Existence and Uniqueness of Mild Solution of certain Nonlinear Integrodifferential Equation

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ABSTRACT

In this paper, by using Banach fixed point theorem, we study the existence and uniqueness of mild and strong solutions of Valera – Fredholm integral equation and Integrodifferential equation.

Keywords: Banach space; Fixed point Theorem; mild and strong solutions.

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1. INTRODUCTION

Existence of mild, strong and classical solutions for differential and Integrodifferential equations in abstract spaces with nonlocal conditions has received much attention in recent years. We refer to the papers of [1,4,3,5]. The conditions on functions used in this work were different from those in [2] as discussed in Refs [6,7]. The main tool employed in the analysis is based on the Banach fixed point theorem. The objective of the present paper is to study the existence of mild and strong solutions of the problem

$$\left[x(t) + g(t, x(t)) \right]' + Ax(t) = f\left(t, x(t), \int_{t_o}^{t} k_1(t,s) k_2(s, x(s))\, ds, \int_{0}^{b} h_1(t,s) h_2(s, x(s))\, ds \right) \tag{1.1}$$

$$x(t_o) + g(t_1, t_2, \dots, t_p, x(\cdot)) = x_0 \tag{1.2}$$

Where $0 \le t_0 < t_1 < t_2 < \dots < t_p \le t_0 + \beta$, $-A$ is the infinitesimal generator of a C_0 Semi group $T(t), t \ge 0$, in a Banach space X and the nonlinear functions:

$$f : \left[t_0, t_0 + \beta \right] \times X \times X \times X \to X, \; g : \left[t_0, t_0 + \beta \right]^p \times X \to X, \; k_1, h_1 : \left[t_0, t_0 + \beta \right] \times \left[t_0, t_0 + \beta \right] \to \left[t_0, t_0 + \beta \right]$$

$$k_2, h_2 : \left[t_0, t_0 + \beta \right] \times \left[t_0, t_0 + \beta \right] \times X \to X$$

and x_0 is a given element of X.

Where X be aBanach space with norm $\left\| \cdot \right\|$. Let $B_r = \left\{ x \in X : \left\| x \right\| \le r \right\} \subset X$ be a closed ball in X and $E = C \left(\left[t_0, t_0 + \beta \right] ; B_r \right)$ denote the complete metric space with metric:

$$d(x,y) = \left\| x - y \right\|_E = \sup_{t \in \left[t_0, t_0 + \beta \right]} \left\{ \left\| x(t) - y(t) \right\| : \; x, y \in E \right\}.$$

This paper is organized as follows. In section 2, we present the preliminaries and hypotheses, this is followed by the results as detailed in Section 3.

2. PRELIMINARIES AND MAIN RESULTS

We shall set forth some Preliminaries and hypotheses that will be used in our subsequent discussion.

Definition 2.1

A continuous solution x of the integral equation

$$x(t) = T(t - t_0) x_0 - T(t - t_0) g(t_1, t_2, ..., t_p, x(\cdot)) + T(t - t_0) g(0, x(0))$$

$$- \int_{t_0}^{t} AT(t - s) g(s, x(s)) ds + \int_{t_0}^{t} T(t - s) f(t, x(t), \int_{t_o}^{s} k_1(s, \tau) k_2(\tau, x(\tau)) d\tau$$

$$, \int_{t_o}^{t_o + \beta} h_1(s, \tau) h_2(\tau, x(\tau)) d\tau) ds, \qquad (2.1)$$

With $t \in [t_0, t_0 + \beta]$, is said to be a mild solution of (1.1)-(1.2) on $[t_0, t_0 + \beta]$.

Definition 2.2

A function x is said to be a strong solution of (1.1)-(1.2) on $[t_0, t_0 + \beta]$

If x is differentiable almost everywhere on $[t_0, t_0 + \beta]$, $x' \in L^1([t_0, t_0 + \beta], X)$ and satisfying (1.1)-(1.2) a.e. on $[t_0, t_0 + \beta]$.

-We list the following hypotheses for our convenience:

(**H1**) There exist continuous $G, P > 0$, such that

$$\left\| g(t_1, t_2, ..., t_p, x_1(.)) - g(t_1, t_2, ..., t_p, x_2(.)) \right\| \leq G \left\| x_1 - x_2 \right\|_E, \qquad for \ x_1, x_2 \in E$$

$$\left\| g(0, x_1) - g(0, x_2) \right\| \leq P \left\| x_1 - x_2 \right\|_E, \qquad , \qquad for \ x_1, x_2 \in E$$

(**H2**) A is the infinitesimal generator of a C_0 semi group $T(t), t \geq 0$ in X such that

$$\left\| T(t) \right\| \leq M,$$

$$\left\| AT(t) \right\| \leq M_1, \qquad for \ some \quad M, M_1 \geq 1.$$

(**H3**) There are constants $L_1, K_1, H_1, G_1, G_2, c_1, c_2$ such that

$$L_1 = \max_{t_o \leq t \leq t_o + \beta} \left\| f(t, 0, 0, 0) \right\|,$$

$$K_1 = \max_{t_o \leq t \leq t_o + \beta} \left\| k_2(s, 0) \right\|,$$

$$H_1 = \max_{t_o \leq t \leq t_o + \beta} \left\| h_2(s, 0) \right\|$$

$$G_1 = \max_{x \in E} \left\| g(t_1, t_2, ..., t_p, x(.)) \right\|,$$

$$G_2 = \max_{x \in E} \left\| g(0, x(0)) \right\|,$$

$$\left\| g(t, x(t)) \right\| \leq c_1 \left\| x \right\| + c_2,$$

(H4) The constants $\|x_0\|, M, M_1, L, K, H, \beta, r, c_1, c_2, \alpha, \gamma$ and satisfy the following two inequalities:

$$M\left[\|x_0\| - G_1 + G_2 + Lr\beta + L\alpha Kr\beta^2 + L\alpha K_1\beta^2 + L\gamma Hr\beta^2 + L\gamma H_1 r\beta^2 + L_1\beta\right] + M_1\left[c_1 r + c_2\right]\beta \le d$$

In the next section, we are going to state the results to be proved in this work.

Theorem 2. 3.

Assume that

(i) Hypotheses (H1)-(H2) hold,

(ii) $f : [t_0, t_0 + \beta] \times X \times X \times X \to X$ is continuous in t on $[t_0, t_0 + \beta]$ and there exists a constant $L > 0$ such that

$$\left\| f(t, x_1, y_1, z_1) - f(t, x_2, y_2, z_2) \right\| \le L\left(\|x_1 - x_2\| + \|y_1 - y_2\| + \|z_1 - z_2\| \right),$$

for $x_i, y_i, z_i \in B_r, i = 1,2$

(iii) $k_1, h_1 : [t_0, t_0 + \beta] \times [t_0, t_0 + \beta] \to [t_0, t_0 + \beta]$ $k_2, h_2 : [t_0, t_0 + \beta] \times [t_0, t_0 + \beta] \times X \to X$

Are continuous in t on $[t_0, t_0 + \beta]$ and there exist positive constants $K, H, L_g, \alpha, \gamma$ such that

$$\left\| k_2(s, x_1) - k_2(s, x_2) \right\| \le K\left([x_1 - x_2] \right),$$

$$\left\| h_2(s, x_1) - h_2(s, x_2) \right\| \le H\left([x_1 - x_2] \right)$$

$$\left\| g(s, x_1) - g(s, x_2) \right\| \le L_g\left([x_1 - x_2] \right)$$

$$\left| k_1(t, s) \right| \le \alpha,$$

$$\left| h_1(t, s) \right| \le \gamma,$$

for $\alpha, \gamma > 0, x_i, y_i \in B_r, i = 1,2$

Then problem (1.1)-(1.2) has a unique mild solution on $[t_0, t_0 + \beta]$

Proof of Theorem:

The operator $F : E \to E$ Is defined by

$$(Fz)(t) = T(t - t_0)\left[x_0 - g(t_1, t_2, ..., t_p, z(.)) + g(0, z(0))\right] - \int_{t_0}^{t} AT(t - s) g(s, z(s)) \, ds$$

$$+ \int_{t_0}^{t} T(t - s) f\left(t, z(t), \int_{t_\circ}^{s} k_1(s, \tau) k_2(\tau, z(\tau)) \, d\tau, \int_{0}^{b} h_1(s, \tau) h_2(\tau, z(\tau)) \, d\tau\right) ds,$$

For $t \in [t_0, t_0 + \beta]$ we show that F maps E into itself for $z \in E, t \in [t_0, t_0 + \beta]$ with the use of hypotheses (H2)-(H4) and assume points (ii)-(iii), we have:

$$\|(Fz)(t)\|$$

$$\leq \|T(t-t_0)x_0\| - \|T(t-t_0)g(t_1,t_2,...,t_p,z(.))\| + \|T(t-t_0)g(0,z(0)\| -$$

$$- \left\| \int_{t_0}^{t} AT(t-s)g(s,z(s))\,ds \right\| + \left\| \int_{t_0}^{t} T(t-s)f(s,z(s), \int_{t_o}^{s} k_1(s,\tau)k_2(\tau,z(\tau))\,d\tau, \right.$$

$$\left. \int_{t_o}^{t_o+\beta} h_1(s,\tau)h_2(\tau,z(\tau))\,d\tau)\,ds \right\|$$

$$\leq M\|x_0\| + MG_1 + MG_2 - M_1\int_{t_0}^{t} \|g(s,z(s))\|\,ds + M\int_{t_0}^{t}[\,\|f(s,z(s), \int_{t_o}^{s} k_1(s,\tau)k_2(\tau,z(\tau))\,d\tau,$$

$$\int_{t_o}^{t_o+\beta} h_1(s,\tau)h_2(\tau,z(\tau))\,d\tau) - f(s,0,0,0)\| + \|f(s,0,0,0)\|\,]ds$$

$$\leq M\|x_0\| + MG_1 + MG_2 + M_1\int_{t_0}^{t}[c_1\|z(s)\| + c_2]ds + M\int_{t_0}^{t}[L(\|z(s) - 0\| + \| \int_{t_o}^{s} k_1(s,\tau)k_2(\tau,z(\tau))\,d\tau - 0\|$$

$$+ \| \int_{t_o}^{t_o+\beta} h_1(s,\tau)h_2(\tau,z(\tau))\,d\tau - 0\|) + \|f(s,0,0,0)\|\,]ds$$

$$\leq M\|x_0\| + MG_1 + MG_2 + M_1\int_{t_0}^{t}[c_1\|z(s)\| + c_2]ds + M\int_{t_0}^{t}[Lr + L|k_1(s,\tau)| \int_{t_o}^{s} \|k_2(\tau,z(\tau) - k_2(\tau,0) + k_2(\tau,0)\|\,d\tau$$

$$+ L|h_1(s,\tau)| \int_{t_o}^{t_o+\beta} \|h_2(\tau,z(\tau) - h_2(\tau,0) + h_2(\tau,0)\|\,d\tau + L_1\,]ds$$

$$\leq M\|x_0\| + MG_1 + MG_2 + M_1[c_1r + c_2]\beta + M\int_{t_0}^{t}[Lr + L\alpha Kr\beta + L\alpha K_1\beta + L\gamma Hr\beta + L\gamma H_1\beta + L_1\,]ds$$

$$\leq M_1[c_1r + c_2]\beta + M[\|x_0\| + G_1 + G_2 + Lr\beta + L\alpha Kr\beta^2 + L\alpha K_1\beta^2 + L\gamma Hr\beta^2 + L\gamma H_1\beta^2 + L_1\beta\,] \leq r$$

Thus, F maps into itself.

Now, for every $z_1, z_2 \in E$, $t \in [t_0, t_0 + \beta]$ and using

(H1),(H2),(H4) and assume points (ii), (iii), we obtain:

$$\left\| (Fz_1)(t) - \left\| (Fz_2)(t) \right\| \right\|$$

$$\leq \left\| T(t-t_0) \right\| \left\| g(t_1, t_2, ..., t_p, z_1(.)) - g(t_1, t_2, ..., t_p, z_2(.)) \right\| + \left\| T(t-t_0) \right\| \left\| g(0, z_1(0)) - g(0, z_2(0)) \right\|$$

$$+ \int_{t_0}^{t} \left\| AT(t-s) \right\| \left\| g(s, z_1(s)) - g(s, z_2(s)) \right\| ds + \int_{t_0}^{t} \left\| T(t-s) \right\| \left\| f\left(s, z_1(s), \int_{t_o}^{s} k_1(s,\tau) k_2(\tau, z_2(\tau)) d\tau, \right.\right.$$

$$\int_{t_o}^{t_o+\beta} h_1(s,\tau) h_2(\tau, z_2(\tau)) d\tau \left.\right) - f\left(s, z_2(s), \int_{t_o}^{s} k_1(s,\tau) k_2(\tau, z_2(\tau)) d\tau, \int_{t_o}^{t_o+\beta} h_1(s,\tau) h_2(\tau, z_2(\tau)) d\tau \right) \left.\right\| ds$$

$$\leq MG \left\| z_1(s) - z_2(s) \right\| + MP \left\| z_1(s) - z_2(s) \right\| + M_1 \int_{t_0}^{t} L_g \left\| z_1(s) - z_2(s) \right\| ds + \int_{t_0}^{t} ML \left[\left\| z_1(s) - z_2(s) \right\| \right.$$

$$+ \left| k_1(s,\tau) \right| \int_{t_o}^{s} \left\| k_2(\tau, z_1(\tau)) - k_2(\tau, z_2(\tau)) \right\| d\tau + \left| h_1(s,\tau) \right| \int_{t_o}^{t_o+\beta} \left\| h_2(\tau, z_1(\tau)) - h_2(\tau, z_2(\tau)) \right\| d\tau \left. \right] ds$$

$$\leq MG \left\| z_1 - z_2 \right\|_E + MP \left\| z_1 - z_2 \right\|_E + M_1 L_g \beta \left\| z_1 - z_2 \right\|_E + ML \left\| z_1 - z_2 \right\|_E \int_{t_0}^{t} \left[1 + \alpha K \int_{t_o}^{s} d\tau + \gamma H \int_{t_o}^{t_o+\beta} d\tau \right] ds$$

$$\leq MG \left\| z_1 - z_2 \right\|_E + MP \left\| z_1 - z_2 \right\|_E + M_1 L_g \beta \left\| z_1 - z_2 \right\|_E + ML \left\| z_1 - z_2 \right\|_E \beta \left[1 + \alpha K \beta + \gamma H \beta \right]$$

$$\leq q \left\| z_1 - z_2 \right\|_E,$$

Where $q = MG + MP + M_1 L_g \beta + ML \beta + ML \alpha K \beta^2 + ML \gamma H \beta^2$ and hence, we obtain:

$$\left\| (Fz_1) - (Fz_2) \right\|_E \leq q \left\| z_1 - z_2 \right\|_E$$

With $0 < q < 1$. This shows that the operator F is a contraction on the complete metric space E. By Banach fixed point theorem, the function F has a unique fixed point in the space E, and this point is the mild solution of problem (1.1)-(1.2) on $[t_0, t_0 + \beta]$, which completes the proof of the theorem 2.3.

Theorem 2.4:

Assume that:

(ⅰ) hypotheses (H1)-(H4) hold,

(ⅱ) X is a reflexive Banach space with norm $\|.\|$ and $x_0 \in D(A)$, the domain of A.

(ⅲ) $g(t_1, t_2, ..., t_p, x(.)) \in D(A)$,

(ⅳ) There exists a constant $L > 0$ such that

$$\left\| f(t, x_1, y_1, z_1) - f(t, x_2, y_2, z_2) \right\| \leq L_2 \left(\left| t_1 - t_2 \right| + \left\| x_1 - x_2 \right\| + \left\| y_1 - y_2 \right\| + \left\| z_1 - z_2 \right\| \right),$$

(ⅴ) There exist positive constants $L_3, K, H, L_g, \alpha, \gamma$ such that

$$L_3 = \max_{t \in [t_o, t_o + \beta]} \left\| f(t, x(t), 0, 0) \right\|$$

$$K_1 = \max_{\tau \in [t_o, t_o + \beta]} \left\| k_2(\tau, x(\tau)) \right\|,$$

$$H_1 = \max_{\tau \in [t_o, t_o + \beta]} \left\| h_2(\tau, x(\tau)) \right\|,$$

$$R = \max_{s, \tau \in [t_o, t_o + \beta]} \left| k_1(s, \tau) \right|,$$

$$S = \max_{s, \tau \in [t_o, t_o + \beta]} \left| h_1(s, \tau) \right|,$$

$$\left\| g(t_1, x_1) - g(t_2, x_2) \right\| \le L_g \left(\left| t_1 - t_2 \right| + \left\| x_1 - x_2 \right\| \right),$$

$$\left| k_1(t_1, s) - k_1(t_2, s) \right| \le \alpha \left| t_1 - t_2 \right|,$$

$$\left| h_1(t_1, s) - h_1(t_2, s) \right| \le \gamma \left| t_1 - t_2 \right|$$

Then x is a unique strong solution of (1.1)-(1.2) on $[t_0, t_0 + \beta]$ for $0 < \theta < t - t_o$ and $t \in [t_0, t_0 + \beta]$

We get:

$$x(t + \theta) - x(t)$$

$$= [T(t + \theta - t_0) - T(t - t_0)] x_0 - [T(t + \theta - t_0) - T(t - t_0)] g(t_1, t_2, \dots, t_p, x(.))$$

$$+ [T(t + \theta - t_0) - T(t - t_0)] g(0, x(0)) - \int_{t_o}^{t_o + \theta} AT(t + \theta - s) g(s, x(s)) ds - \int_{t_o + \theta}^{t + \theta} AT(t + \theta - s) g(s, x(s)) ds$$

$$+ \int_{t_o}^{t} AT(t - s) g(s, x(s)) ds + \int_{t_o}^{t_o + \theta} T(t + \theta - s) f\left(s, x(s), \int_{t_o}^{s} k_1(s, \tau) k_2(\tau, x(\tau)) d\tau, \int_{t_o}^{t_o + \beta} h_1(s, \tau) h_2(\tau, z_2(\tau)) d\tau\right) ds$$

$$+ \int_{t_o + \theta}^{t + \theta} T(t + \theta - s) f\left(s, x(s), \int_{t_o}^{s} k_1(s, \tau) k_2(\tau, x(\tau)) d\tau, \int_{t_o}^{t_o + \beta} h_1(s, \tau) h_2(\tau, z_2(\tau)) d\tau\right) ds$$

$$- \int_{t_o}^{t} T(t - s) f\left(s, x(s), \int_{t_o}^{s} k_1(s, \tau) k_2(\tau, x(\tau)) d\tau, \int_{t_o}^{t_o + \beta} h_1(s, \tau) h_2(\tau, z_2(\tau)) d\tau\right) ds$$

$$= T(t - t_0) [T(\theta) - I] x_0 - T(t - t_0) [T(\theta) - I] g(t_1, t_2, \dots, t_p, x(.)) + T(t - t_0) [T(\theta) - I] g(0, x(0))$$

$$- \int_{t_o}^{t_o + \theta} AT(t + \theta - s) g(s, x(s)) ds - \int_{t_o}^{t} AT(t - s) [g(s + \theta, x(s + \theta)) - g(s, x(s))] ds$$

$$+ \int_{t_o}^{t_o + \theta} T(t + \theta - s) \left[f\left(s, x(s), \int_{t_o}^{s} k_1(s, \tau) k_2(\tau, x(\tau)) d\tau, \int_{t_o}^{t_o + \beta} h_1(s, \tau) h_2(\tau, z_2(\tau)) d\tau\right) - f(s, x(s), 0, 0) + f(s, x(s), 0, 0) \right] ds$$

$$+ \int_{t_o}^{t} T(t - s) \left[f\left(s + \theta, x(s + \theta), \int_{t_o}^{s + \theta} k_1(s + \theta, \tau) k_2(\tau, x(\tau)) d\tau, \int_{t_o}^{t_o + \beta} h_1(s + \theta, \tau) h_2(\tau, x(\tau)) d\tau\right) \right.$$

$$\left. - f\left(s, x(s), \int_{t_o}^{s} k_1(s, \tau) k_2(\tau, x(\tau)) d\tau, \int_{t_o}^{t_o + \beta} h_1(s, \tau) h_2(\tau, x(\tau)) d\tau\right) \right] ds.$$

Using the assumption the fact $\left\| [T(\theta) - I]x \right\| = \theta \left\| Ax \right\| + 0(\theta)$, we obtain

$$\left\| x(t+\theta) - x(t) \right\|$$

$$\leq M \left[\theta_{\in 1} + \theta \left\| Ax_0 \right\| \right] + M \left[\theta_{\in 2} + \theta \left\| Ag(t_1, t_2, ..., t_p, x(.)) \right\| \right] + M \left[\theta_{\in 3} + \theta \left\| Ag(0, x(0)) \right\| \right] - M_1 \int_{t_o}^{t_o+\theta} \left\| g(s, x(s)) \right\| ds$$

$$- M_1 \int_{t_o}^{t} \left\| g(s+\theta, x(s+\theta)) - g(s, x(s)) \right\| ds$$

$$+ M \int_{t_o}^{t_o+\theta} \left\| f\left(s, x(s), \int_{t_o}^{s} k_1(s,\tau)k_2(\tau, x(\tau))d\tau, \int_{t_o}^{t_o+\beta} h_1(s,\tau)h_2(\tau, z_2(\tau))d\tau\right) - f(s, x(s), 0, 0) \right\| + \left\| f(s, x(s), 0, 0) \right\|] ds$$

$$+ M \int_{t_o}^{t} [\left\| f\left(s+\theta, x(s+\theta), \int_{t_o}^{s+\theta} k_1(s+\theta, \tau)k_2(\tau, x(\tau))d\tau, \int_{t_o}^{t_o+\beta} h_1(s+\theta, \tau)h_2(\tau, x(\tau))d\tau\right) \right.$$

$$\left. - f\left(s, x(s), \int_{t_o}^{s} k_1(s,\tau)k_2(\tau, x(\tau))d\tau, \int_{t_o}^{t_o+\beta} h_1(s,\tau)h_2(\tau, x(\tau))d\tau\right) \right\|] ds$$

$$\leq M \left[\theta_{\in 1} + \theta \left\| Ax_0 \right\| \right] + M \left[\theta_{\in 2} + \theta \left\| Ag(t_1, t_2, ..., t_p, x(.)) \right\| \right] + M \left[\theta_{\in 3} + \theta \left\| Ag(0, x(0)) \right\| \right] - M_1 [c_1 \left\| x(s) \right\| + c_2] \theta$$

$$- M_1 L_g \theta \beta - M_1 L_g \int_{t_o}^{t} \left\| x(s+\theta) - x(s) \right\| ds + ML_2 \alpha\beta K_1 \theta + ML_2 \gamma\beta\theta H_1 + ML_3 \theta + ML_2 \theta\beta$$

$$+ ML_2 \int_{t_o}^{t} [\left\| x(s+\theta) - x(s) \right\| ds + ML_2 \alpha\beta^2 K_2 \theta + ML_2 \alpha\beta\theta K_2 + ML_2 \gamma\beta H_2 \theta$$

$$\leq V \theta + \left[ML_2 - M_1 L_g \right] \int_{t_o}^{t} [\left\| x(s+\theta) - x(s) \right\| ds .$$

Where $\varepsilon 1, \varepsilon 2, \varepsilon 3 > 0$ and

$$V = M \left[\varepsilon 1 + \left\| Ax_0 \right\| + \varepsilon 2 + \left\| Ag(t_1, t_2, ..., t_p, x(.)) \right\| + \varepsilon 3 + \left\| Ag(0, x(0)) \right\| + L_2 \alpha\beta K_1 + L_2 \gamma\beta H_1 + L_3 + L_2 \beta + L_2 \alpha\beta^2 K_2 \right.$$

$$\left. + L_2 \alpha\beta K_2 + L_2 \gamma\beta H_2 \right] - M_1 \{ _1 [c_1 \left\| x(s) \right\| + c_2] + L_g \beta \}$$

Which is independent of θ and $t \in \left[t_0, t_0 + \beta \right]$. we obtain

$$\left\| x(t+\theta) - x(t) \right\| \leq V \theta e^{\left[ML_2 - M_1 L_g \right] \beta} , \text{ for } t \in \left[t_0, t_0 + \beta \right].$$

Therefore, x is Lipschitz continuous on $\left[t_0, t_0 + \beta \right]$. The Lipschitz continuity of x on $\left[t_0, t_0 + \beta \right]$ combined with (iv) and (v) of theorem (2.4) implies

$$t \rightarrow f\left(t, x(t), \int_{t_o}^{s} k_1(s,\tau)k_2(\tau, x(\tau))d\tau , \int_{t_o}^{t_o+\beta} h_1(s,\tau)h_2(\tau, x(\tau))d\tau\right)ds ,$$

Is Lipschitz continuous on $[t_0, t_0 + \beta]$, observe that the equation

$$\left[y(t) + g(t, y(t)) \right]' + Ay(t) = f\left(t, x(t), \int_{t_o}^{t} k_1(t, s)k_2(s, x(s))ds , \int_{0}^{b} h_1(t, s)h_2(s, x(s))ds\right)$$

$$y(t_o) + g(t_1, t_2, ..., t_p, x(\cdot)) + x_0$$

Has a unique strong solution $y(t)$ on $\left[t_0, t_0 + \beta\right]$ satisfying the equation

$$y(t) = T(t - t_0) x_0 - T(t - t_0) g(t_1, t_2, ..., t_p, x(\cdot)) + T(t - t_0) g(0, x(0)) - \int_{t_0}^{t} AT(t - s) g(s, x(s)) \, ds$$

$$+ \int_{t_0}^{t} T(t - s) f\left(t, x(t), \int_{t_0}^{s} k_1(s, \tau) k_2(\tau, x(\tau)) \, d\tau, \int_{t_0}^{t_0 + \beta} h_1(s, \tau) h_2(\tau, x(\tau)) \, d\tau\right) ds$$

$$= x(t), \quad t \in [t_0, t_0 + \beta].$$

Consequently, x(t) is strong solution of initial value problem (1.1)-(1.2) on $\left[t_0, t_0 + \beta\right]$ .this completes the proof of theorem 2.4

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