International Journal of Enhanced Research in Science, Technology & Engineering
ISSN: 2319-7463, Vol. 4 Issue 7, July-2015

Development Augmented Lagrange Multiplier for Solving Constrained Optimization

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ABSTRACT

In this paper, we develop the algorithm of Augmented Lagrange Multiplier for solving non linear problem. The new algorithm is satisfies The global convergence and its prove effective when compared with other established algorithms in this filed.

Keywords: Nonlinear Constraint Optimization, Exterior-interior point, Penalty Method, Augmented Lagrange Multiplier Method.

1. INTRODUCTION

We consider nonlinear optimization problems of the form

$$Min\ f(x) \quad \text{…….(1)}$$

Subject to

$$h_j(x) = 0\ (j = 1,2, \dots \dots . J) \quad \text{…….(2)}$$

$$g_j(x) \geq 0 (j = 1,2, \dots . . m) \quad \text{…….(3)}$$

Where, $f_i(x)$,$h_j(x)$ and $g_k(x)$ are functions of the design vector

$$x = (x_{1,} x_2, \dots \dots \dots , x_n)^T \quad \text{…….(4)}$$

Here the components x_i of x are called design or decision variables, and they can be real continuous discrete or the mixed of these two.

The functions $f_i(x)$ where $i = 1,2, \dots \dots N$ are called the objective functions or simply cost functions ,and in the case of $N = 1$,there is only a single objective, the space spanned by the decision by the decision variables is called the design space or search space R^n,while the space formed by the objective function values is Called the solution space or response space , the equalities for h_j and inequalities g_j are called constraints. [1]

2. EQUALITY CONSTRINTS

Consider the following equality-constrained problem:

$$Minmize\ f(x) \quad \text{…….(5)}$$

Subject to

$$h_j(x) = 0\ (j = 1,2, \dots \dots . J) \quad \text{…….(6)}$$

Equality constraints are mathematically neat and easy to handle. Numerically, they require more effort to satisfy. They are also more restrictive on the design as they limit the region from which the solution can be obtained . The symbol representing equality constraints in the abstract model is h. there may be more than one equality constraint in the design problem .A vector representation for equality constraints is introduce though the following representation .[H],$[h_1, h_2, \dots . , h_J]$,and h_j:$j = 1,2, \dots . . , J$ are ways of identifying the equality constraint. The dependence on the design variable X is omitted for convenience. Note that the length of the vector is J. An important reason for distinguishing the equality and inequality constraints is that they are manipulated differently in the search for optimal solution. The number n of design variables in the problem must be greater than the number of equality constraints $l$for optimization to take place. If n equal to l,then the problem will be solved without reference to the objective . In mathematical terms the number of equations matches the number of unknowns. If n is less than J, then you have an over determined set of relations which could result in an inconsistent problem definition. The set of equality constraints must be linearly independent. Broadly, this implies that you cannot obtain one of the constraints from elementary arithmetic operation on the remaining constraints. This serves to ensure that the mathematical search for solution will not fail. These techniques are based on methods from linear algebra in the standard format for optimization problems. [2]

3. IN EQUALITY CONSTRINTS

Consider the following inequality-constrained problem:

$$Minmize\ f(x) \qquad \text{…….(7)}$$

Subject to

$$g_j(x) \geq 0 (j = 1,2, \ldots .. m) \qquad \text{…….(8)}$$

Inequality Constrains appear more naturally in problem formalization. Provide more flexibility in design selection. The symbol representing inequality constrains in the abstract model is g. There may be more than one inequality constrains in the design problem. The vector representation for inequality constrains is similar to what we have seen before. Thus [G],[$g_1, g_{2,\ldots,}g_m$] and $g_j = 1,2, \ldots., m$ are ways of identifying the inequality constrains. m represents the number of inequality constrains .All design functions explicitly or implicitly depend on the design (or independent) variable X. g is used to describe both less than equal to(≤) and greater than or equal to (≥) constrains. Strictly greater than (>) and strictly less than (<) are not used much in optimization because the solution are usually expected to lie at the constraint boundary, In the standard format, all problems are expressed with the ≤ relationship. Moreover, the right-hand side of the ≤ sign is 0. In the case of inequality constrains a distinction is made as to whether the design variable lie on the constraint boundary or in the interior of the region boundary the constraint. If the set of design variables lie on the boundary of the constraint, this expresses the fact that constraint is satisfied with strict equality, that is $g = 0$. The constraint acts like an equality constraint. In optimization terminology, this particular constraint is referred to as an active constraint .If the set design variables do not lie on the boundary, that is, they lie inside the region of the constraints, they are considered inactive constraints. Mathematically, the constraint satisfied the region g<0. An inequality constrains can therefore be either active or inactive [2]

4. EXTERIOR– INTERIOR – POINT ALGORTHM

The techniques we have been discussing are sometimes called exterior-point algorithm. If we limit our initial consideration to inequality constrained minimization problems in which the minimum is on a boundary, We recognize that our previous algorithm frequently starts outside(exterior to) the feasible region with an objective function below that of the constrained minimum .We are finally forced to accept a feasible solution with a higher value of the objective function .An alternative, "inside-out" approach to this problem would be the selection of an initial point that is feasible but has an objective function higher than the constrained minimum. A produce that dose this, and approaches the constrained minimum while maintaining feasibility, is called an interior –point algorithm [3]

5. METHOD OF LAGRANGE MULTIPLIERS

The method of Lagrange multipliers converts a constraint problem to an unconstrained

$$Min\ f(x) \qquad \text{…….(9)}$$

Subject to

$$h_j(x) = 0\ (j = 1,2, \ldots \ldots . J) \qquad \text{…….(10)}$$

$$g_j(x) \geq 0 (j = 1,2, \ldots .. m)$$

to reformulate the above problem as the minimization of the following function

$$L(x, \lambda, w) = f(x) + \sum_{j=1}^{J} \lambda_j \big(h_j(x)\big) + \sum_{j=1}^{m} w_j \big(g_j(x)\big) \qquad \text{…….(11)}$$

where $\lambda_j (j = 1,2, \ldots .. J)$, $w_j (j = 1,2, \ldots .. m)$ Lagrange multiples

the optimally requires that following stationary condition hold

$$\frac{\partial L}{\partial x_i} = \frac{\partial f}{\partial x_i} + \sum_{j=1}^{J} \lambda_j \frac{\partial g_j}{\partial x_i} \qquad i = 1,2, \ldots ., n \qquad \text{…….(12)}$$

And

$$\frac{\partial L}{\partial \lambda_j} = h_j = 0 \qquad j = 1,2, \ldots ., J \qquad \text{…….(13)}$$

$$\frac{\partial L}{\partial w_j} = g_j = 0 \qquad j = 1,2, \ldots ., m \qquad \text{…….(14)}$$

$J + m + n$ equation will determine the n components of x and J, m

Lagrange multiplier. As $\frac{\partial L}{\partial h_j} = \lambda_j, \frac{\partial L}{\partial g_j} = w_j$ we can consider λ_j, w_j as the rate of the equality and inequality $L(x, \lambda_j, w_j)$ As the function of h_j and g_j .[4]

6. PENALITY METHOD

For a non linear optimization problem with equality and in equality constrains, a common method of incorporating constrains is the penalty method. For the optimization problem

$$Min\, f(x) \quad \text{…….(15)}$$

Subject to

$$h_j(x) = 0 \; (j = 1,2, \dots \dots . J) \quad \text{…….(16)}$$

$$g_j(x) \geq 0 (j = 1,2, \dots . . m)$$

The idea to define a penalty function so that the constrained problem is transformed into an un constrained problem. Now we define

$$f(x, \mu_k) = f(x) + \mu_k \sum_{j=1}^{J} h_j^2(x) + \mu_k \sum_{j=1}^{m} max\,(0, g(x)) \quad \text{…….(17)}$$

where μ_k is the penalty parameter

7. THE AUGGMENTED LAGRANGE MULTIPLIERS METHOD (ALM)

combines the Lagrange multiplier and the penalty function methods.
This problem can be solved by combining the procedures of the two preceding sections. The augmented Lagrange function, in this case, is defined as:

$$A(x, \lambda, w, \mu_k) = f(x) + \sum_{j=1}^{J} \lambda_j\, h_j(x) + \sum_{j=1}^{m} w_j \alpha_j + \mu_k \sum_{j=1}^{J} h_j^2(x) + \mu_k \sum_{j=1}^{m} \alpha_j^2 \quad \text{…….(18)}$$

$$\alpha_j = Max \left\{ g_j(x), -\frac{w_j}{2\,\mu_k} \right\} \quad \text{…….(19)}$$

where μ_k is the penalty parameter .It can be noted the function A reduces to the Lagrange if $\mu_k = 0$ [1]

8. AUGGMENTED LAGRANGE MULTIPLIERS METHOD (PHRAUG)

The constraints defined by $h(x) = 0$ and $g(x) \geq 0$ will be include in the augment Lagrange defined. Given $x \in R^{n}$ $w \in R^{m}$,$w \geq 0$ we define the (Powell -Hestenes –Rockafellar).[5][6][7]
Augment given Lagrange by:

$$L(x, \lambda, w) = f(x) + \frac{\mu}{2}\left[\sum_{j=1}^{J} \left(h_j(x) + \frac{\lambda_j}{\mu}\right)^2\right] + \sum_{j=1}^{m} max\, \left\{0, g_j(x) + \frac{w_j}{\mu}\right\}^2 \quad \text{…….(20)}$$

PHP-like augmented Lagrange method are based on the iterative minimization of $L(x, \lambda, w)$ with respect to $x \in \Omega$ followed by convenient updates of w, λ, μ.[8]

9. OUTLINE OF THE STANDARD AUGGMENTED LAGRANGE ALGORITHM

- choose a tolerance $\varepsilon = 10^{-5}$ starting point x_0 =0 ,initial penalty parameter $\mu_0 = 1$ and initial Lagrange multipliers $\lambda_0 = 0$
- perform unconstrained optimization on the augment lagrangian function
- set $\lambda^{k+1} = \lambda^k + 2\,\mu_k\, h(x)$ and $w^{k+1} = w^k + 2\,\mu_k\, g(x)$
- Increase $\mu_{k+1} = 2\,\mu_k$ if $\|\lambda_{k+1} - \lambda_k\| < 0.5$
- check the convergence criteria. if $\|x_{k+1}^* - x_k^*\| < \varepsilon$ then stop. Otherwise, set $x_0 = x_k^*$ and return to step 2.

10. THE NEW MODIFIED OF AUGGMENTED LAGRANGE METHOD (MALM)

the basic idea is the same a small value of μ forces the minimize of L to lie closes to the feasible set, while, at the same time, values of x that reduce f are preferred. The advantage of the augmented Lagrangian approach is that by including an explicit estimate of the Lagrange multiplier, it is not necessary to decrease μ to zero in order to obtain convergence, and so various numerical problems are avoided. I will assume that x^* is a local minimizer and $\lambda^*$and w^* is the corresponding Lagrange multipliers. The new formal is defined:

$$A(x,y,\lambda,w) = f(x) - \sum_{j=1}^{J} \lambda_j\, h_j(x) + \sum_{j=1}^{J} \frac{1}{\mu_j}(h_j(x))^2 - \sum_{j=1}^{m} w_j\big(g_j(x) - y_j\big) + \sum_{j=1}^{m} \mu_j \frac{1}{g_j(x)-y_j} - \mu_j \frac{1}{y_j} \quad \text{…….(21)}$$

NMAL like augmented Lagrange methods are based on the iterative minimization of $AL(x,\lambda,w)$ with respect to $x\epsilon\Omega$ followed by convenient updates of λ,w and μ

$$\lambda^* = \lambda^* - \frac{2}{\mu} h(x) \quad \text{…….(22)}$$

$$w^* = w + \frac{\mu}{(g(x)-y)^2} \quad \text{…….(23)}$$

11. NEW ALGORITHM

- starting point x_0=0 ,initial penalty parameter $\mu_0 = 1$ and initial Lagrange multipliers λ_0, w_0
- set k=1 and compute $d_1 = -H_1 g_1$
- compute the vale unconstrained optimization on the augment Lagrange function by using the eq.(21)
- set $d_k = -H_1 g_1$ if update H_k by using BFGS
- set$x_{k+1} = x_k + \lambda_k d_k$
- check the convergence criteria. if $\|x^*_{k+1} - x^*_k\| < \varepsilon$ then stop and return to step8, Else update
  $\lambda^* = \lambda^* - \frac{2}{\mu} h(x)$
  $w^* = w + \frac{\mu}{(g(x)-y)^2}$
- set $x_0 = x^*_k$, $k = k+1$ and return to step

12. THE CONVERGENCE ANALISIS OF NEW MODIFIED AUGGMENTED LAGRANGE MULTIPLIER METHOD (NMALM)

The convergence analysis of the augmented Lagrange method is similar to that of the quadratic penalty method, but significantly more complicated because there are three parameters (λ;w;μ) instead of just one. As a straightforward generalization of the previous method, I can define.

$$F=\begin{bmatrix}\partial_x l\\ \partial_y l\\ \partial_\lambda l\\ \partial_w l\end{bmatrix}=\begin{bmatrix}\partial f(x^*) - \lambda\,\partial h(x^*) - w\partial g(x) + \frac{2}{\mu}\, h(x^*)\partial h(x^*) - \mu\frac{\partial g(x^*)}{(g(x^*)-y)^2}\\ w + \frac{\mu}{y^2} + \frac{\mu}{(g(x)-y)^2}\\ -h(x^*)\\ -(g(x^*)-y)\end{bmatrix} \quad \text{…….(24)}$$

and solve for $(x;\, \lambda_+)$, $(x;\, w_+)$ regarding λ and w and μ as parameters. First of all, assuming as usual that x^*,λ^*,w^* a local minimizer-Lagrange multiplier pair

$$F = \begin{bmatrix}\partial f(x^*) - \lambda\,\partial h(x^*) - w\partial g(x) + \frac{2}{\mu}\, h(x^*)\partial h(x^*) - \mu\frac{\partial g(x^*)}{(g(x^*)-y)^2}\\ w + \frac{\mu}{y^2} + \frac{\mu}{(g(x^*)-y)^2}\\ -h(x^*)\\ -(g(x^*)-y)\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix} \quad \text{…….(25)}$$

for all μ > 0. Moreover, the Jacobin of F (with respect to the variables x λ_+, w_+) is

$$J(x,\lambda_+,w_+,\lambda,w,\mu) = \begin{bmatrix}\partial_{xx} l & \frac{-2\mu\,\partial g(x)}{(g(x)-y)^3} & -\partial h(x) & -\partial g(x)\\ \frac{-2\mu\,\partial g(x)}{(g(x)-y)^3} & \frac{-2\mu}{y^3} + \frac{2\mu}{(g(x)-y)^3} & 0 & 1\\ -\partial h(x) & 0 & 0 & 0\\ -\partial g(x) & 1 & 0 & 0\end{bmatrix} \quad \text{…….(26)}$$

Assuming x^* is a nonsingular point of the NLP, the matrix

$$J(x^*\lambda^*w^*,\lambda,w,\mu)=\begin{bmatrix} \partial_{xx}l & \frac{-2\mu\,\partial g(x^*)}{(g(x^*)-y)^3} & -\partial h(x^*) & -\partial g(x^*) \\ \frac{-2\mu\,\partial g(x^*)}{(g(x^*)-y)^3} & \frac{-2\mu}{y^3}+\frac{2\mu}{(g(x^*)-y)^3} & 0 & 1 \\ -\partial h(x^*) & 0 & 0 & 0 \\ -\partial g(x^*) & 1 & 0 & 0 \end{bmatrix} \quad ....(27)$$

as μ→0. Therefore, there exists $\hat{\mu}>0$ such that $J(x^*\lambda^*w^*,\lambda,w,\mu)$ is nonsingular for all $\mu\in[0,\hat{\mu}]$.

The implicit function theorem1 then implies that there exists a neighborhood N of λ^* and w^* such that there exist functions x,λ_+ and x,w_+defined on N × [0;$\hat{\mu}$] such that

- $x(\lambda^*,\mu)=x^*, \lambda_+(\lambda^*,\mu)=\lambda^*$ for all $\mu\in[0,\hat{\mu}]$
- $x(w^*,\mu)=x^*, w_+(w^*,\mu)=w^*$ for all $\mu\in[0,\hat{\mu}]$

$f(x(\lambda,w,\mu),\lambda_+(\lambda,\mu),w_+(w,\mu),\lambda,w,\mu)=0$

Then The functions x,λ_+, w_+ satisfy

$$\partial f(x)-\lambda_+\,\partial h(x)-w_+\partial g(x)+\frac{2}{\mu}\,h(x)\partial h(x)-\mu\frac{\partial g(x)}{(g(x)-y)^2}=0 \quad ....(28)$$

$$w+\frac{\mu}{y^2}+\frac{\mu}{(g(x)-y)^2}=0 \quad ....(29)$$

$$-h(x)=0 \quad ....(30)$$

$$-(g(x)-y)=0 \quad ....(31)$$

By solving the eq.(28)

$$\partial f(x)=\lambda\,\partial h(x)+w\,\partial g(x)-\frac{2}{\mu}\,h(x)\,\partial h\partial(x)+\mu\,\frac{\partial h(x)}{(g(x)-y)^2} \quad ....(32)$$

$$\partial f(x)=\partial h(x)\left[\lambda-\frac{2}{\mu}\,h(x)\right]+\partial g(x)\left[w+\frac{\mu}{(g(x)-y)^2}\right] \quad ....(33)$$

We get on

$$\mu^*=\frac{\mu}{y}$$

$$\lambda^*=\lambda^*-\frac{2}{\mu}h(x)$$

$$w^*=w^*+\frac{\mu}{(g(x)-y)^2}$$

substituting this into (32) then produces

$$\partial f(x)-\lambda^*-\frac{2}{\mu}h(x)\;\partial h(x)-w^*+\frac{\mu}{(g(x)-y)^2}g(x)+\frac{2}{\mu}\,h(x)\partial h(x)-\mu\frac{\partial g(x)}{(g(x)-y)^2}=0 \quad ....(34)$$

Rearranging this last equation shows that

$\partial L\ (x\ (\lambda,w,\mu)\ )=0$ ....(35)

in other words, x(λ,μ) ,x(w,μ) a stationary point of L(x(λ,w,μ))for each $\lambda\in N, w\in N$ and each $\mu\in[0,\hat{\mu}]$.

Since

$$\partial^2 L=\partial^2 f-\lambda\,\partial^2 h(x)+\frac{2}{\mu}[h(x)\,\partial^2 h(x)+\partial h(x)\,\partial h(x)]-w\partial^2 g(x)\quad-\mu\frac{(g(x)-y)^2\partial^2 g(x)-\partial g(x)\;2(g(x)-y)\partial g(x)}{(g(x)-y)^4} \quad ....(36)$$

13. THEOREM

Suppose f : $R^n\rightarrow R$ and g : $R^n\rightarrow R^m$ are twice continuously differentiable and x*

is a local minimize of the NLP

$$Minmize\ f(x)$$

Subject to $h_j(x)=0\ (j=1,2,\ldots\ldots.J)$

$$g_j(x)=0,(j=1,\ldots..m) \quad ....(37)$$

If x^* is a nonsingular point and λ^* and w^* are the corresponding Lagrange multiplier, then there exists $\hat{\mu}>0$ and ∈ > 0 and a function $x: N\times[0,\hat{\mu}\,]\rightarrow R^n, N=B_\epsilon(\lambda^*,w^*)$ with the following properties:

- x is continuously differentiable;
- $x\ (\lambda^*,\mu)=x^*$ and $x\ (w^*,\mu)=x^*$ for all $\mu\epsilon[0,\hat{\mu}]$;
- $x(\lambda^*,\mu)$ and $x(w^*,\mu)$ is the unique local minimize of $L(\lambda,w,\mu)\ in\ N$;

However, since λ^* and w^* are unknown, the condition $\lambda\rightarrow\lambda^*, w\rightarrow w^*$ cannot be enforced directly. Instead, the augmented Lagrangian method updates λ using the results of the unconstrained minimization :$\lambda\leftarrow\lambda_+(\lambda;\mu)\ and$

$w \leftarrow w_+(w; \mu)$ It is necessary to prove, then, that updating λ and w in this manner produces a sequence of Lagrange multiplier estimates converging to λ^*, w^* Since λ_+, w_+ is a continuously differentiable function of λ,μ

and $\lambda_+(\lambda^*, \mu) = \lambda^*$, $w_+(w^*, \mu) = w^*$ I can write
$\lambda_+(\lambda, \mu) = \lambda^* + \int_0^1 \nabla \lambda_+(\lambda^* + t(\lambda - \lambda^*); \mu)^T (\lambda - \lambda^*) dt$
Using the triangle inequality for integrals, it follows that
$\|\lambda_+(\lambda, \mu) - \lambda^*\| \le \int_0^1 \|\nabla \lambda_+(\lambda^* + t(\lambda - \lambda^*), \mu)^T\| \, \|(\lambda - \lambda^*)\| dt \le c(\mu)\|(\lambda - \lambda^*)\|$ …….(38)
where C(μ) is an upper bound for . $\|\nabla \lambda + (.; \mu)^T\|$ Similarly
$w_+(w, \mu) = w^* + \int_0^1 \nabla w_+(w^* + t(w - w^*), \mu)^T (w - w^*) dt$
$\|w_+(w, \mu) - w^*\| \le \int_0^1 \|\nabla w_+(w^* + t(w - w^*), \mu)^T\| \, \|(w - w^*)\| dt \le D(\mu)\|(w - w^*)\|$ …….(39)
where D(μ) is an upper bound for $\|\nabla w_+(.; \mu)^T\|$ Similarly
$x(\lambda, \mu) = x^* + \int_0^1 \nabla x_+(\lambda^* + t(\lambda - \lambda^*), \mu)^T (\lambda - \lambda^*) dt$
$\|x(\lambda, \mu) - x^*\| \le \int_0^1 \|\nabla x_+(\lambda^* + t(\lambda - \lambda^*), \mu)^T\| \, \|(\lambda - \lambda^*)\| dt \le c(\mu)\|(\lambda - \lambda^*)\|$ …….(40)
where C(μ) is an upper bound for $\|\nabla x_+(\lambda, \mu)^T\|$
$x(w, \mu) = x^* + \int_0^1 \nabla x_+(w^* + t(w - w^*), \mu)^T (w - w^*) dt$
$\|x(w, \mu) - w^*\| \le \int_0^1 \|\nabla x_+(w^* + t(w - w^*), \mu)^T\| \, \|(w - w^*)\| dt \le D(\mu)\|(w - w^*)\|$ …….(41)
where D(μ) is an upper bound for $\|\nabla x_+(w, \mu)^T\|$
The function x, y_+, λ_+, w_+ are defined by the equation.
$\partial f(x) - \lambda_+ \partial h(x) - w_+ \partial g(x) + \frac{2}{\mu} h(x) \partial h(x) - \mu \frac{\partial g(x)}{(g(x) - y)^2} = 0$
$w + \frac{\mu}{y^2} + \frac{\mu}{(g(x) - y)^2} = 0$
$-h(x) = 0$
$-g(x) - y = 0$
Differentiating these equations with respect to y, λ and w and simplifying the results yields
$\partial_{xx} l \, \partial x^T - \frac{2\mu \, \partial g(x)}{(g(x) - y)^3} \partial y^T - \partial h(x) \partial \lambda_+^T - \partial g(x) \partial w_+^T = 0$
$\frac{-2\mu \, \partial g(x)}{(g(x) - y)^3} \partial x^T - \frac{2\mu}{y^3} + \frac{2\mu}{(g(x) - y)^3} \partial y^T = 0$
$-\partial h(x) \, \partial x^T = 0$
$-\partial g(x) \partial x^T - \partial y = 0$

$$J(x, y, \lambda, w) \begin{bmatrix} \nabla x(x, y, \lambda, w)^T \\ \nabla y(x, y, \lambda, w)^T \\ \nabla \lambda(x, y, \lambda, w)^T \\ \nabla w(x, y, \lambda, w)^T \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad …….(42)$$

$J(x, y, \lambda, w)$as $\lambda \to \lambda^*$ and $w \to w^*$, it follows that
$\|J(x, y, \lambda, w)^T\|$
is bounded above for all λ and w sufficiently close to λ^* and w^*. Therefore, from

$$\begin{bmatrix} \nabla x^T \\ \nabla y^T \\ \nabla \lambda^T \\ \nabla w^T \end{bmatrix} = J(x, y, \lambda, w)^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad …….(43)$$

I can deduce that there exist $\hat{\mu} > 0$ and $M > 0$ such that, for all $\mu \in (0, \hat{\mu})$

$\|\nabla x^T\| \le \mu M, \|\nabla \lambda^T\| \le \mu M$
$\|\nabla x^T\| \le \mu M, \|\nabla w^T\| \le \mu M$
Using μM in place of C(μ) and D(μ) above, I obtain
$\|\lambda_+(\lambda; \mu) - \lambda^*\| \le \mu M, \|\lambda - \lambda^*\|$
$\|x_+(\lambda; \mu) - x^*\| \le \mu M, \|\lambda - \lambda^*\|$
$\|w_+(w; \mu) - \lambda^*\| \le \mu M, \|w - w^*\|$
$\|x_+(w; \mu) - x^*\| \le \mu M, \|w - w^*\|$

RESULT AND CONCLUSION

Several standard non-linear constrained test functions were minimized to compare the new algorithm with standard algorithm see (Appendix, A) with 1<n<3 and 1<$g_i(x)$ <4 and 1<$h_i(x)$ < 2.

This paper includes two parts. Is considered as the comparative performance of the following algorithm:

- mixed Equality-Inequality- constrained problem of the augment Lagrange method (MAXAUG)
- New modified Augmented Lagrange method mixed (Equality-Inequality- constrained) problem with Lagrange method (New MAXAUG) Augmented Lagrange method

All the result are obtained using (Laptop).All programs are written in visual FORTRAN language and for all cases the stopping criterion taken to be $\|x_i - x_{i-1}\| < \varepsilon$
All the algorithms in this paper use the same ELS strategy which is the quadratic interpolation technique .
The comparative performance for all of these algorithms are evaluated by considering NOF,NOG,NOI and NOC where NOF is the number of function evolution and NOI is the number of iteration and NOG is the number of gradient evolution and NOC number of constrained evolution .
In table (1) we have compared of two algorithms (MAXAUG) and (New MAXAUG).

Table (1)

Test fn.			**MAXAUG**			**New MAXAUG**		
	NOF	**NOG**	**NOI**	**NOC**	**NOF**	**NOG**	**NOI**	**NOC**
1-	165	8	2	1	585	39	2	2
2-	806	52	2	1	792	76	3	3
3-	248	5	2	1	128	3	2	1
4-	41	2	2	1	170	5	3	1
5-	469	43	3	3	462	36	3	2
6-	719	52	5	9	692	46	3	3
7-	550	37	2	1	423	33	2	1
8-	595	44	2	1	507	39	3	3
9-	45	5	5	7	33	2	2	1
10-	202	24	5	9	347	32	2	1
11-	633	40	5	9	466	39	2	1
12-	163	3	2	1	39	5	2	1
13-	157	8	2	1	149	7	2	1
14-	33	2	2	1	93	3	2	1
15-	312	6	2	1	210	4	3	2
T.O	5156	331	43	47	5033	369	36	25

14. Appendix

1-$min f(x) = (x_1 - 3)^2 + (x_2 - 2)^2$
s.t
$x_1^2 - x_2^2 + 5 = 0$
$x_1 + 2x_2 - 4 \leq 0$

2- $min f(x) = x_1^2 - x_1 x_2 + x_2^2$
s.t
$x_1^2 + x_2^2 - 4 = 0$
$2x_1 - x_2 + 2 < 0$

3- $min f(x) = (x_1 - 2)^2 + (x_2 - 1)^2$
s.t
$x_1 - 2x_2 = -1$
$-\frac{x_1^2}{4} + x_2^2 + 1 \geq 0$

4- $min f(x) = x_1^3 + 2x_2^2 x_3 + 2x_3$
s.t
$x_1^2 + x_2 + x_3^2 = 4$
$x_1^2 - x_2 + 2x_3 \leq 2$

5- $min f(x) = e^1 - x_1 x_2 + x_2^2$
s.t
$x_1^2 + x_2^2 = 4$
$2x_1 + x_2 \leq 2$

6- $minf(x) = x_1^3 - 3x_1x_2 + 4$
s.t
$-2x_1 + x_2^2 = 5$
$5x_1 + 2x_2 \geq 18$

7- $minf(x) = -e^{-x_1-x_2}$
s.t
$x_1^2 + x_2^2 - 4$
$x_1 - 1 \geq 0$

8- $minf(x) = -x_1^2 + 2x_1x_2 + x_2^2 - e^{-x_1-x_2}$
s.t
$x_1^2 + x_2^2 - 4 = 0$
$x_1 + x_2 \leq 1$

9- $minf(x) = -x_1x_2x_3$
s.t
$20 - x_1 \geq 0$
$11 - x_2 \geq 0$
$42 - x_3 \geq 0$
$72 - x_1 - 2x_2 - 2x_3 \geq 0$

10- $minf(x) = (x_1 - 1)^2 + x_2 - 2$
s.t
$x_2 - x_1 = 1$
$x_1 + x_2 \geq 2$

11- $minf(x) = {x_1}^2 + {x_2}^2$
s.t
$x_1 - 3 = 0$
$x_2 - 2 \leq 0$

12-$minf(x) = \frac{1}{4000}({x_1}^2 + {x_2}^2) - \cos\left(\frac{x_1}{\sqrt{1}}\right) cos\left(\frac{x_2}{\sqrt{2}}\right) + 1$
s.t
$x_1 - 3 = 0$
$x_2 - 2 \leq 0$

13- $minf(x) = (x_1 - 2)^2 + \frac{1}{4}{x_2}^2$
s.t
$2x_1 + 3x_2 = 4$
$x_1 - \frac{7}{2}x_2 \leq 1$

14- $minf(x) = -x_1\, x_2$
s.t
$20x_1 + 15x_2 - 30 = 0$
$\frac{{x_1}^2}{4} + {x_2}^2 - 1 \leq 0$

15- $minf(x) = {x_1}^4 - 2{x_1}^2x_2 + {x_1}^2 + x_1\,{x_2}^2 - 2x_1 + 4$
s.t
${x_1}^2 + {x_2}^2 - 2 = 0$
$0.25\,{x_1}^2 + 0.75{x_2}^2 - 1 \leq 0$

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