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Connected Domination on Linear and Double Hexagonal Chains

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Abstract: Let G be connected graph with vertex set V (G) and the edge set E (G). A subset D of G is called a connected dominating set if every vertex v ϵ V – D is adjacent to at least one vertex in D and the induced subgraph < D > is connected. The connected domination number $\gamma_C(G)$ is the minimum cardinality taken over all connected domination sets of G.

Keywords: connected dominating set, linear hexagonal chain, double hexagonal chain.

1. Introduction

Let G = (V (G), E (G)) be a simple, undirectedand nontrivial graph without isolated vertices. A set $D \subseteq V(G)$is called a connected dominating set if every vertex v ϵ V – D is adjacent to at least one vertex in D and the induced subgraph< D > is connected. The connected domination number $\gamma_C(G)$is the minimum cardinality taken over all connected domination sets of G. Hexagonal systems are geometric objects obtained by arranging mutually congruent regular hexagons in the plane. They are of considerable importance in theoretical chemistry because they are natural graph representation of benzenoid hydrocarbons [1]. Each vertex in hexagonal system is either of degree two or of degree three. Vertex shared by three hexagons is called an internal vertex of the respective hexagonal system. We call hexagonal system catacondensed if it does not possess internal vertices, otherwise we call it pericondensed. A hexagonal chain is a catacondensed hexagonal system in which every hexagon is adjacent to at most two hexagons. Linear hexagonal chain is hexagonal chain which is a graph representation of linear polyacene. The linear hexagonal chain with h hexagons will be denoted by B_h. A double hexagonal chain consists of 2 condensed identical hexagonal chains. Since chemical structures are conveniently represented by graphs, where atoms correspond to vertices and chemical bounds correspond to edges, many physical and chemical properties of molecules are well correlated with graph theoretical invariants.In this paper we discuss the connected domination on linear hexagonal chain $B_h$that is illustrated in figure 1, and on double hexagonal chain B_{2h} which is represented in figure 2.

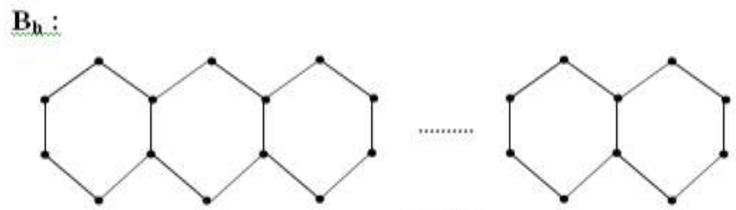


Figure 1

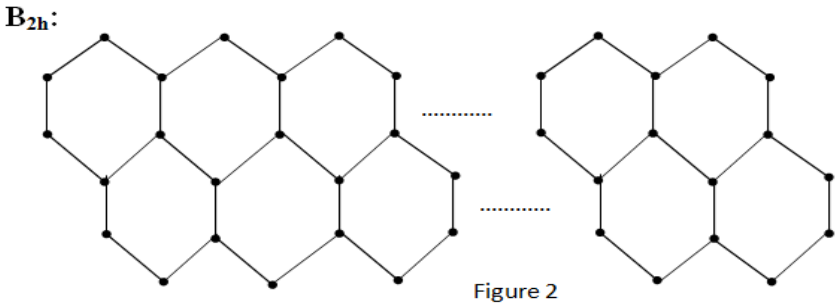


Figure 2

For isomorphic graphs the connected domination number is equal. Therefore, each hexagon will be represented with its isomorphic graph that is illustrated in figure 3.

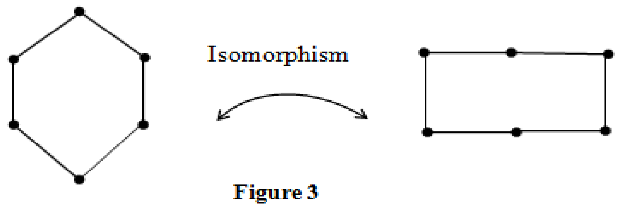
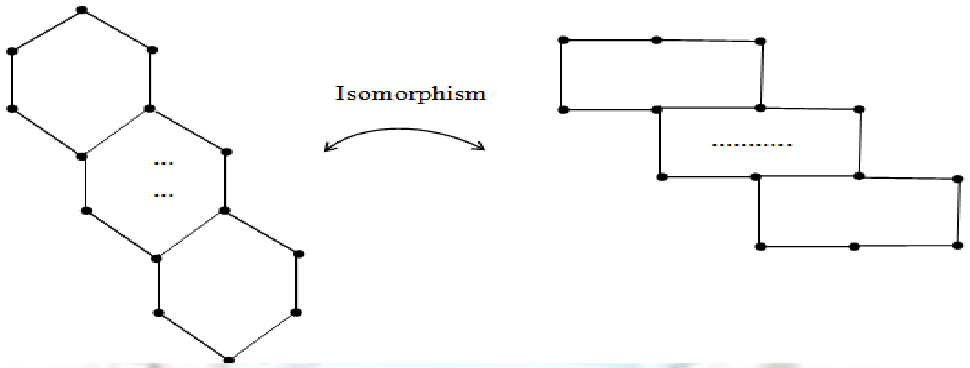
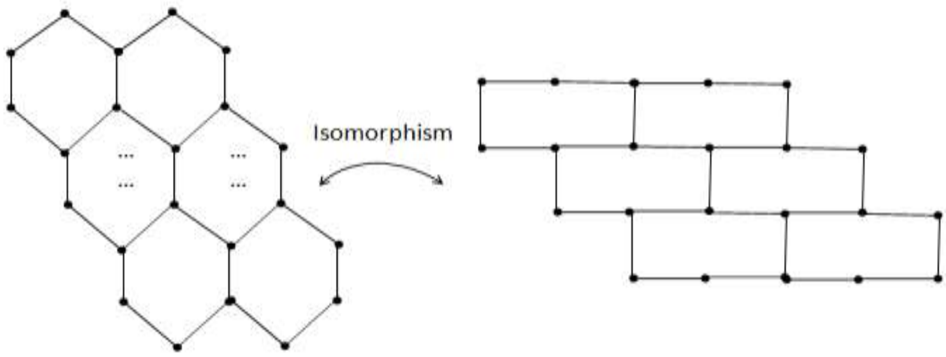
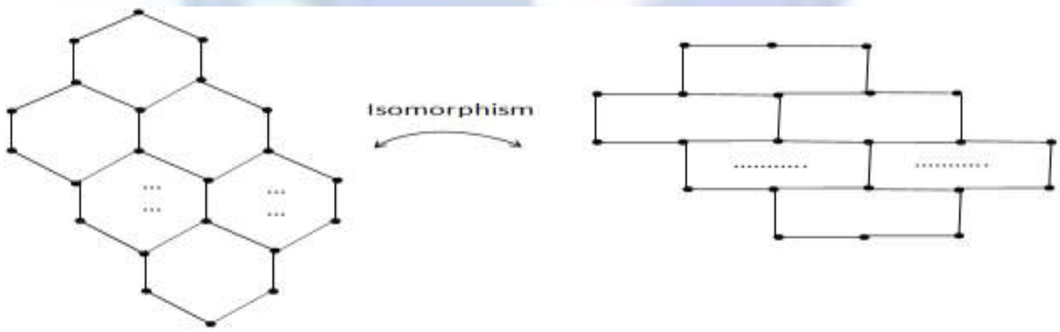


Figure 3

Various isomorphic representations of B_h and B_{2h} are illustrated in the following figures:







2. Connected Domination Number on Linear Hexagonal Chains

LetB$_h$be the linearhexagonal chain with h hexagons represented in figure 1. The structure in figure 1 is isomorphic with the one illustrated in figure 4.

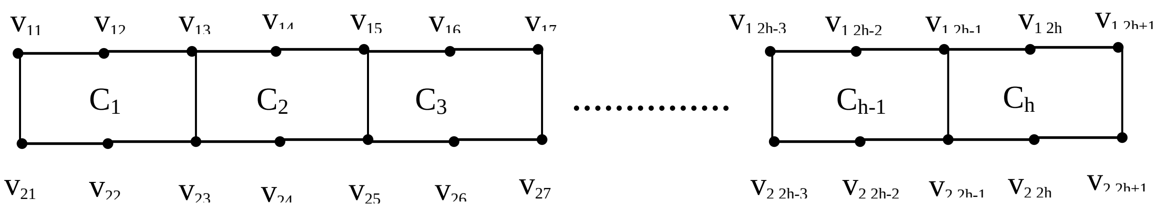


Figure 4

Theorem 2.1: Let B_h be the linear hexagonal chain with h hexagons. Then $\gamma_C(B_h) = \begin{cases} \frac{5h+2}{2} & \text{if h is even} \\ \frac{5h+3}{2} & \text{if h is odd} \end{cases}$

Proof: Let C_1, C_2, C_3,……C_h be the cycles of B_h represented in figure 3. Let D be any minimum connected domination set of B_h.
For dominating the vertices of C_1, D must contain at least 4 vertices in which two vertices are common with C_2. For dominating the vertices of C_2, D must contain at least two more vertices of C_2 in which one of the vertex lies on both C_2 and C_3. For dominating the vertices of C_3, D must contain at least three vertex in which two vertices are common to C_3 and C_4. For dominating the vertices of C_4, D must contain at least two more vertex in which one of the vertex lies on both C_4 and C_5.

Proceeding like this, it is clear that D must contain at least 4 vertices of C_1, 2 vertices of C_2 not on C_1, 3 vertices of C_3 not on C_2, 2 vertices of C_4 not on C_3, 3 vertices of C_5 not on C_4,………………,

2 vertices of C_h not on C_{h-1} if h is even.

Therefore, $|D| = 4 + 2\left(\frac{h}{2}\right) + 3\left(\frac{h-2}{2}\right) = \frac{5h+2}{2}$

If h is odd, from the last cycle of C_h, D must contain 3 vertices of C_h not on C_{h-1}, therefore

$|D| = 4 + 2\left(\frac{h-1}{2}\right) + 3\left(\frac{h-1}{2}\right) = \frac{5h+3}{2}$

The diagrammatic representation of $\gamma_C(B_h)$ is in figure 5.

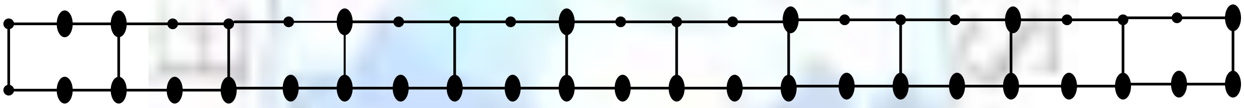


Figure 5

Therefore, $\gamma_C(B_h) = \begin{cases} \frac{5h+2}{2} & \text{if h is even} \\ \frac{5h+3}{2} & \text{if h is odd} \end{cases}$

3. Connected Domination Number on Double Hexagonal Chains

Let B_{2h} be the double hexagonal chain with h hexagons on each linear chain which is represented in figure 2. The structure in figure 2 is isomorphic with the one illustrated in figure 6.

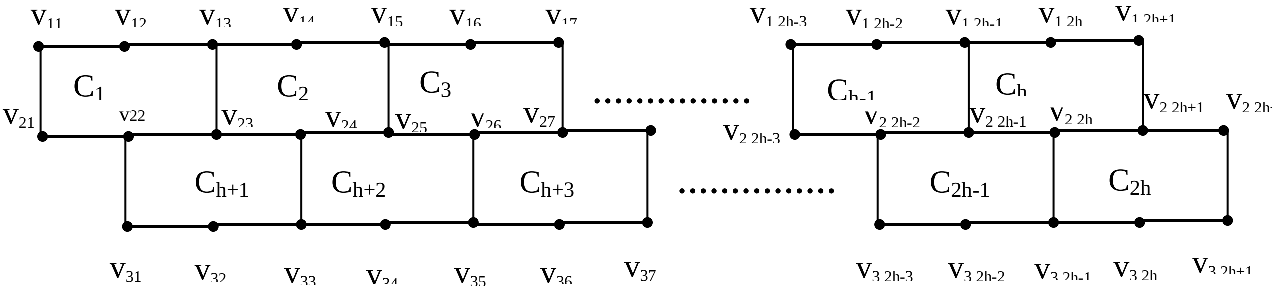


Figure 6

Theorem 3.1. Let B_{2h} be the double hexagonal chain. Then $\gamma_C(B_{2h}) = \begin{cases} 3h+2 & \text{if h is even} \\ 3h+3 & \text{if h is odd} \end{cases}$

Proof:
Let $C_1, C_2, C_3, \ldots\ldots, C_h$ be the cycles of the first layer and $C_{h+1}, C_{h+2}, \ldots\ldots, C_{2h}$ be the cycles of the second layer. Let D be any minimum connected domination set of B_{2h}.

For dominating the vertices of C_1, D must contain at least 4 vertices in which two vertices are common with C_2 and C_{h+1}. For dominating the vertices of C_2, D must contain at least two more vertices of C_2 in which one of the vertex lies on both C_2 and C_{h+1}. For dominating the vertices of C_{h+1}, D must contain one more vertex which is common to C_{h+1} and C_{h+2}. For dominating the vertices of C_{h+2}, D must contain one more vertex in C_{h+2} which is common to C_3.

Proceeding like this, it is clear that for the first layer D must contain at least 4 vertices of C_1, 2 vertices of C_2 not on C_1, 2 vertices of C_3 not on C_2, 2 vertices of C_4 not on C_3, ,………………,
2 vertices of C_h not on C_{h-1}.

For the second layer of cycles, D must contain at least one vertex of each cycle if h is even. If h is odd, D must contain at least one vertex of each cycle except the last cycle C_{2h}. For dominating the vertices of C_{2h}, D must contain at least two vertices of C_{2h}.

Therefore, $|D| = 4 + 2(h-1) + h = 3h + 2$ if h is even and $|D| = 4 + 2(h-1) + h - 1 + 2 = 3h + 3$ if h is odd
The diagrammatic representation of $\gamma_C(B_{2h})$ is in figure 7.

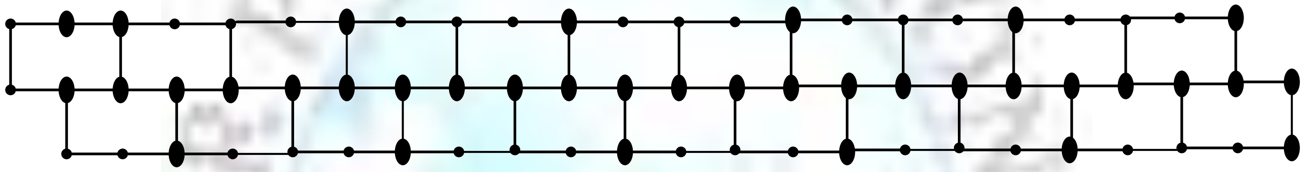


Figure 7

Therefore, $\gamma_C(B_{2h}) = \begin{cases} 3h+2 & \text{if h is even} \\ 3h+3 & \text{if h is odd} \end{cases}$.

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