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Effect of Linear Interaction and Generation of Sideband in the Magnetosphere

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Abstract: Common view is that non-linear interaction between whistler mode signals from a VLF transmitter and energetic electrons can successfully explain the generation of sideband spacings of 1-30 kHz in the magnetosphere. However, VLF transmitter experiments conducted at Siple show that long keydown signals injected into the magnetosphere often generate sidebands as a result of nonlinear interaction with energetic particles. The spectral characteristics of observed sidebands are quite varied from ~ 2 to 100 Hz, but it bears no simple relationship to the carrier amplitudes. It is shown that such small spacing need wave magnetic field amplitude of 1.5 pT, which is lower than threshold for non-liner amplification. The results indicate that linear amplification is able to explain such sideband spacings.

Keywords: ELF/VLF, Sideband spacing, Trapped particles, Wave-particle interaction, VLF Emissions.

Introduction

Wave-particle interactions (WPI) between whistler mode ELF/VLF waves and energetic electrons play an important role in many magnetospheric processes such as charged particle precipitation, X-ray generation, D-and E-region perturbations, auroral display as well as light/heating. A lot of work has been done in this field by Stanford, University group using natural whistlers generated by lightning discharges and ELF/VLF/plasmaspheric hiss besides man-made signals. Siple Station VLF transmitter signals have been utilised since 1973 to probe Earth's magnetosphere plasma. For a long time, Siple experiments have been devoted to describe various properties of the received signals as well as their interpretation in terms of whistler-mode signal path location, propagation characteristics and other magnetospheric processes. The phenomenon of side band generation is the result of WPI in the magnetosphere and this can be observed in transmitter signals received aboard satellites or at recording stations on the ground. Assuming both static and dynamic magnetic fields occurring over the time scales of interest, experimental and theoretical methods have been adopted for studying wave-particle interaction (WPI) in the magnetosphere. In order to investigate the effect of dynamic rnagnetospheric processes on

WPT, the response of the interaction mechanism to time dependent perturbations in the magnetic field has been tested using global field compressions. The applicability of linear or quasi-linear, and nonlinear amplification mechanisms has been examined. Dowden et al. have studied both linear and non-linear amplification mechanisms using spectrograms / phasograms of 6.6 kHz signals which were transmitted from Anchorage, Alaska by means of a transportable (TVLF) station comprising of VLF transmitter and balloon-lofted antenna, and received in the conjugate areas located at Dunedin, New Zealand and Campbell Island. The experiment was performed in August-September 1973 and radiated power of transmitted signals was 93 watt. Such results have been analysed, explaining the differences between linear and nonlinear mechanisms as: (i) Linear amplification mechanism is most obvious when long trains of whistlers are observed sometimes with increasing amplitude in successive hops. Mid-latitude hiss appears to be a consequence of amplification gained over several hops. Measurements using artificial VLF signals indicate amplification of up to 25dB/hop. Sometimes, fairly narrow band (~1 kHz) linear amplification is frequency dependent and varies over period of several minutes. However, an essential feature of a linear mechanism is that the received signal is directly proportional to the input signal.

(ii) In non-linear amplification, the output (from the magnetospheric amplifier) amplitude is not related to the input amplitude provided it is above some threshold. For nonlinear amplification, input wave magnetic field amplitude (Bw) has to be above threshold of around 5 pT as reported by Inan and Juan et al. The output amplitude may be very large, but even when it is not; the distinguishing feature of the non-linear amplification mechanism is that the output amplitude takes a finite time to grow. Sometimes this is the only obvious feature; it appears as slowness in the response to a transmitter pulse or amplitude modulation or phase reversal modulation. The response to a pulse of adequate length (~1s) is as follows:

(a) Initially or for the first 0.1s (approx), the output wave grows quasi-exponentially with an amplification of up to100 dB/s. This is followed by rapid amplitude fluctuations which are sometimes.

(b) At the pulse end (and sometimes before the pulse end), the output wave may continue as a self-sustaining emission and so emissions produced by the transmitter pulse are known as artificially stimulated emission (ASE). Well before the stage of free emission, the output wave appears to be at a discernibly higher frequency than the transmitted (eventually ~ 100 Hz).

Method for computation of sideband Spacing

Ikeda studied the possibility of sideband generation in the whistler mode via a non-linear Doppler-shifted cyclotron interaction between energetic electrons and the whistler mode carrier signal. The energetic electrons resonate with the quasi monochromatic whistler mode signal to generate sidebands as well as broaden the transmitted carrier frequency. He derived the following formula for sideband spacing for $\omega_h > \omega$, where, ω_h, is the angular cyclotron frequency of energetic electrons in the Earth's magnetic field, and ω, the wave angular frequency:

$$F_{spacig} = 0.063n.\sqrt{(k.V_{\perp.}\Omega_{BW}} \tag{1}$$

Where, n, is the order of sideband generation; k, the wave number; $V_{\perp.}$ perpendicular speed of the resonant electrons; and Ω_{BW} (= e.Bw/m), the angular frequency of wave (trapping) magnetic field (also known as wave gyrofrequency), e and m are electronic charge and mass, respectively. The wave number (k) can be calculated as:

$$k = \omega.\mu / c \tag{2}$$

where, c, is the speed of light and the refractive index of the medium, which for the whistler mode can be evaluated as:

$$\mu = \surd\{\omega_p^2 / \omega(\omega_h - \omega)\} \tag{3}$$

where, ω_p, is the plasma frequency of the electrons. The resonant velocity of the electrons (V_R can thus be calculated as:

$$V_R = c.(\omega_h - \omega)^{3/2}(\omega_{p.}\ \surd\omega) \tag{4}$$

The electron density [N(e)] required to calculate the angular plasma frequency in the equatorial plane of the L shell considered at L = 4.23 (where the duct between Siple and Roberval was found to be located26) was taken to be 313 el.cm^{-3}. This value corresponds to that obtained using the diffusive equilibrium mode1 which has been used earlier by Singh and Singh and Singh et al. The energetic electron gyrofrequency f_H $(= \omega_H / 2\pi)$ at the equator on a field line specified by L can be computed from following equation for a dipolar geomagnetic field:

$$f_H(\text{in kHz}) = 873.6/L^3$$

Results and Discussion

As the experiment conducted from Siple had two interacting signal frequencies of 4.02 and 4.44 kHz $(f = \omega / 2\pi)$, these frequencies were chosen for calculations. The values of the sideband spacing were calculated at not only 5 pT (believed to be the minimum wave magnetic amplitudel9 in the equatorial plane to cause non-linear amplification) but also at 1 and 3 pT. Equation 1 clearly indicates that minimum spacing occurs for n = 1 and low pitch angles (a) since $V_\perp$ = V Sinα; only those pitch angles for which $a \geq a_0$, where a_0 is the edge of loss cone (i.e. a_0 is the half loss cone pitch angle) will contribute to wave growth hence, $a = 15^0, 25^0$ and 35^0 is adopted. Table I shows the $F_{spacing}$ at two considered frequencies. It is clear from the Table that as pitch angle increases, the sideband spacing increases, but decreases as the frequency increases. It is also inferred from the Table 1 that for generation of a 2 Hz sideband spacing one needs:

(i) Wave magnetic amplitude (B_w) to be around/below 3 pT, and
(ii) Resonant electrons having pitch angle values lower than 15^0.

As shown by Park, the duct was located at L = 4.23. At this location, the loss cone pitch angle, a_0, is found to be 5.03^0. Figure 1 depicts variation of $F_{spacing}$ with wave amplitude (B_w) for $\alpha = \alpha_o = 5.03^0$. It is evident from fig. 1 & 2, that 2Hz spacing will be generated by $B_w < 1.5$ pT at different frequencies which is suggested that these spacings were generated by linear cyclotron resonance between the transmitted VLF signals and energetic electrons with pitch angles close to the edge of the loss cone. The present prediction can be checked using a linearity test. Inan and Man et al have utilized this test to discuss non-linear wave-particle interactions between energetic electrons and whistler mode waves taking place in the equatorial plane of magnetosphere of the 60° geomagnetic field line (L = 4).

Cornilleau-Wehrlin & Gendrin and Raghuram et al have shown that to explain the generation of sidebands associated with received VLF signals at L = 3.8, the wave magnetic amplitude should be of the order of 4-5 pT and this phenomenon cannot be explained by the linear mechanism. Whereas these workers have done the calculations at $\alpha = 45^0$, in the present study pitch angle was considered just equal to equatorial half loss cone pitch angle ($\alpha \sim \alpha_o$) as it is the minimum pitch angle required for wave growth ($\alpha \geq \alpha_o$). The methods of Inan and film et al were followed in the present study which considers:

(i) Ratio of resonance and trapping times

Resonance time (t_R) is the time taken by the energetic electron in traversing the resonant/interaction region (Z), close to the equator, i.e.

$$t_R = Z/V_R \ldots \tag{6}$$

The trapping time (t_T) for which electrons is trapped in the potential well of the wave can be expressed as:

$$t_T = 2\pi / \omega_T \tag{7}$$

with

$$\omega_T = \sqrt{(k.V_{\perp.}\Omega_{BW})} \tag{8}$$

The t_R and t_T values at L = 4.23 for a transmitted frequency of 4.02 kHz are found to be 23 and 190 ms, respectively producing the ratio of t_R and t_T to be 0.12. For the case of transmitted frequency of 4.44 kHz, these values are 25 and 195 ms, respectively producing the ratio of these two parameters as 0.13. According to Inan et al the ratio t_R / t_T should be greater than 1 for non-linear amplification but in the present case it is much below 1 indicating a linear interaction is taking place in generating 2 Hz side band spacing. Interaction length (Z) depends upon a parameter β, which is function of loss cone pitch angle (α_0), wave frequency and electron's gyrofrequency and is given as:

$$\beta = 1.5 + [\tan^2 a_0 (\omega_H - \omega_H) / 2\omega_H \tag{9}$$

Useful parameters related to this test of linearity are provided in Table 2.

(ii) Evaluation of linearity parameter 'Nark et al have provided a parameter (p) based on wave and geomagnetic field inhomogeneity values which can be adopted to test whether linear processes are taking place or not. This parameter depends upon k, tan $a, \Omega_{BW}, and \partial\omega_H / \partial_Z$, where

$$\partial\omega_H / \partial_Z = 9.\omega_H (Z / LR)^2 \tag{10}$$

R is the geocentric distance of the interaction region. It was found that value of this parameter, p is an order of magnitude less than the threshold for non-linear processes.

Figures and Tables

Table 1: Sideband spacing ($F_{spacing}$, Hz) computed at four different pitch angles (15^0,25^0,35^0, 40°) and signals of 4.02 and 4.44 kHz. The wave magnetic amplitude (Bw) is taken to be 1,3,5 pT

	f= 4.02kHz			f=4.44kHz		
B_w	1	3	5	1	3	5
15^0	3.0	5.2	6.7	2.9	5.0	6.5
25^0	3.9	6.8	8.8	3.8	6.6	8.6
35^0	4.8	8.4	10.8	4.7	8.1	10.5
40^0	5.7	10	12.9	5.6	9.6	12.4

Table 2: Various data used in the calculation of number of Trap Oscillation (N).

L = 4. 23
n = 313electrons/cm^3
f_H = 11.54 kHz
f = 4.02 kHz & f = 4.44 kHz
k = 2. 43(10^{-3}) m^{-1} , k = 2. 63(10^{-3}), m^{-1}
μ = 28.9 , μ = 28.3
V_R = 1. 94(10^7) m/s, V_R = 1. 69(10^7)m/s
Ω(B_w) = 0. 2637 rad/s, Ω(B_w) = 0. 2637 rad/s
L_{int} = 45. 830 km L_{int} = 49. 695 km
ωT = 33.11 rad/s ωT = 32.16 rad/s
Z= 442 & 422 km
β = 1.5

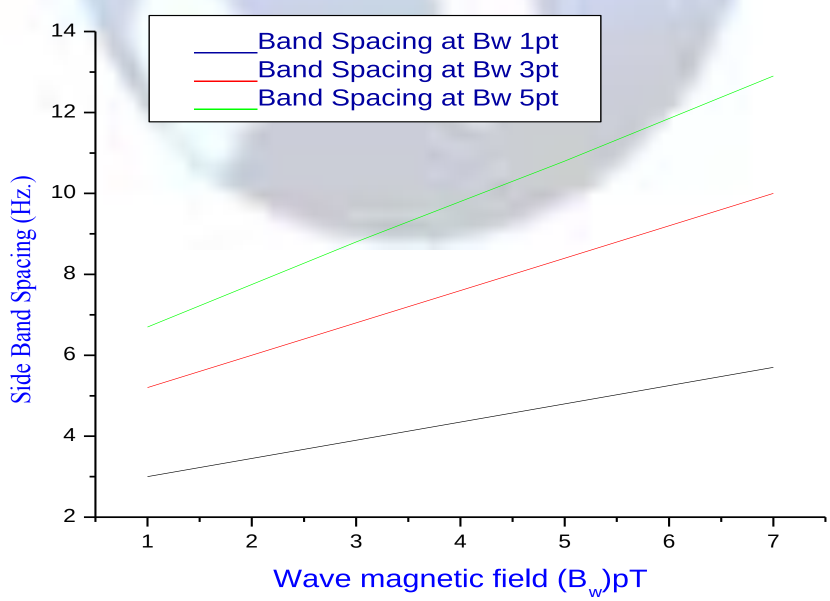

Fig. 1. Variation of sideband spacings (Hz) with wave magnetic amplitude Bw (pT) at frequency 4.02kHz.

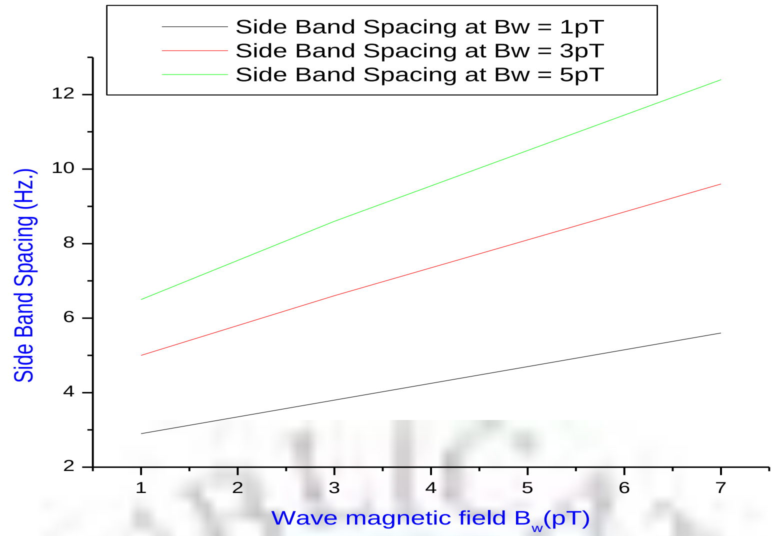

Fig. 2. Variation of sideband spacings (Hz) with wave magnetic amplitude Bw (pT) at frequency 4.44kHz.

Conclusion

Controlled transmitter experiments show that sideband generation is a fairly common phenomenon in the magnetosphere. Since sideband spacing is only a small fraction of the carrier frequency (typically ~ 1%), especially high resolution is needed to identify sidebands. For this reason, no statistical description of the occurrence of sideband is available at present. However, based on the selected examples illustrated above as well as other studies not included in this paper, it is estimated that perhaps up to 50% or more of magnetospherically amplified transmitter signals lasting more than ~1 s show evidence of sideband structure. Many theoretical analyses also indicate that sideband generation is an unavoidable consequence of nonlinear wave-particle interactions.

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