

# Effect of Switching Angle on Magnitude of Inrush Current and its Second Harmonic of Power Transformer

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## ABSTRACT

The study of Inrush Current in Power Transformer is very essential, because it causes unwanted/ false operation of differential relay. The magnitude of Inrush current and its harmonics depend on the instant angle at which transformer is switch on. In this paper by using Simulink model of transformer we found the increasing/decreasing trend of magnitude of inrush current and its second harmonic.

**Keywords:** Inrush current, Second Harmonics, Magnetizing Flux.

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## 1. INTRODUCTION

Literally “Inrush” means “that rushes in”

An inrush current is the surge of transient current that rushes in a transformer when transformer is energized. These currents are of high magnitude, harmonic-rich generated when transformer cores are driven into saturation. According to Faraday’s law of Electromagnetic Induction the voltage induced across the winding is given as

$$e = Nd\phi/dt, \text{ where } \phi \text{ is the flux in the core.}$$
$$e = E \sin \omega t = d\phi/dt \Rightarrow \phi = \int e \cdot dt = E \int \sin \omega t \cdot dt$$

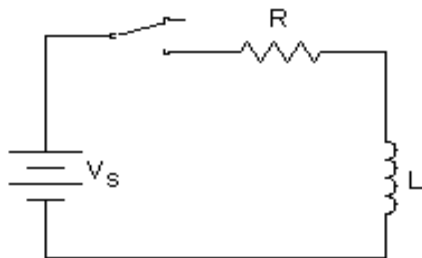
Hence the flux will be integral of the voltage wave. If the transformer is switched on at the instant of voltage zero, the flux wave is initiated from the same origin as voltage waveform, the value of flux at the end of first half cycle of the voltage waveform will be,

$\phi'_m = \left(\frac{E}{\omega}\right) \int_0^\pi \omega \sin \omega t \cdot dt = \phi_m \int_0^\pi \sin \omega t \cdot d(\omega t) = 2\phi_m$ , where  $\phi_m$  is the maximum value of steady state flux. That means flux become double to its maximum value. The transformer core is generally saturated just above the maximum steady state value of flux. But during switching on the transformer the maximum value of flux will jump to double of its steady state maximum value. As, after steady state maximum value of flux, the core becomes saturated, the current required to produced rest of flux will be very high. So transformer primary will draw a very high peaky current from the source which is called magnetizing inrush current in transformer or simply inrush current in transformer.

A harmonic is a signal or wave whose frequency is an integral (whole-number) multiple of the frequency of some reference signal or wave. The term can also refer to the ratio of the frequency of such a signal or wave to the frequency of the reference signal or wave.

## 2. EXPRESSION OF TRANSIENT CURRENT IN TRANSFORMER

Transformer at no load can be considering as series R-L circuit.



**Figure (1): Transformer at no load**

Consider the application of a sinusoidal voltage to a transformer that is modelled very simply as a coil of  $N$  turns to which a sinusoidal voltage is suddenly applied by closing a switch.

Let the applied voltage given by

$$V_s = V_m \sin(\omega t + \alpha)$$

When the switch is closed we can write the voltage equation by KVL as

$$V_R + V_L = 0$$

$$R_i + L \frac{di}{dt} = V_m \sin(\omega t + \alpha)$$

Where,  $i$  is the instantaneous current

The above equation can be solved by a number of methods. The solution consists of two parts, namely

$$i = i_n + i_f$$

In this equation  $i_n$  is called the natural response and is the general solution of the homogeneous equation (input set to zero). The component  $i_f$  is called the forced response and is a particular solution of equation.

From circuit theory,

$$i_n = K e^{-\frac{R}{L}t}$$

Where  $k$  is an arbitrary constant to be determined from initial conditions. The forced response is given by

$$i_f = \frac{\vartheta_m}{Z} \sin(\omega t + \alpha - \phi)$$

Where

$$Z = \sqrt{R^2 + (\omega L)^2}$$

$$\phi = \tan^{-1} \frac{\omega L}{R}$$

The complete solution (total response) is therefore given by

$$i = i_n + i_f = K e^{-\frac{R}{L}t} + \frac{\vartheta_m}{Z} \sin(\omega t + \alpha - \phi)$$

The initial conditions are applied to the total response. In our case the initial conditions are those existing at the time of closing the switch. Since the current is just before closing the switch, the initial conditions are

$$i = 0 \text{ At } t = 0$$

Putting these values of  $I$  and  $t$  in equation, we get

$$0 = K e^0 + \frac{\vartheta_m}{Z} \sin(0 + \alpha - \phi)$$

This gives

$$K = -\frac{\vartheta_m}{Z} \sin(\alpha - \phi)$$

Substituting the value of  $K$  in equation, we obtained the equation that gives the magnitude of inrush current as a function of time can be expressed as

$$i = i_n + i_f = \frac{\vartheta_m}{Z} \sin(\omega t + \alpha - \phi) - \frac{\vartheta_m}{Z} \sin(\alpha - \phi) e^{-\frac{R}{L}t}$$

The component  $i_n$  is also learn as d. c. offset component, since it is unidirectional. The d. c. component decays exponential with a time constant  $L/R$ .

The angle  $\alpha$  denotes the instant in the voltage cycle, at which the switch is closed, i.e.  $\alpha$  = Switching angle

$\vartheta_m$  is the maximum applied voltage

$R$  is the core loss resistance

$L$  is the core magnetizing inductance

$t$  is time in second

$$\phi = \tan^{-1} \frac{\omega L}{R}$$

### 3. EXPRESSION OF MAGNETIZING FLUX IN TRANSFORMER

Equivalent circuit of Transformer with secondary open shown in figure

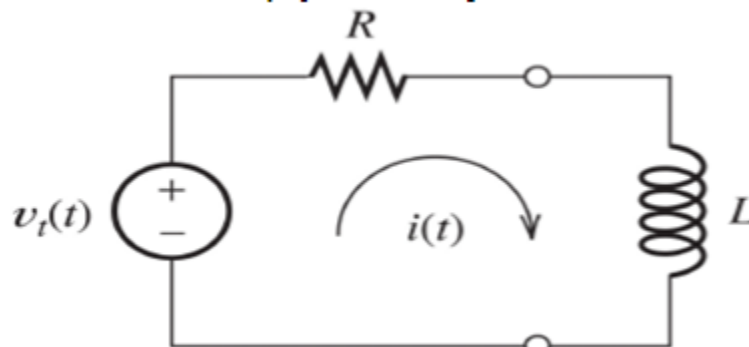


Figure (2): Equivalent circuit of Transformer with secondary open

The transformer winding is connected to a supply voltage  $v(t)$  and secondary is made open.

Let the applied voltage given by

$$\vartheta(t) = \vartheta_p \sin(\omega t + \phi)$$

The applied voltage is expressed as a function of flux in the core and primary current.

The applied voltage is given by:

$$\vartheta = Ri + N \frac{d\phi(t)}{dt}$$

By neglecting the core loss and resistance equation (2) now becomes

$$\vartheta(t) = N \frac{d\phi(t)}{dt}$$

Solve for value of  $\phi(t)$

$$\begin{aligned} \phi(t) &= \int \frac{\vartheta(t)}{N} dt \\ \phi(t) &= \frac{1}{N} \int \vartheta_p \sin(\omega t + \phi) dt \\ \phi(t) &= \frac{\vartheta_p}{N} \int_{-\infty}^t \sin(\omega t + \phi) dt \\ \phi(t) &= \frac{\vartheta_p}{N} \int_{-\infty}^0 \sin(\omega t + \phi) dt + \frac{\vartheta_p}{N} \int_0^t \sin(\omega t + \phi) dt \\ &= \phi(0) + \frac{\vartheta_p}{N_w} [-\cos(\omega t + \phi)]_0^t \\ &= \phi(0) - \frac{\vartheta_p}{N_w} [\cos(\omega t - \cos \phi)] \end{aligned}$$

By using this relation  $\{\cos(A + B) = \cos A \cos B - \sin A \sin B\}$ , above equation become

$$= \phi(0) - \frac{\vartheta_p}{N_w} [\text{Cos}\omega t \text{Cos}\phi - \text{Sin}\omega t \text{Sin}\phi - \text{Cos}\phi]$$

Taking common  $\cos\phi$  term in above equation, we get

$$= \phi(0) - \frac{\vartheta_p}{N_w} [\cos\phi[\cos\omega t - 1] - \sin\omega t \sin\phi]$$

The nature of the transformer flux as a function of time during a transient switching on or energization is given by equation.

$$\phi(t) = \frac{\theta_p}{N_w} [\sin \omega t \sin \phi + \cos \phi [1 - \cos \omega t] + \phi(0)]$$

Where,

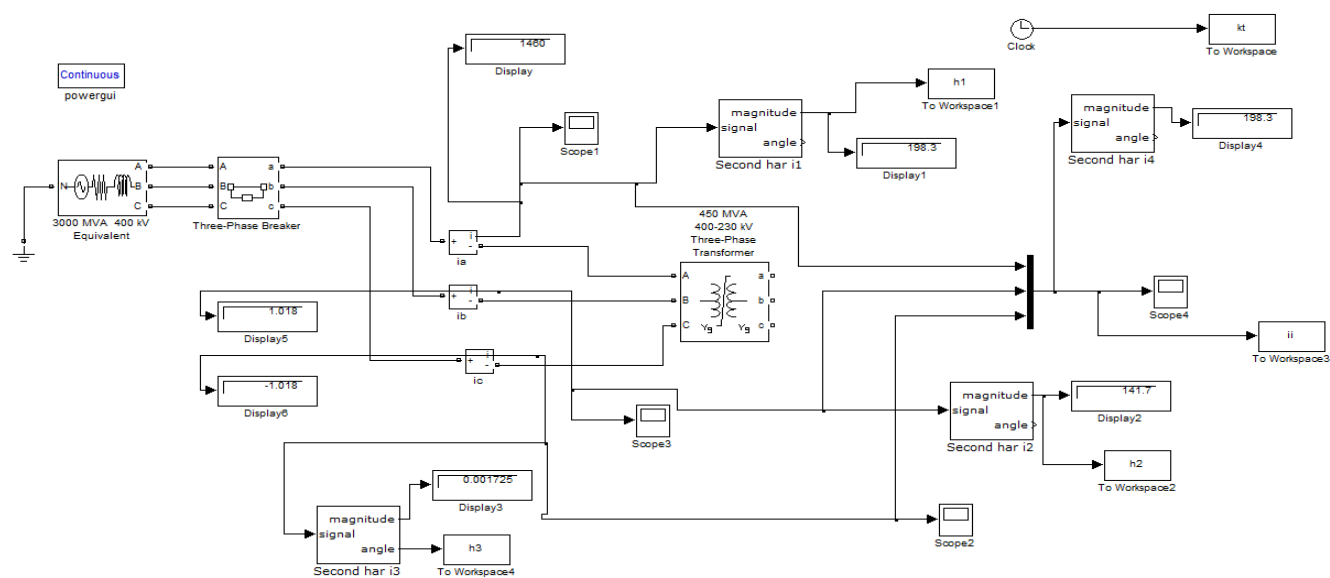
 $\vartheta_p$  Max value of voltage

$\omega$  Angular frequency

N No. of terms in transformer coils

 $\phi$  Switching angle

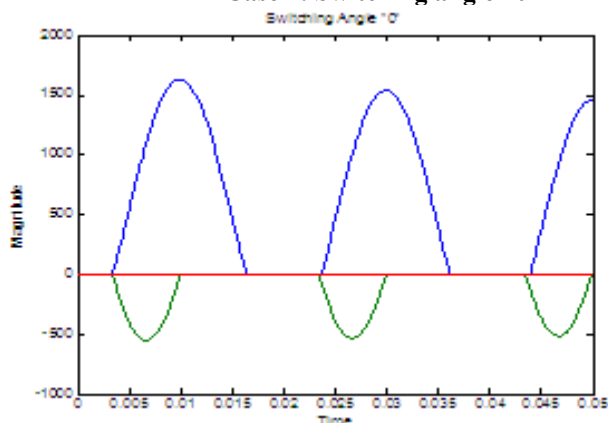
#### 4. MATLAB SIMULINK MODEL OF POWER TRANSFORMER



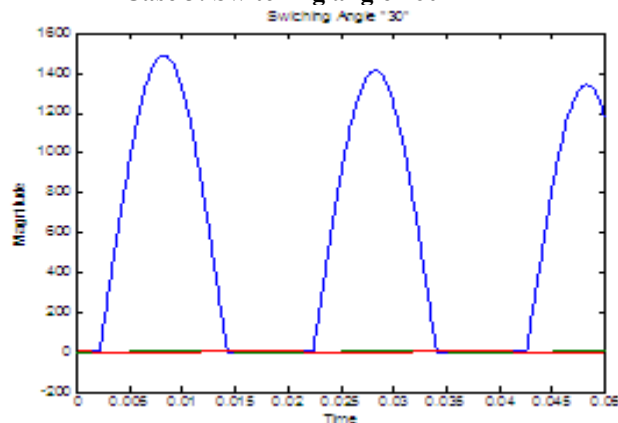
**Figure (3): Matlab Simulink Model of Transformer**

## 5. EFFECT OF SWITCHING ANGLE VARIATION ON MAGNITUDE OF INRUSH CURRENT THROUGH MATLAB SIMULATION

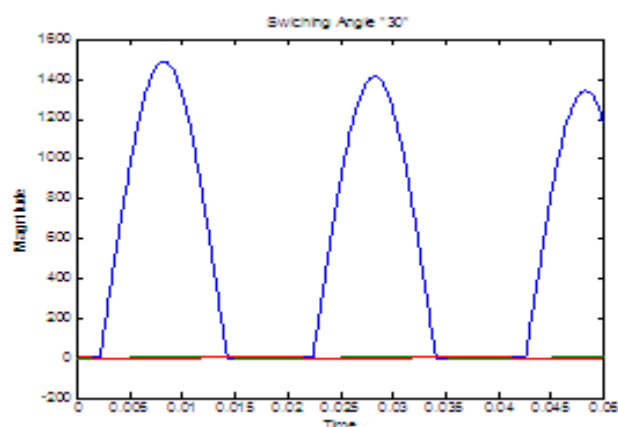
### Case 1: Switching angle “0”



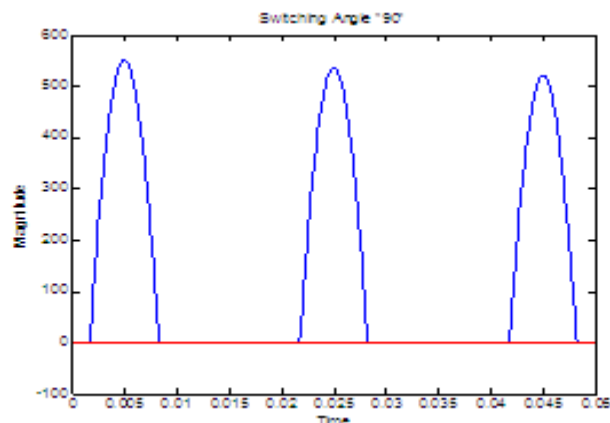
### Case 3: Switching angle “60”



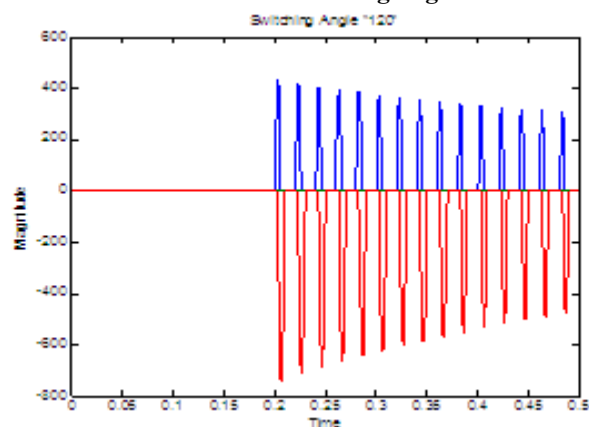
Case 2: Switching angle “30”



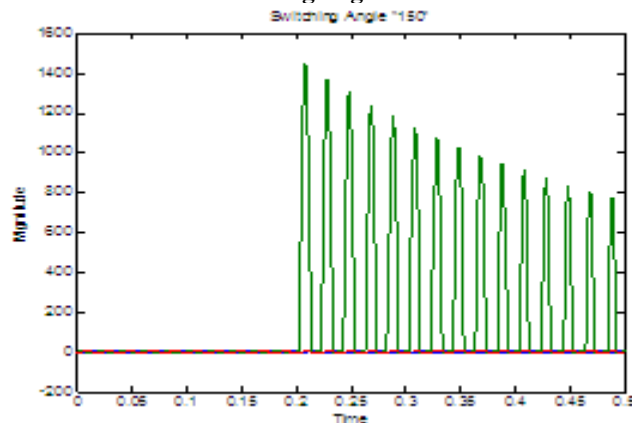
Case 4: Switching angle “90”



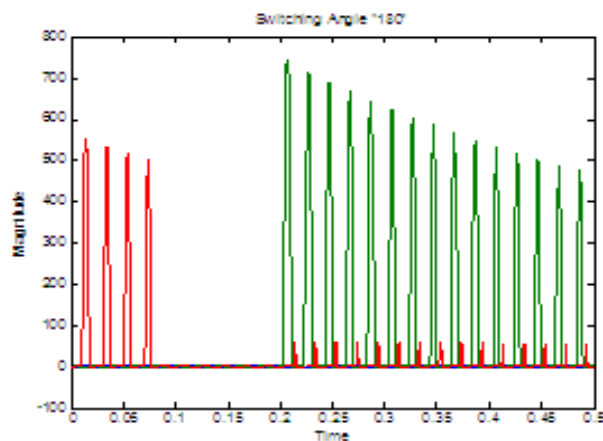
Case 5: Switching angle “120”



Case 6: Switching angle “150”



Case 7: Switching angle “180”



## 6. MAGNITUDE OF INRUSH CURRENT AT DIFFERENT SWITCHING ANGLE SHOWN IN THE TABLE BELOW.

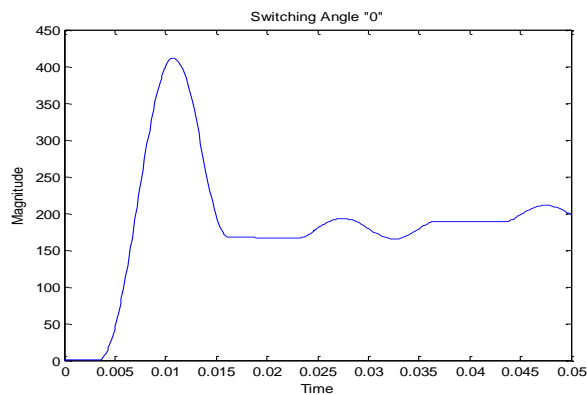
Table (1): Magnitude of Inrush current at different switching angle

S.NO	ANGLE(degree)	INRUSH CURRENT(A) (peak value)
1	0	1650

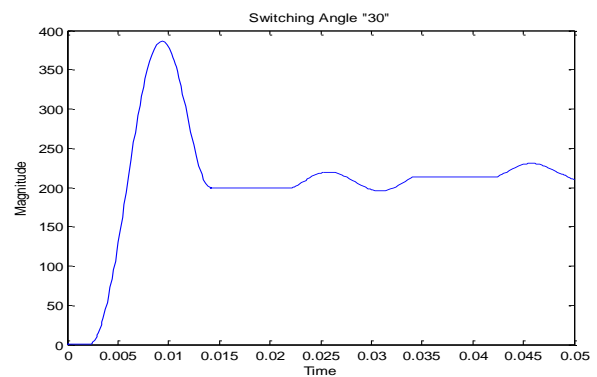
2	30	1530
3	60	1300
4	90	550
5	120	775
6	150	1400
8	180	795

## 7. EFFECT OF SWITCHING ANGLE VARIATION ON THE MAGNITUDE OF SECOND HARMONIC COMPONENT IN INRUSH CURRENT THROUGH MATLAB SIMULATION.

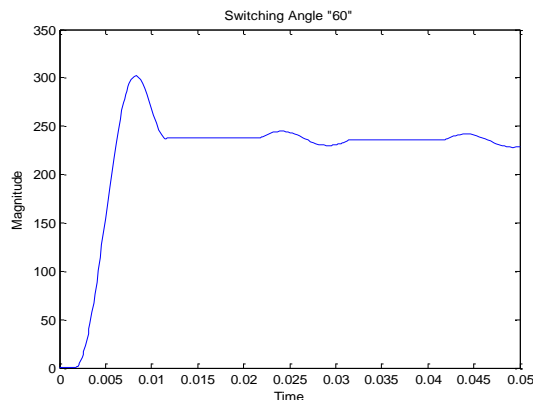
**Case 1: Switching angle "0"**



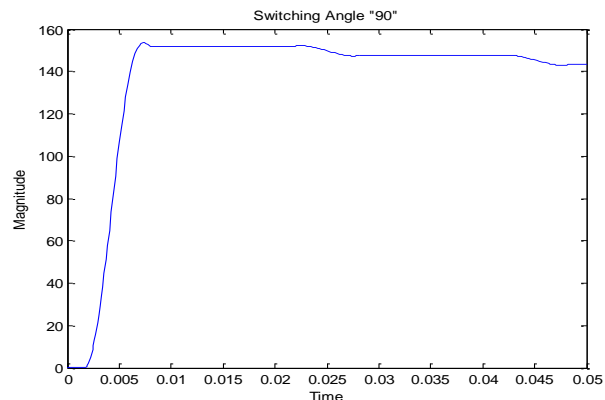
**Case 2: Switching angle "30"**



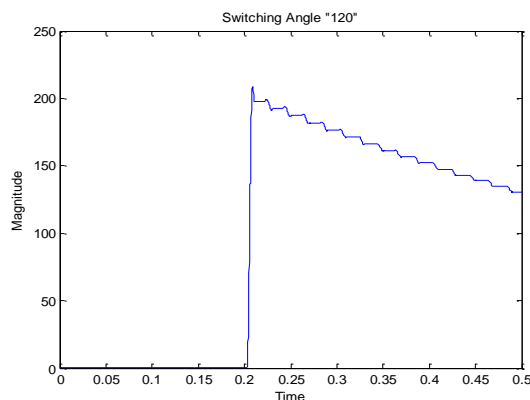
**Case 3: Switching angle "60"**



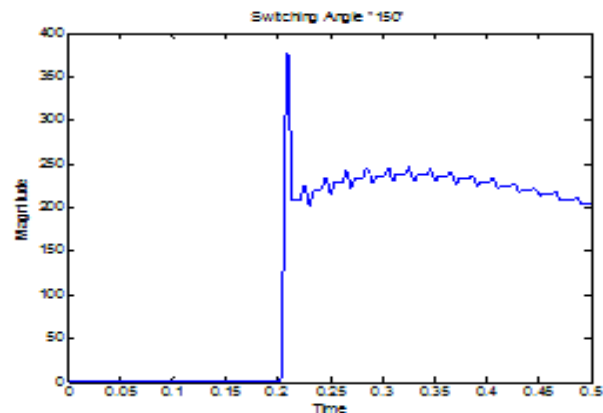
**Case 4: Switching angle "90"**



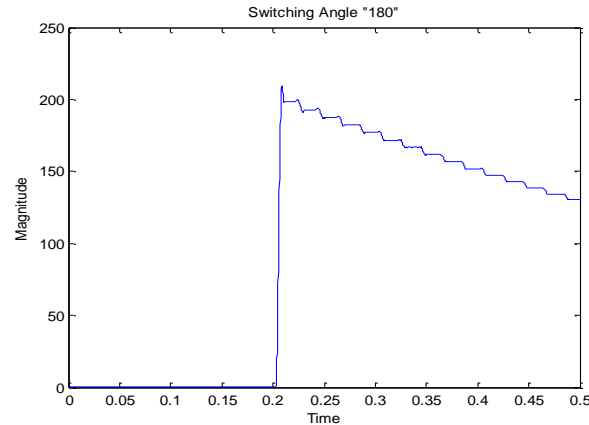
**Case 5: Switching angle "120"**



**Case 6: Switching angle "150"**



### Case 7: Switching angle "180"



## 8. MAGNITUDE OF SECOND HARMONIC PRESENT IN INRUSH CURRENT AT DIFFERENT SWITCHING ANGLE

Table (2): Magnitude of Second Harmonic component in Inrush current at different switching angles

S.NO	Switching Angle (Degree)	Magnitude of Second Harmonic in Inrush Current
1	0	410
2	30	370
3	60	300
4	90	150
5	120	200
6	150	370
8	180	215

## 9. MAGNITUDE OF SECOND HARMONIC PRESENT IN CURRENTS IN NORMAL CONDITION AT DIFFERENT SWITCHING ANGLE

Table (3): Magnitude of second harmonic present in currents in Normal condition

S.NO	Switching Angle (degree)	Magnitude of Second Harmonic in Normal Current
1	0 degree	0.007269
2	30 degree	0.07979
3	60 degree	0.007269
4	90 degree	0.07979
5	120 degree	0.007269
6	150 degree	0.07979
7	180 degree	0.003117

## CONCLUSION

Thus the inrush current phenomenon is investigated on a three phase transformer by using the MATLAB Simulink model transformers. It is seen that the peak value of magnetizing inrush current is very high as compared to the normal magnetizing current and when the transformer was operated at an angle of 90 degree the value of the inrush current was found out to be minimal. It is seen that the of second harmonic magnitude in inrush current is very high as compared to its magnitude in normal condition current and when the transformer was operated at an angle of 90 degree the value of the Second harmonic magnitude was found out to be minimal.

## **Appendix I:**

Source: 3000 MVA  
Phase-to-phase rms voltage (v): 400 kV  
Frequency: 50Hz  
Transformer Power: 450 MVA  
Step down: 500 kV/ 230 kV  
Frequency: 50 Hz

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