only in creation of particles but also in formation of sounds, tunes (and as a consequence in formation of speech) and in many other areas.

Investigation of emergent phenomena in our time leaded many researchers to discovery that the development of complex systems does not run in "gradual changes" but in "sharp jumps". These jumps were described (not too convincingly and rather from "a geometrical point of view") by biologists, e.g., by Kauffman [6] (transfer from chaos to order, concepts of attractors), Ho [7] (organisms as coherent systems), Reid [8] (emergence as a tool for evolution by experiments), Markos [9] (added factors of biosemiotics) and from "time point of view" by researchers from "soft sciences", e.g., by Renfrew [10] (sharp change of structure in societies in The Cyclades and the Aegean in The Third Millennium BC), Gould and Eldridge [11] (discoveries in paleontology), To this list is necessary to add "and by many others" however the space of this article is limited.

In the context of our recent works [13], [14], [15], [16] is pleasant to see that Laughlin introduces some emergent situations that we described and detected by means of violation of structural invariants (e.g., violation of symmetries, super symmetries or structural invariants in hydrodynamics (Re) in Table 1 in paper [16] and then in works [13], [14], [15]).

The factor which was for Laughlin very important and may be it changed slightly his point of view was the "collective behavior" and the condition of many elements in observed system. Collective behavior has already been pointed out in works of Herbert Haken [4] and in many works exploring synchronization phenomena of large groups of living elements (fish in ocean [8] , birds in long way travels, spontaneous light impulses of fire flies). The same conditions of many elements we find nowadays in papers with topics of Genetic Algorithms, Genetic Programming and Evolution Computing [33, 34], [36] or in papers with multiagent problems [5]. The condition of many elements justified uncertainties in cases when it was not good look in the system. Similarly was emphasized the condition of enormously long time needed for the change of the systems. (Especially explanation of changes in biology needed billions elements in an interaction and if it is possible - billions of years. Both this represents small "graspability" fogged by doubtful estimations of probability.) But we show that the condition of many elements is a secondary one and that the emergence arises from a certain amount of elements in compartment as a certain immanent property of a complex system. Similarly, the condition of "infinitely long" time seems to be completely misleading way.

In this paper we want to discuss the statements of R. Laughlin in a wider context of disciplines and to compare his conclusions with our result in detecting emergent situations [13], [14], [15], [16].

2. RELATED WORKS

Papers introduced in this Section are not the best works of their authors. However – because I want to attract an interest in the described field of problems I introduce works that are available and easy for study.

Robert Laughlin approaches the complex systems as systems which are able to change their structure (and execute emergence phenomena) from inside of the system not explicitly excited by external actions. (Though these external influences (regarding their intensity) is impossible to neglect.) Robert Laughlin approaches the complex systems as systems which are able to change their structure (and execute emergence phenomena) from inside the system not explicitly excited by external actions. The portion between external and internal initiators of emergence phenomenon is specific one for the way of complex system modelling. From this point of view we may find the related works and characterize them by essential markers: in micro world – calibration groups [22], superstrings, supersymmetry [3], M-theory [23, 24]; in macro world – philosophy [8], matroids and Ramsey theory of graph [13-16], concept lattices [12, 37], genetic algorithms and genetic programming [33, 34], cellular automata [36]. Literature sources [29, 30] point out one approach of computational chemistry that is near to understanding complex systems with some temporary lines of research [18, 19].

Another problem line in discussion of the foundation of emergent phenomena is the duration and trend of emergent changes. In all works cited in the introduction are clear two factors (another will be introduced in Section 3):

- Sharp changes of structure (not changes of function).
- Relatively short time of emergent change (considered to a relevant time of complex system existence).

As examples of such emergent changes we introduce cases mentioned in Introduction as changes complex systems observed by “soft sciences”. (Examples from “hard sciences” will be explored in Section 5.)

David Renfrew demonstrated in his doctoral thesis [10] that absolute reconstruction (may be damage) of Cyclad Civilization (CC) was done during fifty years. Comparing with 800 year duration of CC it represents 5% of the time of CC existence. Similarly Gould and Eldredge [11] pointed out relatively short time of substantial changes seen on trilobites and slugs. They came with idea of jump evolution changes separated with long phases of equilibrium.

Literature sources [29-30] point out one approach of computational chemistry that is near to understanding complex systems with some temporary lines of research. The first formulation of concepts of symmetry have been done in crystallography and works [18, 19] deals with analysis of special crystals though they are not concentrated on the symmetry.

There exists a large number of sources referring artificial intelligence in connection with complex systems and especially with emergent phenomena in creative activities and problem solving. From this field we name only one source [28] in which are references pointed out research line managed by John Gero.

3. EMERGENCE AS AN ORGANIZATION PRINCIPLE

Some properties of „Complex system“ seem to be stable nowadays. Here there are:

- Many elements in mutual interactions,
- Multidimensionality,
- Quasi stability in state changes,
- Nonlinear characteristics,
- Self-organizing processes,
- Emergent behavior,
- Motions in the border of chaos,
- Non stochastic future,
- Inclination to network and multi agent organizations.

Similarly the concept „Emergent situation“ has a certain conceptual background [1], [8], [16]:

Emergent situations of type A – weak emergent situations.

The causes of these situations and their output forms (outputs, shapes) are known. They can be recognized and their appearance can be predicted. Examples of processes and systems that generate such situations are: the Belousov-Zhabotinsky reaction; environments for initiating solitons; the oregonator; the brusselator. They all belong to the field of Synergetics.

Emergent Situations of class B

The causes of these situations are not known, but their output forms are known. Such situations have the following properties:

b1) The situation appears suddenly without an explicit association with situations of the previous relevant "time-space" context of the system. (The reason of it may be in insufficient evidence of possible previous situations (cognitive reason) or the system changes its structure "from inside". In most cases we assume a mixture of both the variants.)

b2) The situation appears as a discrete concept meaningful in the mind of the observer, e.g., a behavior (of a group of termites), an object (a photograph), a shape (e.g., a design of a sculpture) or a property (super conductivity).

b3) The global reason for the appearance of this type of situation is a violation of the system structure (not a violation of the system function). In other words, the situation is induced by a jump in the system structure (the author of [8] speaks about "saltatory" changes).

b4) The detailed reasons and the internal causes of the appearance of the situation are not known (and therefore is impossible to propose a complete prediction model).
(However "the shape" of the emergent situation is known - i.e., it is known how it looks like.)

b5) The appearance of a situation of this type can be detected.
Situations belonging to this class include: a change in behavior strategy in a swarm colony; the appearance of floods; the appearance of rough waves; traffic jams.

Emergent Situations of class C

Neither the causes nor the output forms of these situations are known. Such situations have the same properties as the situations from class B), with the exception of item b4), which has the following content:

c4) No model of a situation of this kind is available before it first operates. (It is not known how the situation looks like.)
Situations that belong to this class include: potential instabilities in ecosystems; the appearance of artifacts in nano-structures; discovery situations in Problem Solving; a violation of symmetries in quantum mechanics.

For some cases of EMSs of type B), and especially of type C), the model of EMSs is unavailable (b4, c4). In these cases is investigated the structure of the environment in which an emergent situation appearance is "anticipated", and the theory of the violation of structural invariants (in the next text) is applied.

Note 3.1*:* Sometimes is discussed the time order of the violation of structural invariants and appearance of EMS. Some people accept an opposite order of changes, i.e., at first appearance of EMS and then the violation of structural invariants. We introduce the "violation of a structural invariant" as a tool for the detection of appearance of EMS (it means in the field of symbols) and not as a natural phenomenon.

As the case where it is seen transparently is, e.g., the elimination of damping in the linear dynamic system of second order with stable behavior. This is a violation of structural invariant (which is the structure of equation of motion). The member with first derivation of output variable disappeared from the equation of motion. And it detects EMS as the jump change of stable behavior into periodical harmonic oscillation.

We may see that characteristics of emergent situations could gradually fulfill Laughlin image about an organization principle. In work [16] was introduced the table with appearance of emergent situations in various applications with their detection by means of structural invariants. Though the results of this work have not been cited by anyone (as I know) the table will be hard to avoid for someone who will deal with emergent situations in complex systems.

The fact that is good known by Laughlin is an emergence of new particles as a consequence of violation of symmetry (as a structural invariant). It is done also by Goldstone's theorem. As for the violation of SUSY (SUperSYmmetry) we anticipate gravitation particles [3, 23-25], and as for the violation of more essential symmetries we can remember axions [27]. Before we reconstruct the table from work [16] we recapitulate what Laughlin understood as emergences:

- Emergence of new particles as a consequence of spontaneous violation of symmetry (e.g., in Weinberg – Glashow – Salam electroweak theory emerge so called intermedial bosons W^+, W- and Z^0 (Higgs′s bosons) or in gases, liquids and solid materials and bodies emergence of particles of sound – phonons.
- Emergence of novel properties (e.g., superconductivity, Debye temperature, consciousness, intelligence).

To this collection belongs a long file of emergent phenomena, emergent situations and emergent components that are not directly related to Laughlin's book and appear in macrostructures:

- Emergent situations in networks of macro world (e.g., floods, traffic jams), [13], [14].
- Emergences of diseases (e.g., diseases of cardio-vascular system; diabetes), [15].
- Emergent situations in problem solving and conceptual design [28, 35].

(Some other will be referred in Table 1 and in the legend to this Table.)

There are also some small interesting confluences in Laughlin's statements. For example – the whole book [1] is a presentation of emergent phenomena, emergent situations and self organizing processes in various fields of physics (from Newton laws, through acoustics, superconductivity till quantum mechanics). On the other hand he told (in work [2]) word for word: " … in environments where appear self organization and emergences do not hold laws of quantum mechanics". (Yes. It is selected from the dialog with a newsman from Der Spiegel however they were words of Nobel Laureate.) Some beginning student of physics could be internally excited by firing question "What is valid from nowadays physics?".

What is not in concordance with Laughlin's image of emergence principle are nano technological structures that are formed by self organization or by self assembly [38]. There appears a natural question "What is more powerful in structuring of matter: self organization and self assembly (as natural procedures of Nature) or emergence (as an organization principle)?".) Very interesting is Laughlin's "meeting" with artificial intelligence. It is worthwhile to read one of many comments to Laughlin's book: " A Different Universe proposes that consciousness and intelligence are emergent properties. Building artificial intelligence based on conventional computer technology is therefore a fool's errand. Computers are machines that are designed and programmed to do specific things. No matter how big or fast the computer is, it will only do what it is programmed to do. If consciousness is indeed an emergent property, it would also, by definition, be insensitive to the underlying physics from which it emerged. So even if carbon-based lifeforms were not present in the universe, life and even intelligence could (in principle) emerge from a completely different underlying physical process, perhaps in a self-organized plasma. The possibility of intelligence without carbon-based life should give proponents of a questionable "finely-tuned universe" based on the silly "anthropic principle" second thoughts."

4. EMERGENT SITUATIONS AND STRUCTURAL INVARIANTS (SIs)

Emergent phenomena in complex systems depend more than in cases of classical systems on cognitive dispositions of the observer. At first we have to say what we consider as "cognitive" and "not cognitive". Simply we can say that all what is consciously registered by our mind is cognitive. It means that all what we observe and in further we process is cognitive. It means that does not exist any "objective reality". Exists only model of reality. There are many models of reality simply ordered according to "a distance of reality". Models that are near to "reality" explain how the processes of "reality" function. Models more distanced of reality introduce how the processes of reality can be represented, signed, computed. For the needs of this paper we will consider only two levels of models: a level very near to "reality" (denoted as NAT) and the level more distanced of "reality" (denoted here as SYMB).
In the next text we will work with a few hypotheses. Here are the first of them.

Hypothesis 1(H1) (in NAT): Emergence and emergent situation are induced by a sharp change of complex system structure ("jump on the structure") . ♦

Hypothesis 2(H2) (in SYMB): In case that we accept H1, an emergent phenomenon appearance may be detected as a sharp violation of some structural characteristics of the complex system description (in our case the violation of so called Structural Invariants). ♦

In [12, 16] there has been introduced concept of structural invariant and we repeat here only essential concepts and relations.

The structure of a complex system is considered as an essential relation that is stable in time and a space and that is not influenced by the function of the system. It means that if this relation is substantially changed then is changed the system (in other words – a system S1 is transferred into another system S2). In this paper is introduced a concept "Structural Invariant" as a component of the structure.

The method proposed in this paper uses a **Model of a Complex System (MCS)** that has in general the following form:

$$MCS = \langle S, \langle \Gamma_1(S), \ldots, \Gamma_n(S) \rangle, \langle Inv(\Gamma_i), \ldots, Inv(\Gamma_p) \rangle \rangle, \quad (1)$$

where S represents a basic set of elements, $\langle \Gamma_1(S), \ldots, \Gamma_n(S) \rangle$ are structures on S considered as relevant for appearance of emergent situations and $\langle Inv(\Gamma_i), \ldots, Inv(\Gamma_p) \rangle$ are invariants on some $\Gamma_1(S), \ldots, \Gamma_n(S)$.

Example 4.1.

Let us consider a set S of regulation elements on water carriers (streams, rivers, ponds) of some hydrological network (that is understood here as a complex system). The assignment of these elements to water carriers (eventually coordinates of these elements and their set up) is a structure of this system. $\Gamma(S) = \langle s_1(re_1, re_2)), s_2(re_3), \ldots, s_n(re_m) \rangle$ where s_i are regulation elements and re_j denotes their parameters (e.g., s_1 – is a valve, re_1 is the specification of the type of the valve and re_2 is the maximal height of opening of the valve).

As an invariant on this structure can be formed $Inv(\Gamma) = \langle s_1(re_1(val(re_1)), re_2(val(re_2))), s_2(re_3(val(re_3))), \ldots, s_n(re_m (val(re_m))) \rangle$, where $val(re_i)$ are values of variables (e.g., $re_1(val(re_1))$ is the value of the valve type (V351) and $re_1 (val(re_2))$ is 0.25 m).
$Inv(\Gamma) = \langle (valve(type(V351), heightmax (0.25)), \ldots \rangle$. ♦

Models for the detection of a **possible appearance of emergent situation (PAES)** have the form:

$$MD(PAES) = \langle MCS, COND_{VInv} \rangle, \quad (2)$$

where $COND_{VInv}$ represents the conditions of violation of MCS structural invariants.

Example 4.2.

Regarding the invariant from Example 4.1., we find, e.g., $COND_{VInv} = \langle Inv(\Gamma) = \langle \emptyset \rangle \rangle$, that can be interpreted as an absolute loss of information about regulation elements and as a consequence of appearance of floods (EMS) in this hydrological network. ♦

There have been explored the following SIs and their violations that induced the appearance of Emergent Situations (EMSs).

Structural invariants:

- **(MB, M)** … **M**atroid **B**ases, **M**atroid, [13-15, 17], [39], [41,3].
- **(HD, RUL)** … **H**asse **D**iagram, Set of evaluated **RUL**es, [12, 37]
- **(DE, IAT)** … **D**egree of **E**mergence , **I**nterpretation **AT**tractors, [35],
- **(ALG TRS)** … **ALG**ebra of **TR**ansformation**S**, [15],
- **(SYMS)** … physical and geometrical **SYM**metrie**S**, [1, 18, 19],
- **(SUSY)**, **(DFM)** … **Su**per **S**ymmetry, **D**ual of **F**ano **M**atroid, [3], [22], [23].

The following Table 1 is actualized table from work [16] (without references to older cases).

Table 1 Structural Invariants, fields of application and causes of emergence

Fields of application →	Networks		Monitoring		Detection of diseases		Problem solving and conceptual design		Physics of micro-world		Super gravity	
Causes of emergence	IND	SI	IND	SI	IND	SI	IND	SI	IND	SI	IND	SI
Formation of new dimension	**I1 I4**	**(M, BM)**			**I5**	**ALG TRS**	**I3 I5**	**DE IAT**	**I5**	**(MB, M)**	**I1 I5**	**SUS Y**

Appearance of new properties			I1 I2	HD RUL	I5	ALG TRS			I5	SYM S	I5	SUS Y
Violation of structural or physical law									I1 I4	SYM S	I1 I5	SUS Y
Restructuring from inside	I1 I2	(MB, M)					I3 I5	DE IAT	I4 I5	SYM S	I4 I5	SUS Y

Legend for Table 1:

DESs (DUXs) … Detection of Emergent Situations (Detection of Unexpected Situations).

Fields of application:

- Networks … DESs in hydrological and transport networks [12], [13], [14], [15], (as are floods and traffic jams),
- Monitoring … DESs in monitoring of ecosystems or bioengineering systems [15], [39] (as are violation and water cycles and changes of local climates),
- Detection of diseases … [15] (as are diseases of cardio-vascular system or diabetes),
- Problem solving and conceptual design … DESs in problem solving and conceptual design [18], [35],
- Physics of micro-world … DESs in physics of elementary particles [3], [20], [21], [22],
- Super Gravity (SUGRA) … ESs in systems of quantum theory of physical field involving gravitation interaction to electroweak, and strong interactions, [23], [24], [25], [26].

Causes of Emergence:

- Formation of new dimension … dimension is considered as an element of some basis (matroid basis, vector space basis),
- Appearance of new properties … appearance of new relevant variables,
- Violation of some structural or physical law … e.g., violation of transitivity in an equivalence relation,
- Restructuring from inside… in cases where restructuring form inside of the system prevails external influences.

INDicators of emergent situations:

- **I1** … very large or very small quantities of standard variables,
- **I2** … unexpected sequences and coincidences of actions in behavior,
- **I3** … non resolvable situations,
- **I4** … unexpected configurations of I/O quantities,
- **I5** … appearance of novel artifacts, properties, particles.

5. EXAMPLES OF APPLICATION OF STRUCTURAL INVARIANTS IN THE DETECTION OF EMERGENT SITUATIONS

(M, BM) – (Matroid, Bases of Matroid)

In this Section are reminded some essential points of invariant (M, BM) that was used, e.g. in, [13], [14], [15], [17]. Emergent phenomena in complex systems depend more than in cases of classical systems on cognitive dispositions of Very important in this case of complex systems description are two factors: level of the description and the basic group (compartment) of complex system elements.

In this paper there are used two descriptive sets: the first **- symptoms** represented by external observational variables (e.g., in ecology - biodiversity, maximum temperature, in biology – morphology of cells, …, in diagnostics of airplanes – sound of the engines, vibrations of wings), and the second **- drivers** represented by internal properties (e.g., in ecology - high velocity in transport layer, decrease of area of landscape vegetation, in biology – genomes, …, in diagnostics of airplanes – composition of fuel, state of jets).

The calculus for the emergent situation in a complex system (that is introduced in the paper) associates two variables for emergent situation – The power $\mathbf{H_P}$ of emergent phenomenon (that is usually computed by quantities of symptoms) and the complexity $\mathbf{H_{COM}}$ of emergent phenomenon (that is associated by drivers).

Essential relations between power H_P and complexity H_{COM} express two equations

$$H_P(\mathbf{B+1}) = H_P(\mathbf{B}) + (u/c)\ \ H_{COM}(\mathbf{B}), \tag{3}$$

$$H_{COM}(\mathbf{B+1}) = H_{COM}(\mathbf{B}) + u\, H_P(\mathbf{B}), \tag{4}$$

where $\mathbf{B} \in \mathbf{BM}$ is a basis of matroid and $\mathbf{B+1}$ is the basis $\mathbf{B}$ extended by one element. Variables $H_P(\mathbf{B})$, $H_P(\mathbf{B+1})$ and $H_{COM}(\mathbf{B})$, $H_{COM}(\mathbf{B+1})$ power and complexity of emergent phenomenon related to compartment with bases $\mathbf{B}$ and $\mathbf{B+1}$. Symbol u denotes the quotient of self organization and c is the limit of this quotient (the best self organisztion). Quotient $(u/c) \in \langle 0, 1\rangle$ represents "intelligence" of self organizing process that will execute the emergent phenomenon. Operating with equation (3) we obtain contribution to power released by emergent phenomenon

$$\Delta H_P(\mathbf{B+1}) = (u/c)\ \ H_{COM}(\mathbf{B}). \tag{5}$$

The contribution to Power of the emergent phenomenon $\Delta H_P(\mathbf{B+1})$ that has an intuitive meaning (e.g., damage of houses by floods) in some level of the description is estimated by quantities of external variables (symptoms) s_i, i = 1, …, n for emergent (s_{iem}) and for nominal (x_{inom}) situations:

$$\Delta H_P(\mathbf{B+1}) = \Big(\sum_{i=1,n} (\omega_i\, (s_{iem}/s_{inom})^2\Big)^{1/2}, \ \text{for}\ \ i=1,\ldots,n, \tag{6}$$

where ω_i are quotients of importance. The contribution to power of emergent phenomenon results in a dimensionless real number expressed here in % (for example, contribution for 20 % is calculated as (120/100) = 1.2) . Equation (5) is associated with equation (4) where $H_{COM}(.)$ is approximated in our case by number of elements of basis $\mathbf{B}$, i.e. #**B.** In order to use expression (5) for computation of the number of matroid basis is necessary to set up the quotient (u/c).

The goal of the method is to compute matroid bases and to find the number of elements (according to Ramsey numbers) that is necessary to add for the extension of some matroid basis by at least one element.

The order of operations in the method is the following one: by quantified actualized symptoms is computed the contribution to power of emergent phenomenon (according to (6)) and from it is computed complexity of emergent phenomenon that determines the number of elements of basis **B** (according to (5)) and by calculus of Ramsey numbers is derived the number of drivers in compartment. The further computations that lead to detection of a possible appearance of an emergent situation (PAES) are realized on the compartment by the relation (8) (in the next text).

Respecting the categories of modeling levels "**NAT**" and "**SYMB**" introduced in Section 4., we indicate the change of structure of complex system compartment as:

$$\textbf{NAT:}\quad S1 \rightarrow (S1 \oplus s) \rightarrow \text{Chaotic phase} \rightarrow \text{SOP} \rightarrow \text{EP} \rightarrow\ S2, \tag{7}$$

S1, S2 are compartments of complex systems. S1 is extended by a sub compartment "s" and goes through a chaotic phase, phase of self organization and phase of emergent phenomenon into S2. Symbol " ⊕" has no specific significance and depends on a real case of method application. SOP symbolizes "Self Organizing Process"
and EP is "Emergent Phenomenon"

$$\textbf{SYMB:}\quad SM1 \rightarrow (SM1 \otimes sm) \rightarrow SM2, \tag{8}$$

where SM1, SM2 are sign models representing S1 and S2, and s*m* is a sign sub model that extends SM1.
In case that SM1 and SM2 are matroids holds the following expression:

$$\textbf{SYMB:}\ \langle X1, \mathbf{BM1}\rangle \rightarrow \langle X1, \mathbf{B1}\rangle \rightarrow \langle X1 \cup E1, \mathbf{B1} \cup e1\rangle \rightarrow \langle X2, \mathbf{B2}\rangle \rightarrow \langle X2, \mathbf{BM2}\rangle, \tag{9}$$

where X1, X2 are carriers of matroids (sets of matroid elements), **BM1** is a set of bases on X1, **B1** is a basis from X1, E1 is a set of elements that extends X1 and e1 is an element that extends basis **B1** into **B2** and **BM2** are bases on X2 .

In continuation of hypotheses H1, H2 from the Introduction we introduce the following three hypotheses that associate the violation of invariant (**M, BM**) with the a Possible Appearance of an Emergent Situation (PAES).

Hypothesis 3(H3) (in NAT): In order to execute emergent phenomenon the complex system increases the number of elements in basic group (compartment) by a minimum number of elements, according to nearest Ramsey number. ♦

Hypothesis 4(H4) (in SYMB): One way how to represent detection of PAES by extension of a matroid basis is to increase the number of interacting elements in basic group (compartment). ♦

Hypothesis 5(H5) (in SYMB): A possible appearance of emergent situation (PAES) is detected as the possibility of extension (reduction) of the basis of the matroid (matroid formed on the compartment of complex system) by at least one element. ♦

These hypotheses describe from one side the assumption about the behavior of a complex system with regard to emergent situations and from the second side they contain the way of processing of information by the user of the proposed method.

Note5.1: According to hypothesis H3 (above) the system selects as optimal those Ramsey numbers for which is needed to add the minimum elements for the extension of basis by one element.

Example 5.1. $\#X = 1600$. ($\#\mathbf{B} = 11$ for $\#X \geq 1597$) and for one element extension of Basis ($\#\mathbf{B} = 12$ for $\#X \geq 1637$) is needed to add at least 40 elements. ♦

A basic rule for the detection of an emergent situation (in relation with expression (15)) is the following one:

$$\text{IF } (\#E1 \geq \min \Delta f(RN)) \Rightarrow (\text{PAES}), \quad (10)$$

where E1 is a set that extends matroid $\langle X1, \mathbf{BM1} \rangle$ (7) and contains at least one element e1 extending basis **B1**. The number "min Δf(RN)" is a minimal difference between further and actual Ramsey number. PAES denotes "a possible appearance of an emergent situation".

As an application of the proposed method there have been computed examples of emergent phenomena in some macroscopic cases as **-** appearance of floods, appearance of traffic jams in highways, sharp changes in termite societies or violation of so called Small Water Cycle [13, 14]. A qualitative concordance in the interpretation of emergent phenomena is possible to find also in fields relatively very distanced of floods and traffic jams. E.g., in the case of the evolution of biological complex systems: model of genome is a network (according to Kauffman [6]) that stays in a state between crystallization and chaos.

It is important that the system that models genome (according to our categorization is in NAT) does not realize an astronomical number of possible configuration of the network but only a small sample. The development of these configurations is managed by so called **attractors** (in SYMB). Kauffman considers as such attractors – the number of cell types. It complains very good to our method where the attractor is the number of elements in matroid bases. Emergence appears by the extension of number of matroid elements in order to extend some of its bases by at least one element.

(HD, RUL) – (Hasse Diagram, Set of evaluated RULes)

Here we show the case where structural invariant (HD, RUL) is extracted from observational data. This invariant was used, e.g., in [12] and [37]. As for a motivation was in [12] introduced example of a classifier for which had been sharply changed configuration of inputs (EMS). As a consequence was its wrong function and the classifier was needed to calibrate. The calibration was executed very often and for sureness even in cases when the classifier worked correctly. In case of application of EMS detector the calibration was executed only in needed cases.

The state diagram (Fig. 1.) represents a monitoring system. This system monitors function of measurement centers in some supervision block. The state diagram has one starting checkpoint (S) and four ending checkpoints q_1, q_2, q_3, q_4.

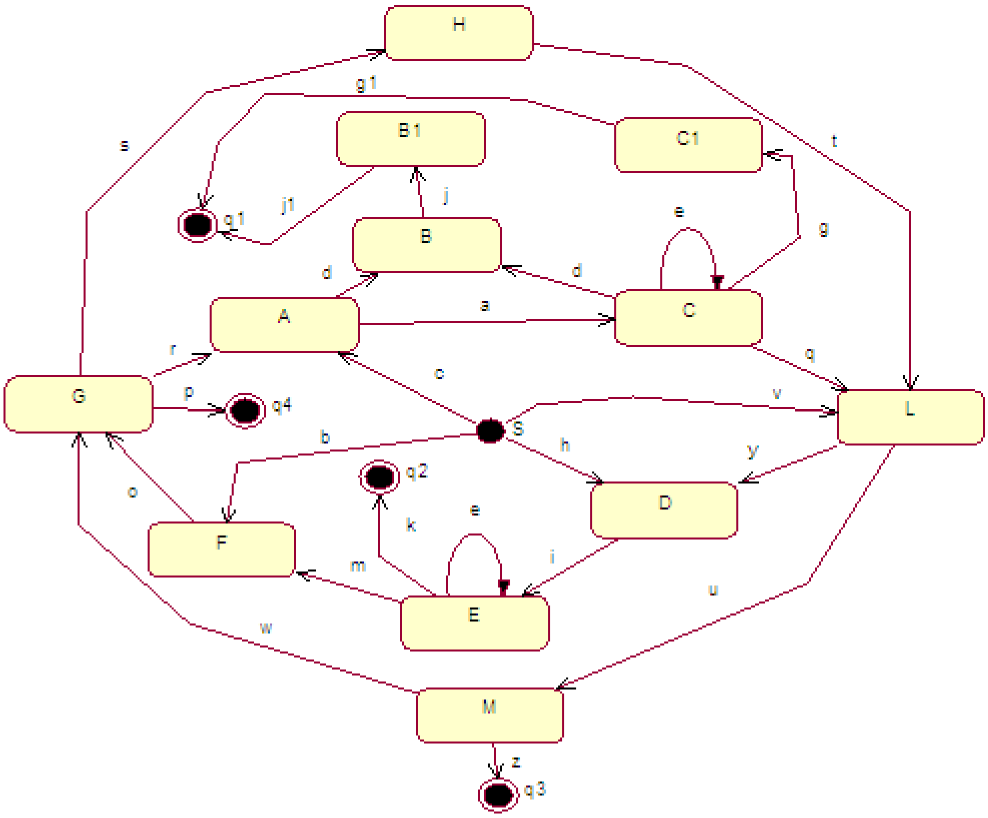

Figure 1: State diagram of monitoring system

Transition between states are denoted here (for simplicity) by small letters (avoiding complicated description of the whole state diagram in UML notation). There are given four situation classes A_1, …, A_4 (sets of state trajectories) that formalize a correct function of the supervision block. Each state trajectory starts in the initial state S and finishes in some final state q_1, q_2, q_3, q_4.

$A_1 \approx \{s_{11} = cdjj_1$, $s_{12} = caedjj_1$, $s_{13} = caeedjj_1$, $s_{14} = caeegg_1\}$, $A_2 \approx \{s_{21} =$ hik, $s_{22} =$ hiek, $s_{23} =$ hieek, $s_{24} =$ himoraqyik$\}$, $A_3 \approx \{s_{31} =$ vuz, $s_{32} =$ vuwstuz$\}$, $A_4 \approx \{s_{41} =$ bop, $s_{42} =$ bostyimop, $s_{42} =$ himop$\}$.

These classes represent various levels of check activities of the monitoring system: A_1 … level of measurement activities of sensors, A_2 … contains a check of the functions of the measurement stations, A_3 … checks the correctness of the function of the whole monitoring system, A_4 … checks the level of external conditions (weather, climate, storm, …) of monitored landscape.

In [12] there was proposed a method for the detection of EMS with structural invariant "Hasse Diagram, Set of evaluated rules". There was used the following sequence of operations: within the framework of A_1, …, A_4 there was developed so called **Datamining context** as a matrix R({A_1, …, A_4}, Set of situations). This context was transformed into form of Hasse diagram. From Hasse diagram there were extracted rules and these rules have been evaluated by characteristics Supp(r_i) (Support for rule r_i) and Conf (r_i) (Confidence for rule r_i).
The result is seen in Table 2. or in Fig..2.

Table 2: Table of rules extracted from Hasse diagram

Rule No. i	**Rule r_i**	**Supp(r_i)**	**Conf (r_i)**
1	**A1 $\Rightarrow$ A2**	**0.2**	**0.5**
2	**A1 $\Rightarrow$ A3**	**0.2**	**0.5**
3	**A1 $\Rightarrow$ A4**	**0.4**	**1**
4	**A2 $\Rightarrow$ A3**	**0.4**	**0.5**
5	**A3 $\Rightarrow$ A4**	**0.4**	**0.66**
6	**A1A2 $\Rightarrow$ A4**	**0.2**	**1**
7	**A2A4 $\Rightarrow$ A1**	**0.2**	**0.33**

8	**A2A3 $\Rightarrow$ A4**	**0.2**	**0.5**
9	**A2A4 $\Rightarrow$ A3**	**0.2**	**0.33**
10	**A1A3 $\Rightarrow$ A4**	**0.2**	**1**
11	**A3A4 $\Rightarrow$ A1**	**0.2**	**0.5**

As an example of an EMS we introduce the trajectory s^* = caquz. The result of monitoring systems for this situation is:

$$(s^* \Theta A_1) \text{OR} (s^* \Theta A_3), \tag{11}$$

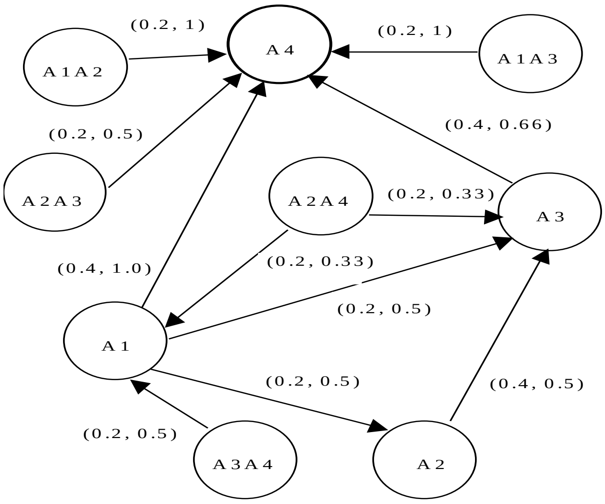

Figure 2: Network of rules for Table 2:

where symbol Θ denotes "the situation belongs to class". The expression (11) is needed to be correctly interpreted. Expression (11) says that " s* belongs to A1 or to A3". And after this is investigated if it is not violated a structural invariant of the system, i.e., if are not violated rules from Table 5.1.
And there is seen that situation s^* = caquz violates rules r_3 and r_5:

$$A_1 \Rightarrow A_4, \tag{12}$$
$$A_3 \Rightarrow A_4. \tag{13}$$

These rules express that when the system is in class A_1 then it will also be in class A_4 and if it is in class A_3 then it will also be in class A_4. However s^* = caquz has no intersection with A_4.

The illustrated application of the proposed method was applied not only for modeling and monitoring the functions of an ecosystem but is also available for many systems with a state diagram description and with the possibility of using a rule-based form for MD(PAES).

(ALG TRS) - ALGebra of TransformationS

Very important are complex systems the description of which is based on structures of transformations [15]. On these structures is possible to recognize algebras of transformations that represent detection structures and invariants for the constructions of *MCS* and *MD(PAES)*.

Model of complex system is in this case

$$\text{MCS} = \langle S, \langle \Gamma_1(S) \equiv \tau(S)\rangle, \langle \text{Inv}(\Gamma_1) \equiv \Xi = \langle T, \tau(S), o \rangle\rangle\rangle, \tag{14}$$

where S is a set of situations, a structure $\Gamma_1(S)$ is a set of transformations $\tau(S)$ and an invariant on this structure is an algebra of transformations $\Xi = \langle T, \tau(S), o \rangle$. T is a set of composed transformations constructed from set of generators in $\tau(S)$ by a binary composition operation "o". The invariant Ξ is understood as all the three components T, $\tau(S)$, o and the violation of each of them induces an emergent situation.
Model MD(PAES) has the form:

$$\text{MD(PAES)} = \langle \text{MCS}, \text{COND}_{\text{VInv}} \equiv (\exists s, \exists \tau^*, ((\tau^* (s) = s^* \notin S) \text{ AND } (\tau^* \notin T))\rangle, \tag{15}$$

where τ^* is a "wrong" composition of transformations and s^* is an emergent situation.
In paper [15] there has been demonstrated the case of detecting emergent situations on ECG diagrams as symptoms of diseases of human cardio-vascular system. The solution was based on invariant (M, BM) and Ramsey theory of graph. Another solution will be introduced now where will be used deeper analysis of algebra of transformations.

The beginning is the same as in work [15].

In Fig. 3 is illustrated correspondence between a part of so called ECG PQRST complex and the states in one cycle of the heart activity. In our state approach description we see 6 transformations that realize this cycle:

$$\tau_U(U(k-1))\rightarrow P(k)\rightarrow\tau_P(P(k))\rightarrow Q(k)\rightarrow \tau_Q(Q(k))\rightarrow R(k)\rightarrow\tau_R(R(k))\rightarrow S(k)\rightarrow\tau_S(S(k))\rightarrow T(k)\rightarrow\tau_T(T(k))\rightarrow U(k), \quad (16)$$

where P, Q, R, S, T, U are phases (states) of ECG signal (according to Fig. 3) and τ_P, τ_Q, τ_R, τ_S, τ_T, τ_U are transformations. Time "k" is related to the whole PQRST cycle and in the following text has no importance and may be neglected. Without a loss of generality we consider about a composition algebra

$$\Xi = \langle T, \{ \tau_P, \tau_Q, \tau_R, \tau_S, \tau_T, \tau_U \}, o \rangle, \quad (17)$$

where "o" is a symbol for the composition of transformations. Expression (16) illustrates transformation activities as, e.g.,

$$P(k)=\tau_U(U(k\text{-}1)),\ Q(k)=\tau_P(P(k)),\ R(k)=\tau_P(Q(k)),\ S(k)=\tau_R o\tau_Q(Q(k)),\ \ldots,\ U(k)=\tau_T o\tau_S o\tau_R(R(k))=\tau_T(\tau_S(\tau_R(R(k)))), \quad (18)$$

While in [15] was considered that the reason of ECG representation of individual heart diseases (emergence phenomena) consists in external deformation transformation (τ_{Q+} , τ_{R+}, τ_{S+}) which attack transformations in (17) in this paper we consider about violation of algebra of transformations $\Xi = \langle T, \tau(S), o \rangle$ by the following way:

$$s_{j+1}= \tau_i (s_j), \quad \text{where } \tau_i \in T \text{ and } s_j, s_{j+1} \in S, \quad (19)$$

$$\tau = \tau_i\, o\tau_k\, o \ldots o\tau_w \text{ where } \tau ,\tau_i, \tau_k, \tau_w\in T, \quad (20)$$

$$\Diamond\exists\tau,\tau = \tau_i\, o\tau_k\, o \ldots o\, \tau_w = 1,\ 1(s) = s \text{ where } \tau ,\tau_i, \tau_k, \tau_w \in T, s \in S, \quad (21)$$

where $\Diamond$ is a symbol of possibility, i.e., "$\Diamond\ \exists\ \tau$" denotes "it may exist such a transformation τ, that …". (The operation "o " is associative, reducible, not commutative and not closed! It means that could exist composition of transformations which fall outside T.)

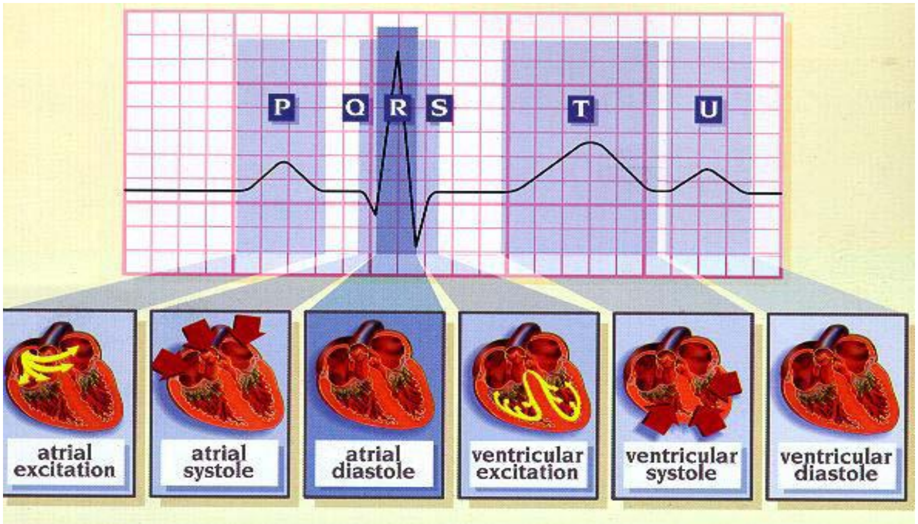

Figure 3: Phases P, Q, R, S, T, U of ECG signal (source [15]).

Such a transformation τ^* ($\tau^* \notin$ T) (according to expression (15)) generates an emergent situation s* (τ^* (s)= s*$\notin$ S).

Frankly said – purely algebraic description of our structural invariant does not assume formation of a situation outside *T*. It means such a situation which has properties considerably different from situations in T. (It is clear that even free algebra is not enough.)

In this point our approach overlaps algebraic approach and we may summarize:

- It is assumed emergent situation from classes B and C – so that it is impossible to model emergent situation.
- For the detection of emergent situation we need testable description of transformations τ_P, τ_Q, τ_R, τ_S, τ_T, τ_U and composition operation "o".
- Testing these transformation and operation "o" we detect deformed transformation or violated composition operation. Both these facts lead to appearance of emergent situation.

Impossibility to include conditions of MD(PAES) in algebraic formalism - it means impossibility to work with situations that appear outside T (and as a consequence the need of external testing algorithms) leads us to decision that combination of matroid theory and Ramsey theory of graph (in [15]) is more suitable for the detection emergent situation on algebra of transformations. Algebraic approach is more suitable in quantum mechanics where the algebras of transformations have the structure of groups and emergent situations may be investigated within the framework of violating symmetries or supersymmetries.

(SYMS) - Physical and Geometrical SYMmetrieS

In this Section we will speak about structural invariant symmetries (SYMS) that represents geometrical and physical symmetries and about special structuring of symmetries in groups of transformations. In physics is the symmetry of a physical system understood as an invariance of its important properties regarding transformations of variables that this system describe (e.g., variables in equations of motion of such a system). The invariance of equations of motion regarding certain transformations leads (according to theorem of E. Noether) to conservation of some characteristic variable (e.g., energy). It was discovered that systems of such transformations have the structure of group. Violation of a symmetry is then the violation of some group (according to previous Section – violation of some algebra of transformations). Each violation of a transformation group induces appearance of emergent phenomena. These phenomena describes Goldstone's theorem: Each spontaneous breaking of symmetry in continuous matters induces appearance of new particles. Let us remind violations of symmetries in laws of micro world – in physics of elementary particles in quantum mechanics.

The effort of union of electromagnetic interaction (with group of symmetries U(1)) with weak interaction (with group of symmetries SU(2)) resulted in “Weinberg – Salam – Glashow” theory of electroweak interaction with group of symmetries SU(2) x U(1). (Group of symmetry of unified interaction has to contain as a subgroups the calibration groups of the original interactions.) Emergence of new particles is seen in the inverse process, i.e., in the division of electroweak interaction into the original interactions. It results in spontaneous violation of symmetry: The group of symmetries is broken up to the subgroup U(1) and a corresponding part of vector bosons (W^+, W^-, Z^o) (that within electroweak interaction had zero matter (energy)) acquire energy $\sim$ 100GeV. It is understood as an emergence of intermedial bosons – (Higgs's bosons).

Emergence of particles of sound (phonons) is observed as a consequence of spontaneous violation of symmetry in gases, liquids and solid materials. This phenomenon is important especially for the formation of speech, music and the most of sound signals in macro world.

SUper SYmmetries (SUSY) and SUper GRAvity (SUGRA)

Super gravity is an ambitious program for union of interactions in quantum theory and consists in union of gravitation interaction with strong, weak and electromagnetic interactions. From the point of view of symmetry was necessary for this purpose to use Lee super algebras and especially those that contain as a sub algebra a group of time-space transformations (e.g., Poincare group). Such a super algebras are called **super symmetrical**.

Within the program of geometrical formulation of super gravity there were developed innovations of Kaluza-Klein unification theory for various dimensions higher than 4. As it was introduced by Witten (e.g., in [23]) – the dimension of initial variety of Kaluza-Klein theory has to have dimension d=11 in order to contain needed group of internal symmetry. It was in concordance with the result for maximal N=8 (N is number of copies of a supersymmetry in an extended suppersymmetric theory) the thory of exte… what is N) super gravity in time space d=4.

An interconnecting bridge between super gravity and super symmetry was the theory of super strings. There was developed 5 essential models of super string theory: Type I, Type IIA, Type IIB, Heterotic SO(32) and Heterotic S_8 x S_8. The investigation of string dualities opened the way for unification of all super string theories into one called M-theory (derived from “Membrane”, “Magic”, “Mystery”). It was also discovered that such an M-theory is possible realize only in D=11 space time. (The number of dimensions needed for M-theory is the same as the number of dimensions needed for super gravity. The connections between both these theories were introduced, e.g., in [23], [24] and [25].)

Strings and super strings are imagined as one dimensional elements (of Planck magnitude 10^{-33}) that oscillate in various frequency modes and in various dimensions (resp. in extra dimensions). Geometrical structure of an internal space determines laws of interactions between elementary particles and quantities of physical constants (as they are charges and masses) that characterizes individual particles. It means that it determines natural laws in an external 3-dimensional space. These basic physical laws which were observed in our Nature and Universe are in this context consequences of more fundamental laws of some unitary theory of super strings. These unitary theories enable many solutions dependently on compactification (folding, ordering, organization) of the internal space.(It means - in the dependence of the folding of extra dimensions between d=3 (d=4) and d=11.) Each of such solution can be interpreted as emergences of an individual universe with various laws visible in our 3+1 space time.

Summarizing: violation of SUSY may result in:

- Emergence of super particles (gravitons and as a consequence the emergence of gravitation waves).
- Emergence of new individual universes.

Note 5.2: As a conclusion of this sub section we quote from [26]: "The standard paradigm for incorporating supersymmetry into a realistic theory is to have underlying dynamics of the theory be supersymmetric, but the ground state of the theory does not respect the fact that symmetry and supersymmetry is broken spontaneously. However **-** the supersymmetry can not be violated permanently by the particles of MSSM (Minimal Supersymmetric Standard Model) as they currently appear. **This means that there is a new sector of the theory that is responsible for the breaking**. The only constraint of this new sector is that it must break supersymmetry permanently and must give superparticles in TeV scale masses."

There is a challenge to investigate this sector as a sector of emergences and emergent situations.

CONCLUSIONS

Writing this paper we tried to keep up as a central line Laughlin's ideas and the book and to move in the field of physics or in a small surrounding of it. Not to be a victim of evolutionary biologists and of discussions about special properties of living organisms. Look at the beginning of such a discussion that could deform our paper. Ho [7]: "Living organisms are coherent systems and not thermodynamic systems." It means that does not hold necessarily causality. For us it means that emergence has its origin in coherent worlds. Coherent systems excel by a brutal connectivity and in each element of the system are concentrated all informations. It means that it does need the concept of the flow of informations and as a consequence it does not need any geometry. And continues the discussion about evolutionary landscapes. Eigen [21]: In space with 360 dimensions it is possible to situate 10^{108} of cubes that have magnitude of edges 0.1 nm. In the surrounding of each point stay in "nearest distance" 10^{19} of neighboring elements." Where there are our considerations about super strings with length 10^{-33} m.

In the proposed paper were introduced concepts of complex systems and emergent situations. There was actualized the table of structural invariants, application fields and reasons of emergent phenomena originally introduced in [16] and there was extended the Laughlin's thesis "Emergence is an organization principle" [1] into fields outside the physics. It was pointed out that it is more powerful principle with wider consequences. The paper corresponds to Laughlin's interest in complex systems and emergences (that was motivated by a final phase of reductionism) and contributes to a starting phase of age of emergences and uncertainties.

6. ACKNOWLEDGMENT

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