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Enumeration of Principal Cyclic Codes of Length 8 over Z_8

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ABSTRACT

In this paper, all principal cyclic codes of length 8 over Z_8 are enumerated on the basis of the structure, generators and degree of minimal degree polynomials of a cyclic code of length 2^k over Z_8. The rank of such codes is also been obtained. It is found that there are 402 principal cyclic codes of length 8 over $\mathbb{Z}_8$, out of which 8 are principal cyclic codes with leading coefficient 4, 67 are principal cyclic codes with leading coefficient $0, 2$ or 6 and 327 are principal monic cyclic codes.

Keywords: Rings, Cyclic Codes, Minimal Degree Polynomial, Rank

INTRODUCTION

A linear code C of length n over a finite commutative ring R is defined as an R-submodule of R^n. A cyclic code C over R of length n is a linear code such that whenever $(c_0, c_1, c_2, \dots, c_{n-1})$ is in C, $(c_{n-1}, c_0, c_1, \dots, c_{n-2})$ also belongs to C. The rank of a code C (denoted by $rank(C)$) over a ring R is defined as the minimum number of generators of C as an $R-$ module [1].

Many authors ([2], [3], [4],[5],[9],[10],[11]) are recently studying cyclic codes over finite rings. The structure of a cyclic code of length $n = 2^k$ over $\mathbb{Z}_8$ as an ideal of the ring $\mathbb{Z}_8[x]/\langle x^n - 1\rangle$ is given in [6]. A distinguished set of generators and rank of such a cyclic code is obtained in [7], using the structure of a cyclic code of length $n = 2^k$ over $\mathbb{Z}_8$ given in [6]. It is proved that the rank of a cyclic code of length $n = 2^k$ over $\mathbb{Z}_8$ is $n - v$, where v is the degree of a minimal degree polynomial in the code. The minimal degree polynomials in a cyclic code play an important role in enumeration of a cyclic code of length 2^k over $\mathbb{Z}_8$. However, for a given code with distinguished form of generators, the value of v is not always obvious. The degree of minimal degree polynomial in a cyclic code of length 2^k over $\mathbb{Z}_4$ was determined by Abualrub et al.in [1]. Garg et al. extended the results of Abualrub et al. and determined the degree of minimal degree polynomial with leading coefficient $2,4$ or 6 in a cyclic code C of length $n = 2^k$ over $\mathbb{Z}_8$ in [8].

In this paper, using the degree of minimal degree polynomials of a cyclic code C of length $n = 2^k$ over $\mathbb{Z}_8$ given by [8], all principal cyclic codes of length 8 over Z_8 are enumerated. The rank of such codes are also determined. There are 402 principal cyclic codes of length 8 over Z_8, out of which 8 are principal cyclic codes with leading coefficient 4, 67 are principal cyclic codes with leading coefficient $0,2$ or 6 and 327 are principal monic cyclic codes.

PRELIMINARIES

A distinguished set of generators of a cyclic code of length $n = 2^k$ over $\mathbb{Z}_8$ has been obtained in [7]. Firstly, we recall those results for further reference.

Let C be cyclic code of length $n = 2^k$ over $\mathbb{Z}_8$. Let $f(x) = q_0(x)$ be a minimal degree polynomial among all monic polynomials in C, $g(x) = 2q_1(x)$ be a minimal degree polynomial among all polynomials in C with leading coefficient 2 or 6 and $h(x) = 4q_2(x)$ be a minimal degree polynomial among all polynomials in C with leading coefficient 4. Let $deg(f(x)) = r$, $deg(g(x)) = s$ and $deg(h(x)) = v$. It is obvious that $r \geq s \geq v$. A unique form for the polynomials $f(x), g(x)$ and $h(x)$ is determined in Theorem 2.1 below. Note that the cyclic code C may or may not contain any monic polynomial or any polynomial with leading coefficient 2 or 6.

Theorem 2.1 *[7] A cyclic code C of length 2^k over $\mathbb{Z}_8$ is generated by one or more polynomials from the set $\{f(x), g(x), h(x)\}$ where*

1. $f(x) = q_0(x) = (x+1)^r + 2\sum_{i=0}^{s-1} \beta_i (x+1)^i + 4\sum_{i=0}^{v-1} \gamma_i (x+1)^i$ with $\beta_i, \gamma_i \in \mathbb{Z}_2$
2. $g(x) = 2q_1(x) = 2(x+1)^s + 4\sum_{i=0}^{v-1} \alpha_i (x+1)^i$ with $\alpha_i \in \mathbb{Z}_2$
3. $h(x) = 4q_2(x) = 4(x+1)^v$

Remark 2.2 *Referring back to Theorem 2.1, the generators of a cyclic code of length 2^k over $\mathbb{Z}_8$ can further be written as*

1. $f(x) = q_0(x) = (x+1)^r + 2(x+1)^c \beta(x) + 4(x+1)^d \gamma(x)$ where $\beta(x)$ and $\gamma(x)$ belong to $\mathbb{Z}_2[x]/\langle x^n - 1\rangle$.
2. $g(x) = 2q_1(x) = 2(x+1)^s + 4(x+1)^e \alpha(x)$ where $\alpha(x) \in \mathbb{Z}_2[x]/\langle x^n - 1\rangle$.

As stated in Theorem 2.1 above, a cyclic code of length 2^k over $\mathbb{Z}_8$ is generated by one or more polynomials from the set $\{f(x), g(x), h(x)\}$. Clearly, the minimal degree polynomial in any cyclic code of length 2^k over $\mathbb{Z}_8$ is $h(x) = 4(x+1)^v$. In case $h(x) = 4(x+1)^v$ is not a generator of the cyclic code of length 2^k over $\mathbb{Z}_8$, then the minimal degree polynomial $h(x) = 4(x+1)^v$ in the code is not obviously known. Therefore, the minimal degree polynomial in the cyclic codes $\langle g(x)\rangle, \langle f(x), g(x)\rangle, \langle f(x)\rangle$ needs to be determined. In case, a cyclic code of length 2^k over $\mathbb{Z}_8$ is generated by $f(x)$ only, the degree of a minimal degree polynomial $g(x) = 2q_1(x)$ in the code is also not obvious. The degree of minimal degree polynomials $g(x)$ and $h(x)$ in a cyclic code of length 2^k over $\mathbb{Z}_8$ has been obtained by Garg et al. in [8]. Using the results of [8], all principal cyclic codes of length 8 over $\mathbb{Z}_8$ are enumerated. The rank of such codes is also determined and tabulated in the next section.

PRINCIPAL CYCLIC CODES OF LENGTH 8 OVER $\mathbb{Z}_8$

Principal cyclic codes of length 8 over $\mathbb{Z}_8$ depend upon the degree of $f(x), g(x)$ and $h(x)$. Depending upon the degrees of $f(x), g(x)$ and $h(x)$, all principal cyclic codes of length 8 over $\mathbb{Z}_8$ are listed in the following table. The rank of such codes are also obtained and tabulated.

Table 1: Principal Cyclic Codes of Length 8 over $\mathbb{Z}_8$

Sr. No.	Code	Rank
1	$\langle 0\rangle$	8
2	$\langle 1\rangle$	8
3	$\langle 2\rangle$	8
4	$\langle 4\rangle$	8
5	$\langle 4(x+1)\rangle$	7
6	$\langle 4(x+1)^2\rangle$	6
7	$\langle 4(x+1)^3\rangle$	5
8	$\langle 4(x+1)^4\rangle$	4
9	$\langle 4(x+1)^5\rangle$	3
10	$\langle 4(x+1)^6\rangle$	2
11	$\langle 4(x+1)^7\rangle$	1
12	$\langle 2(x+1)\rangle$	7
13	$\langle 2(x+1)+4\rangle$	7
14	$\langle 2(x+1)^2\rangle$	6
15	$\langle 2(x+1)^2+4\rangle$	6
16	$\langle 2(x+1)^2+4+4(x+1)\rangle$	6
17	$\langle 2(x+1)^2+4(x+1)\rangle$	6
18	$\langle 2(x+1)^3\rangle$	5
19	$\langle 2(x+1)^3+4+4(x+1)+4(x+1)^2\rangle$	5
20	$\langle 2(x+1)^3+4\rangle$	5
21	$\langle 2(x+1)^3+4+4(x+1)\rangle$	5
22	$\langle 2(x+1)^3+4(x+1)\rangle$	5

23	$\langle 2(x+1)^3+4(x+1)+4(x+1)^2\rangle$	5
24	$\langle 2(x+1)^3+4(x+1)^2\rangle$	5
25	$\langle 2(x+1)^3+4+4(x+1)^2\rangle$	5
26	$\langle 2(x+1)^4\rangle$	4
27	$\langle 2(x+1)^4+4\rangle$	4
28	$\langle 2(x+1)^4+4+4(x+1)\rangle$	4
29	$\langle 2(x+1)^4+4+4(x+1)+4(x+1)^2\rangle$	4
30	$\langle 2(x+1)^4+4+4(x+1)+4(x+1)^2+4(x+1)^3\rangle$	4
31	$\langle 2(x+1)^4+4+4(x+1)^2\rangle$	4
32	$\langle 2(x+1)^4+4+4(x+1)^2+4(x+1)^3\rangle$	4
33	$\langle 2(x+1)^4+4+4(x+1)^3\rangle$	4
34	$\langle 2(x+1)^4+4(x+1)\rangle$	4
35	$\langle 2(x+1)^4+4(x+1)+4(x+1)^2\rangle$	4
36	$\langle 2(x+1)^4+4(x+1)+4(x+1)^2+4(x+1)^3\rangle$	4
37	$\langle 2(x+1)^4+4(x+1)+4(x+1)^3\rangle$	4
38	$\langle 2(x+1)^4+4(x+1)^2\rangle$	4
39	$\langle 2(x+1)^4+4(x+1)^2+4(x+1)^3\rangle$	4
40	$\langle 2(x+1)^4+4(x+1)^3\rangle$	4
41	$\langle 2(x+1)^5\rangle$	4
42	$\langle 2(x+1)^5+4\rangle$	5
43	$\langle 2(x+1)^5+4+4(x+1)\rangle$	5
44	$\langle 2(x+1)^5+4+4(x+1)+4(x+1)^2\rangle$	5
45	$\langle 2(x+1)^5+4+4(x+1)^2\rangle$	5
46	$\langle 2(x+1)^5+4(x+1)\rangle$	3
47	$\langle 2(x+1)^5+4(x+1)+4(x+1)^2\rangle$	3
48	$\langle 2(x+1)^5+4(x+1)+4(x+1)^2+4(x+1)^3\rangle$	3
49	$\langle 2(x+1)^5+4(x+1)+4(x+1)^2+4(x+1)^3+4(x+1)^4\rangle$	3
50	$\langle 2(x+1)^5+4(x+1)+4(x+1)^2+4(x+1)^4\rangle$	3
51	$\langle 2(x+1)^5+4(x+1)+4(x+1)^3\rangle$	3
52	$\langle 2(x+1)^5+4(x+1)+4(x+1)^3+4(x+1)^4\rangle$	3
53	$\langle 2(x+1)^5+4(x+1)+4(x+1)^4\rangle$	3
54	$\langle 2(x+1)^5+4(x+1)^2\rangle$	4
55	$\langle 2(x+1)^5+4(x+1)^2+4(x+1)^3\rangle$	4
56	$\langle 2(x+1)^5+4(x+1)^3\rangle$	4
57	$\langle 2(x+1)^6\rangle$	4
58	$\langle 2(x+1)^6+4\rangle$	6
59	$\langle 2(x+1)^6+4+4(x+1)\rangle$	6
60	$\langle 2(x+1)^6+4(x+1)\rangle$	5
61	$\langle 2(x+1)^6+4(x+1)+4(x+1)^2\rangle$	5
62	$\langle 2(x+1)^6+4(x+1)^2\rangle$	2
63	$\langle 2(x+1)^6+4(x+1)^2+4(x+1)^3\rangle$	3
64	$\langle 2(x+1)^6+4(x+1)^2+4(x+1)^3+4(x+1)^4\rangle$	3
65	$\langle 2(x+1)^6+4(x+1)^2+4(x+1)^4\rangle$	2
66	$\langle 2(x+1)^6+4(x+1)^2+4(x+1)^4+4(x+1)^5\rangle$	2
67	$\langle 2(x+1)^6+4(x+1)^2+4(x+1)^5\rangle$	2
68	$\langle 2(x+1)^6+4(x+1)^3\rangle$	4
69	$\langle 2(x+1)^7\rangle$	4
70	$\langle 2(x+1)^7+4\rangle$	7
71	$\langle 2(x+1)^7+4(x+1)\rangle$	6
72	$\langle 2(x+1)^7+4(x+1)^2\rangle$	5
73	$\langle 2(x+1)^7+4(x+1)^3\rangle$	1
74	$\langle 2(x+1)^7+4(x+1)^3+4(x+1)^4\rangle$	3
75	$\langle 2(x+1)^7+4(x+1)^3+4(x+1)^5\rangle$	2

76	$\langle 2(x+1)^7+4(x+1)^3+4(x+1)^6\rangle$	1
77	$\langle (x+1)\rangle$	7
78	$\langle (x+1)+2\rangle$	7
79	$\langle (x+1)+4\rangle$	7
80	$\langle (x+1)+2+4\rangle$	7
81	$\langle (x+1)^2\rangle$	6
82	$\langle (x+1)^2+4\rangle$	6
83	$\langle (x+1)^2+4+4(x+1)\rangle$	6
84	$\langle (x+1)^2+4(x+1)\rangle$	6
85	$\langle (x+1)^2+2+2(x+1)\rangle$	6
86	$\langle (x+1)^2+2\rangle$	6
87	$\langle (x+1)^2+2(x+1)\rangle$	6
88	$\langle (x+1)^2+2+4\rangle$	6
89	$\langle (x+1)^2+2+4+4(x+1)\rangle$	6
90	$\langle (x+1)^2+2+4(x+1)\rangle$	6
91	$\langle (x+1)^2+2+2(x+1)+4\rangle$	6
92	$\langle (x+1)^2+2+2(x+1)+4(x+1)\rangle$	6
93	$\langle (x+1)^2+2+2(x+1)+4+4(x+1)\rangle$	6
94	$\langle (x+1)^2+2(x+1)+4\rangle$	6
95	$\langle (x+1)^2+2(x+1)+4(x+1)\rangle$	6
96	$\langle (x+1)^2+2(x+1)+4+4(x+1)\rangle$	6
97	$\langle (x+1)^3\rangle$	6
98	$\langle (x+1)^3+4\rangle$	6
99	$\langle (x+1)^3+4(x+1)\rangle$	6
100	$\langle (x+1)^3+4+4(x+1)\rangle$	6
101	$\langle (x+1)^3+2\rangle$	7
102	$\langle (x+1)^3+2(x+1)\rangle$	6
103	$\langle (x+1)^3+2(x+1)^2\rangle$	6
104	$\langle (x+1)^3+2+2(x+1)\rangle$	7
105	$\langle (x+1)^3+2+2(x+1)^2\rangle$	7
106	$\langle (x+1)^3+2+2(x+1)+2(x+1)^2\rangle$	7
107	$\langle (x+1)^3+2(x+1)+2(x+1)^2\rangle$	6
108	$\langle (x+1)^3+2(x+1)+4(x+1)^2\rangle$	5
109	$\langle (x+1)^3+2+4\rangle$	7
110	$\langle (x+1)^3+2+2(x+1)+4\rangle$	7
111	$\langle (x+1)^3+2+2(x+1)^2+4\rangle$	7
112	$\langle (x+1)^3+2+2(x+1)+2(x+1)^2+4\rangle$	7
113	$\langle (x+1)^3+2(x+1)+4\rangle$	5
114	$\langle (x+1)^3+2(x+1)+4(x+1)\rangle$	5
115	$\langle (x+1)^3+2(x+1)+4+4(x+1)\rangle$	5
116	$\langle (x+1)^3+2(x+1)+4+4(x+1)+4(x+1)^2\rangle$	5
117	$\langle (x+1)^3+2(x+1)+4(x+1)+4(x+1)^2\rangle$	5
118	$\langle (x+1)^3+2(x+1)+4+4(x+1)^2\rangle$	5
119	$\langle (x+1)^3+2+2(x+1)+2(x+1)^2+4\rangle$	7
120	$\langle (x+1)^3+2(x+1)+2(x+1)^2+4\rangle$	5
121	$\langle (x+1)^3+2(x+1)+2(x+1)^2+4+4(x+1)+4(x+1)^2\rangle$	5
122	$\langle (x+1)^3+2(x+1)+2(x+1)^2+4(x+1)\rangle$	5
123	$\langle (x+1)^3+2(x+1)+2(x+1)^2+4(x+1)^2\rangle$	5
124	$\langle (x+1)^3+2(x+1)+2(x+1)^2+4+4(x+1)\rangle$	5
125	$\langle (x+1)^3+2(x+1)+2(x+1)^2+4+4(x+1)^2\rangle$	5
126	$\langle (x+1)^3+2(x+1)+2(x+1)^2+4(x+1)+4(x+1)^2\rangle$	5
127	$\langle (x+1)^3+2(x+1)^2+4\rangle$	6
128	$\langle (x+1)^3+2(x+1)^2+4+4(x+1)\rangle$	6

129	$\langle (x+1)^3 + 2(x+1)^2 + 4(x+1) \rangle$	6
130	$\langle (x+1)^4 \rangle$	6
131	$\langle (x+1)^4 + 4 \rangle$	6
132	$\langle (x+1)^4 + 4(x+1) \rangle$	6
133	$\langle (x+1)^4 + 4 + 4(x+1) \rangle$	6
134	$\langle (x+1)^4 + 2 \rangle$	6
135	$\langle (x+1)^4 + 2 + 2(x+1) \rangle$	7
136	$\langle (x+1)^4 + 2 + 2(x+1) + 2(x+1)^2 \rangle$	7
137	$\langle (x+1)^4 + 2 + 2(x+1) + 2(x+1)^2 + 2(x+1)^3 \rangle$	7
138	$\langle (x+1)^4 + 2 + 2(x+1)^2 \rangle$	4
139	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 2(x+1)^3 \rangle$	5
140	$\langle (x+1)^4 + 2 + 2(x+1)^3 \rangle$	6
141	$\langle (x+1)^4 + 2 + 2(x+1) + 2(x+1)^3 \rangle$	7
142	$\langle (x+1)^4 + 2(x+1) \rangle$	7
143	$\langle (x+1)^4 + 2(x+1) + 2(x+1)^2 \rangle$	7
144	$\langle (x+1)^4 + 2(x+1) + 2(x+1)^3 \rangle$	7
145	$\langle (x+1)^4 + 2(x+1) + 2(x+1)^2 + 2(x+1)^3 \rangle$	7
146	$\langle (x+1)^4 + 2(x+1)^2 \rangle$	4
147	$\langle (x+1)^4 + 2(x+1)^2 + 2(x+1)^3 \rangle$	5
148	$\langle (x+1)^4 + 2 + 4 \rangle$	6
149	$\langle (x+1)^4 + 2 + 4 + 4(x+1) \rangle$	6
150	$\langle (x+1)^4 + 2 + 4(x+1) \rangle$	6
151	$\langle (x+1)^4 + 2 + 2(x+1) + 4 \rangle$	7
152	$\langle (x+1)^4 + 2 + 2(x+1) + 2(x+1)^2 + 4 \rangle$	7
153	$\langle (x+1)^4 + 2 + 2(x+1) + 2(x+1)^2 + 2(x+1)^3 + 4 \rangle$	7
154	$\langle (x+1)^4 + 2 + 2(x+1) + 2(x+1)^3 + 4 \rangle$	7
155	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 4 \rangle$	4
156	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 4 + 4(x+1) \rangle$	6
157	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 4(x+1) \rangle$	6
158	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 4 + 4(x+1)^2 \rangle$	4
159	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 4 + 4(x+1)^3 \rangle$	4
160	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 4 + 4(x+1) + 4(x+1)^2 + 4(x+1)^3 \rangle$	4
161	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 4 + 4(x+1) + 4(x+1)^2 \rangle$	4
162	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 4 + 4(x+1) + 4(x+1)^3 \rangle$	4
163	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 4 + 4(x+1)^2 + 4(x+1)^3 \rangle$	4
164	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 4(x+1) + 4(x+1)^2 \rangle$	4
165	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 4(x+1) + 4(x+1)^3 \rangle$	4
166	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 4(x+1) + 4(x+1)^2 + 4(x+1)^3 \rangle$	4
167	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 4(x+1)^2 + 4(x+1)^3 \rangle$	4
168	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 4(x+1)^3 \rangle$	4
169	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 2(x+1)^3 + 4 \rangle$	5
170	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 2(x+1)^3 + 4 + 4(x+1) \rangle$	5
171	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 2(x+1)^3 + 4 + 4(x+1) + 4(x+1)^2 \rangle$	5
172	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 2(x+1)^3 + 4 + 4(x+1)^2 \rangle$	5
173	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 2(x+1)^3 + 4(x+1) \rangle$	5
174	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 2(x+1)^3 + 4(x+1) + 4(x+1)^2 \rangle$	5
175	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 2(x+1)^3 + 4(x+1)^2 \rangle$	5
176	$\langle (x+1)^4 + 2 + 2(x+1)^3 + 4 \rangle$	6
177	$\langle (x+1)^4 + 2 + 2(x+1)^3 + 4 + 4(x+1) \rangle$	6
178	$\langle (x+1)^4 + 2 + 2(x+1)^3 + 4(x+1) \rangle$	6
179	$\langle (x+1)^4 + 2(x+1) + 4 \rangle$	7
180	$\langle (x+1)^4 + 2(x+1) + 2(x+1)^2 + 4 \rangle$	7
181	$\langle (x+1)^4 + 2(x+1) + 2(x+1)^3 + 4 \rangle$	7

182	$\langle (x+1)^4 + 2(x+1) + 2(x+1)^2 + 2(x+1)^3 + 4 \rangle$	7
183	$\langle (x+1)^4 + 2(x+1)^2 + 4 \rangle$	4
184	$\langle (x+1)^4 + 2(x+1)^2 + 4 + 4(x+1) \rangle$	4
185	$\langle (x+1)^4 + 2(x+1)^2 + 4 + 4(x+1) + 4(x+1)^2 + 4(x+1)^3 \rangle$	4
186	$\langle (x+1)^4 + 2(x+1)^2 + 4 + 4(x+1)^2 \rangle$	4
187	$\langle (x+1)^4 + 2(x+1)^2 + 4 + 4(x+1)^2 + 4(x+1)^3 \rangle$	4
188	$\langle (x+1)^4 + 2(x+1)^2 + 4 + 4(x+1)^3 \rangle$	4
189	$\langle (x+1)^4 + 2(x+1)^2 + 4(x+1) \rangle$	4
190	$\langle (x+1)^4 + 2(x+1)^2 + 4(x+1) + 4(x+1)^2 \rangle$	4
191	$\langle (x+1)^4 + 2(x+1)^2 + 4(x+1)^2 \rangle$	4
192	$\langle (x+1)^4 + 2(x+1)^2 + 4(x+1)^2 + 4(x+1)^3 \rangle$	4
193	$\langle (x+1)^4 + 2(x+1)^2 + 4(x+1)^3 \rangle$	4
194	$\langle (x+1)^4 + 2(x+1)^2 + 2(x+1)^3 + 4 \rangle$	5
195	$\langle (x+1)^4 + 2(x+1)^2 + 2(x+1)^3 + 4 + 4(x+1) \rangle$	5
196	$\langle (x+1)^4 + 2(x+1)^2 + 2(x+1)^3 + 4 + 4(x+1) + 4(x+1)^2 \rangle$	5
197	$\langle (x+1)^4 + 2(x+1)^2 + 2(x+1)^3 + 4(x+1) \rangle$	5
198	$\langle (x+1)^4 + 2(x+1)^2 + 2(x+1)^3 + 4(x+1)^2 \rangle$	5
199	$\langle (x+1)^4 + 2(x+1)^2 + 2(x+1)^3 + 4(x+1) + 4(x+1)^2 \rangle$	5
200	$\langle (x+1)^5 \rangle$	5
201	$\langle (x+1)^5 + 4 \rangle$	5
202	$\langle (x+1)^5 + 4(x+1) \rangle$	5
203	$\langle (x+1)^5 + 4(x+1)^2 \rangle$	5
204	$\langle (x+1)^5 + 4 + 4(x+1) \rangle$	5
205	$\langle (x+1)^5 + 4 + 4(x+1)^2 \rangle$	5
206	$\langle (x+1)^5 + 4(x+1) + 4(x+1)^2 \rangle$	5
207	$\langle (x+1)^5 + 4 + 4(x+1) + 4(x+1)^2 \rangle$	5
208	$\langle (x+1)^5 + 2 \rangle$	8
209	$\langle (x+1)^5 + 2 + 2(x+1) \rangle$	8
210	$\langle (x+1)^5 + 2 + 2(x+1) + 2(x+1)^2 \rangle$	8
211	$\langle (x+1)^5 + 2 + 2(x+1)^2 \rangle$	8
212	$\langle (x+1)^5 + 2(x+1) \rangle$	6
213	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^2 \rangle$	7
214	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^2 + 2(x+1)^3 \rangle$	7
215	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^2 + 2(x+1)^3 + 2(x+1)^4 \rangle$	7
216	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 \rangle$	3
217	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 2(x+1)^4 \rangle$	5
218	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^2 + 2(x+1)^4 \rangle$	7
219	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^4 \rangle$	6
220	$\langle (x+1)^5 + 2(x+1)^2 \rangle$	6
221	$\langle (x+1)^5 + 2(x+1)^2 + 2(x+1)^3 \rangle$	6
222	$\langle (x+1)^5 + 2(x+1)^3 \rangle$	4
223	$\langle (x+1)^5 + 2(x+1) + 4 \rangle$	6
224	$\langle (x+1)^5 + 2(x+1) + 4 + 4(x+1) \rangle$	6
225	$\langle (x+1)^5 + 2(x+1) + 4(x+1) \rangle$	6
226	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^2 + 4 \rangle$	7
227	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^2 + 2(x+1)^3 + 4 \rangle$	7
228	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^2 + 2(x+1)^3 + 2(x+1)^4 + 4 \rangle$	7
229	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4 \rangle$	5
230	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4 + 4(x+1) \rangle$	5
231	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4 + 4(x+1) + 4(x+1)^2 \rangle$	5
232	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4 + 4(x+1)^2 \rangle$	5
233	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4(x+1) \rangle$	4
234	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4(x+1) + 4(x+1)^2 \rangle$	4

235	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4(x+1) + 4(x+1)^3 \rangle$	4
236	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4(x+1)^2 \rangle$	3
237	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4(x+1)^2 + 4(x+1)^3 \rangle$	3
238	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4(x+1)^2 + 4(x+1)^3 + 4(x+1)^4 \rangle$	3
239	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4(x+1)^2 + 4(x+1)^4 \rangle$	3
240	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4(x+1)^3 + 4(x+1)^4 \rangle$	3
241	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4(x+1)^3 \rangle$	3
242	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4(x+1)^4 \rangle$	3
243	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 2(x+1)^4 + 4 \rangle$	3
244	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 2(x+1)^4 + 4 + 4(x+1) \rangle$	4
245	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 2(x+1)^4 + 4 + 4(x+1) + 4(x+1)^2 \rangle$	4
246	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 2(x+1)^4 + 4(x+1) + 4(x+1)^2 \rangle$	5
247	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^2 + 2(x+1)^4 + 4 \rangle$	7
248	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^4 + 4 \rangle$	6
249	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^4 + 4 + 4(x+1) \rangle$	6
250	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^4 + 4(x+1) \rangle$	6
251	$\langle (x+1)^5 + 2(x+1)^2 + 4 \rangle$	6
252	$\langle (x+1)^5 + 2(x+1)^2 + 4 + 4(x+1) \rangle$	6
253	$\langle (x+1)^5 + 2(x+1)^2 + 4(x+1) \rangle$	6
254	$\langle (x+1)^5 + 2(x+1)^2 + 2(x+1)^3 + 4 \rangle$	6
255	$\langle (x+1)^5 + 2(x+1)^2 + 2(x+1)^3 + 4 + 4(x+1) \rangle$	6
256	$\langle (x+1)^5 + 2(x+1)^2 + 2(x+1)^3 + 4(x+1) \rangle$	6
257	$\langle (x+1)^5 + 2(x+1)^3 + 4 \rangle$	4
258	$\langle (x+1)^5 + 2(x+1)^3 + 4 + 4(x+1) \rangle$	4
259	$\langle (x+1)^5 + 2(x+1)^3 + 4 + 4(x+1) + 4(x+1)^2 \rangle$	4
260	$\langle (x+1)^5 + 2(x+1)^3 + 4 + 4(x+1) + 4(x+1)^2 + 4(x+1)^3 \rangle$	4
261	$\langle (x+1)^5 + 2(x+1)^3 + 4 + 4(x+1) + 4(x+1)^3 \rangle$	4
262	$\langle (x+1)^5 + 2(x+1)^3 + 4(x+1) + 4(x+1)^2 \rangle$	4
263	$\langle (x+1)^5 + 2(x+1)^3 + 4(x+1) + 4(x+1)^2 + 4(x+1)^3 \rangle$	4
264	$\langle (x+1)^5 + 2(x+1)^3 + 4(x+1) + 4(x+1)^3 \rangle$	4
265	$\langle (x+1)^5 + 2(x+1)^3 + 4(x+1)^2 + 4(x+1)^3 \rangle$	4
266	$\langle (x+1)^5 + 2(x+1)^3 + 4(x+1)^2 \rangle$	4
267	$\langle (x+1)^5 + 2(x+1)^3 + 4(x+1) \rangle$	4
268	$\langle (x+1)^5 + 2(x+1)^3 + 4(x+1)^3 \rangle$	4
269	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 2(x+1)^4 + 4 + 4(x+1)^2 \rangle$	3
270	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 2(x+1)^4 + 4 + 4(x+1)^2 + 4(x+1)^3 \rangle$	3
271	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 2(x+1)^4 + 4 + 4(x+1)^2 + 4(x+1)^3 + 4(x+1)^4 \rangle$	3
272	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 2(x+1)^4 + 4 + 4(x+1)^2 + 4(x+1)^4 \rangle$	3
273	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 2(x+1)^4 + 4(x+1) \rangle$	5
274	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 2(x+1)^4 + 4(x+1)^2 \rangle$	5
275	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 2(x+1)^4 + 4 + 4(x+1) + 4(x+1)^2 + 4(x+1)^3 \rangle$	4
276	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 2(x+1)^4 + 4 + 4(x+1) + 4(x+1)^3 \rangle$	4
277	$\langle (x+1)^6 \rangle$	4
278	$\langle (x+1)^6 + 4 \rangle$	4
279	$\langle (x+1)^6 + 4 + 4(x+1) \rangle$	4
280	$\langle (x+1)^6 + 4 + 4(x+1)^2 \rangle$	4
281	$\langle (x+1)^6 + 4 + 4(x+1)^3 \rangle$	4
282	$\langle (x+1)^6 + 4 + 4(x+1) + 4(x+1)^2 \rangle$	4
283	$\langle (x+1)^6 + 4 + 4(x+1) + 4(x+1)^2 + 4(x+1)^3 \rangle$	4
284	$\langle (x+1)^6 + 4 + 4(x+1) + 4(x+1)^3 \rangle$	4
285	$\langle (x+1)^6 + 4 + 4(x+1)^2 + 4(x+1)^3 \rangle$	4
286	$\langle (x+1)^6 + 4(x+1) \rangle$	4
287	$\langle (x+1)^6 + 4(x+1) + 4(x+1)^2 \rangle$	4

288	$\langle (x+1)^6+4(x+1)+4(x+1)^2+4(x+1)^3 \rangle$	4
289	$\langle (x+1)^6+4(x+1)+4(x+1)^3 \rangle$	4
290	$\langle (x+1)^6+4(x+1)^2 \rangle$	4
291	$\langle (x+1)^6+4(x+1)^2+4(x+1)^3 \rangle$	4
292	$\langle (x+1)^6+4(x+1)^3 \rangle$	4
293	$\langle (x+1)^6+2 \rangle$	8
294	$\langle (x+1)^6+2+2(x+1) \rangle$	8
295	$\langle (x+1)^6+2(x+1) \rangle$	7
296	$\langle (x+1)^6+2(x+1)+2(x+1)^2 \rangle$	7
297	$\langle (x+1)^6+2(x+1)^2 \rangle$	6
298	$\langle (x+1)^6+2(x+1)^2+2(x+1)^3 \rangle$	6
299	$\langle (x+1)^6+2(x+1)^2+2(x+1)^3+2(x+1)^4 \rangle$	6
300	$\langle (x+1)^6+2(x+1)^2+2(x+1)^4 \rangle$	2
301	$\langle (x+1)^6+2(x+1)^2+2(x+1)^4+2(x+1)^5 \rangle$	5
302	$\langle (x+1)^6+2(x+1)^2+2(x+1)^5 \rangle$	6
303	$\langle (x+1)^6+2(x+1)^3 \rangle$	5
304	$\langle (x+1)^6+2(x+1)+4 \rangle$	7
305	$\langle (x+1)^6+2(x+1)+2(x+1)^2+4 \rangle$	7
306	$\langle (x+1)^6+2(x+1)^2+4 \rangle$	4
307	$\langle (x+1)^6+2(x+1)^2+4+4(x+1) \rangle$	5
308	$\langle (x+1)^6+2(x+1)^2+4+4(x+1)+4(x+1)^2 \rangle$	5
309	$\langle (x+1)^6+2(x+1)^2+4+4(x+1)^2 \rangle$	2
310	$\langle (x+1)^6+2(x+1)^2+4+4(x+1)^2+4(x+1)^3 \rangle$	3
311	$\langle (x+1)^6+2(x+1)^2+4+4(x+1)^3 \rangle$	4
312	$\langle (x+1)^6+2(x+1)^2+4(x+1) \rangle$	6
313	$\langle (x+1)^6+2(x+1)^2+4+4(x+1)^2+4(x+1)^4 \rangle$	2
314	$\langle (x+1)^6+2(x+1)^2+4+4(x+1)^2+4(x+1)^4+4(x+1)^5 \rangle$	2
315	$\langle (x+1)^6+2(x+1)^2+4+4(x+1)^2+4(x+1)^5 \rangle$	2
316	$\langle (x+1)^6+2(x+1)^2+2(x+1)^3+4 \rangle$	6
317	$\langle (x+1)^6+2(x+1)^2+2(x+1)^3+4+4(x+1) \rangle$	6
318	$\langle (x+1)^6+2(x+1)^2+2(x+1)^3+4(x+1) \rangle$	6
319	$\langle (x+1)^6+2(x+1)^2+2(x+1)^3+2(x+1)^4+4 \rangle$	6
320	$\langle (x+1)^6+2(x+1)^2+2(x+1)^3+2(x+1)^4+4+4(x+1) \rangle$	6
321	$\langle (x+1)^6+2(x+1)^2+2(x+1)^3+2(x+1)^4+4(x+1) \rangle$	6
322	$\langle (x+1)^6+2(x+1)^2+2(x+1)^4+4 \rangle$	6
323	$\langle (x+1)^6+2(x+1)^2+2(x+1)^4+4+4(x+1) \rangle$	6
324	$\langle (x+1)^6+2(x+1)^2+2(x+1)^4+4(x+1) \rangle$	5
325	$\langle (x+1)^6+2(x+1)^2+2(x+1)^4+4(x+1)+4(x+1)^2 \rangle$	5
326	$\langle (x+1)^6+2(x+1)^2+2(x+1)^4+4(x+1)^2 \rangle$	4
327	$\langle (x+1)^6+2(x+1)^2+2(x+1)^4+4(x+1)^2+4(x+1)^3 \rangle$	4
328	$\langle (x+1)^6+2(x+1)^2+2(x+1)^4+4(x+1)^3 \rangle$	3
329	$\langle (x+1)^6+2(x+1)^2+2(x+1)^4+4(x+1)^3+4(x+1)^4 \rangle$	3
330	$\langle (x+1)^6+2(x+1)^2+2(x+1)^4+4(x+1)^4 \rangle$	2
331	$\langle (x+1)^6+2(x+1)^2+2(x+1)^4+4(x+1)^4+4(x+1)^5 \rangle$	2
332	$\langle (x+1)^6+2(x+1)^2+2(x+1)^4+4(x+1)^5 \rangle$	2
333	$\langle (x+1)^6+2(x+1)^2+2(x+1)^4+2(x+1)^5+4 \rangle$	6
334	$\langle (x+1)^6+2(x+1)^2+2(x+1)^4+2(x+1)^5+4+4(x+1) \rangle$	6
335	$\langle (x+1)^6+2(x+1)^2+2(x+1)^4+2(x+1)^5+4(x+1) \rangle$	4
336	$\langle (x+1)^6+2(x+1)^2+2(x+1)^4+2(x+1)^5+4(x+1)+4(x+1)^2 \rangle$	4
337	$\langle (x+1)^6+2(x+1)^2+2(x+1)^4+2(x+1)^5+4(x+1)+4(x+1)^2+4(x+1)^3 \rangle$	4
338	$\langle (x+1)^6+2(x+1)^2+2(x+1)^4+2(x+1)^5+4(x+1)^2 \rangle$	5
339	$\langle (x+1)^6+2(x+1)^2+2(x+1)^5+4 \rangle$	5
340	$\langle (x+1)^6+2(x+1)^2+2(x+1)^5+4+4(x+1) \rangle$	4

341	$\langle (x+1)^6+2(x+1)^2+2(x+1)^5+4+4(x+1)+4(x+1)^2\rangle$	2
342	$\langle (x+1)^6+2(x+1)^2+2(x+1)^5+4+4(x+1)+4(x+1)^2+4(x+1)^3\rangle$	3
343	$\langle (x+1)^6+2(x+1)^2+2(x+1)^5+4+4(x+1)+4(x+1)^2+4(x+1)^3+4(x+1)^4\rangle$	3
344	$\langle (x+1)^6+2(x+1)^2+2(x+1)^5+4+4(x+1)+4(x+1)^2+4(x+1)^4\rangle$	2
345	$\langle (x+1)^6+2(x+1)^2+2(x+1)^5+4+4(x+1)+4(x+1)^2+4(x+1)^4+4(x+1)^5\rangle$	2
346	$\langle (x+1)^6+2(x+1)^2+2(x+1)^5+4+4(x+1)+4(x+1)^2+4(x+1)^5\rangle$	2
347	$\langle (x+1)^6+2(x+1)^2+2(x+1)^5+4+4(x+1)^2\rangle$	5
348	$\langle (x+1)^6+2(x+1)^2+2(x+1)^5+4(x+1)\rangle$	6
349	$\langle (x+1)^6+2(x+1)^3+4\rangle$	5
350	$\langle (x+1)^6+2(x+1)^3+4+4(x+1)\rangle$	5
351	$\langle (x+1)^6+2(x+1)^3+4+4(x+1)+4(x+1)^2\rangle$	5
352	$\langle (x+1)^6+2(x+1)^3+4+4(x+1)^2\rangle$	5
353	$\langle (x+1)^6+2(x+1)^3+4(x+1)\rangle$	5
354	$\langle (x+1)^6+2(x+1)^3+4(x+1)+4(x+1)^2\rangle$	5
355	$\langle (x+1)^6+2(x+1)^3+4(x+1)^2\rangle$	5
356	$\langle (x+1)^7\rangle$	4
357	$\langle (x+1)^7+4\rangle$	4
358	$\langle (x+1)^7+4+4(x+1)\rangle$	4
359	$\langle (x+1)^7+4+4(x+1)+4(x+1)^2\rangle$	4
360	$\langle (x+1)^7+4+4(x+1)+4(x+1)^2+4(x+1)^3\rangle$	4
361	$\langle (x+1)^7+4+4(x+1)+4(x+1)^3\rangle$	4
362	$\langle (x+1)^7+4+4(x+1)^2\rangle$	4
363	$\langle (x+1)^7+4+4(x+1)^3\rangle$	4
364	$\langle (x+1)^7+4+4(x+1)^2+4(x+1)^3\rangle$	4
365	$\langle (x+1)^7+4(x+1)\rangle$	4
366	$\langle (x+1)^7+4(x+1)+4(x+1)^2\rangle$	4
367	$\langle (x+1)^7+4(x+1)^2\rangle$	4
368	$\langle (x+1)^7+4(x+1)^2+4(x+1)^3\rangle$	4
369	$\langle (x+1)^6+4(x+1)^3\rangle$	4
370	$\langle (x+1)^7+4(x+1)+4(x+1)^2+4(x+1)^3\rangle$	4
371	$\langle (x+1)^7+4(x+1)+4(x+1)^3\rangle$	4
372	$\langle (x+1)^7+2\rangle$	8
373	$\langle (x+1)^7+2(x+1)\rangle$	7
374	$\langle (x+1)^7+2(x+1)^2\rangle$	6
375	$\langle (x+1)^7+2(x+1)^3\rangle$	6
376	$\langle (x+1)^7+2(x+1)^3+2(x+1)^4\rangle$	5
377	$\langle (x+1)^7+2(x+1)^3+2(x+1)^5\rangle$	2
378	$\langle (x+1)^7+2(x+1)^3+2(x+1)^6\rangle$	6
379	$\langle (x+1)^7+2(x+1)+4\rangle$	7
380	$\langle (x+1)^7+2(x+1)^2+4\rangle$	6
381	$\langle (x+1)^7+2(x+1)^2+4(x+1)\rangle$	6
382	$\langle (x+1)^7+2(x+1)^3+4\rangle$	7
383	$\langle (x+1)^7+2(x+1)^3+4(x+1)\rangle$	4
384	$\langle (x+1)^7+2(x+1)^3+4(x+1)+4(x+1)^2\rangle$	5
385	$\langle (x+1)^7+2(x+1)^3+4(x+1)+4(x+1)^3\rangle$	4
386	$\langle (x+1)^7+2(x+1)^3+2(x+1)^4+4\rangle$	7
387	$\langle (x+1)^7+2(x+1)^3+2(x+1)^4+4(x+1)\rangle$	5
388	$\langle (x+1)^7+2(x+1)^3+2(x+1)^4+4(x+1)+4(x+1)^2\rangle$	5
389	$\langle (x+1)^7+2(x+1)^3+2(x+1)^6+4\rangle$	7
390	$\langle (x+1)^7+2(x+1)^3+2(x+1)^6+4(x+1)\rangle$	5
391	$\langle (x+1)^7+2(x+1)^3+2(x+1)^6+4(x+1)+4(x+1)^2\rangle$	4
392	$\langle (x+1)^7+2(x+1)^3+2(x+1)^5+4\rangle$	6
393	$\langle (x+1)^7+2(x+1)^3+2(x+1)^5+4+4(x+1)\rangle$	6

394	$\langle (x+1)^7+2(x+1)^3+2(x+1)^5+4(x+1)\rangle$	5
395	$\langle (x+1)^7+2(x+1)^3+2(x+1)^5+4(x+1)^2\rangle$	4
396	$\langle (x+1)^7+2(x+1)^3+2(x+1)^5+4(x+1)^2+4(x+1)^3\rangle$	4
397	$\langle (x+1)^7+2(x+1)^3+2(x+1)^5+4(x+1)^3\rangle$	3
398	$\langle (x+1)^7+2(x+1)^3+2(x+1)^5+4(x+1)^3+4(x+1)^4\rangle$	2
399	$\langle (x+1)^7+2(x+1)^3+2(x+1)^5+4(x+1)^4\rangle$	2
400	$\langle (x+1)^7+2(x+1)^3+2(x+1)^5+4(x+1)^4+4(x+1)^5\rangle$	2
401	$\langle (x+1)^7+2(x+1)^3+2(x+1)^5+4(x+1)^5\rangle$	2
402	$\langle (x+1)^7+2(x+1)^3+2(x+1)^6+4(x+1)+4(x+1)^2+4(x+1)^3\rangle$	4

CONCLUSION

Using the structure, generators and degree of minimal degree polynomials of a cyclic code of length 2^k over Z_8, all principal cyclic codes of length 2^k over Z_8 are determined along with their rank. It has been found that there are 402 principal cyclic codes of length 8 over $\mathbb{Z}_8$, out of which 8 are principal cyclic codes with leading coefficient 4, 67 are principal cyclic codes with leading coefficient 0,2 or 6 and 327 are principal monic cyclic codes.

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