International Journal of Enhanced Research in Science Technology & Engineering, ISSN: 2319-7463
Vol. 3 Issue 11, November-2014, pp: (257-267), Impact Factor: 1.252, Available online at: www.erpublications.com

Robust Control of Twin rotor system with Parameter Dependent Performance

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Abstract: This paper presents a robust Hinfinity control with parameter dependent performance of a highly nonlinear multi-input multi-output and strongly coupled dynamics system which is the twin rotor system for the purpose to stabilize the system in elevation and azimuth angles, improve tracking performance, disturbance rejection and handles the coupling problem. Simulation results showed that the controllers exhibit good performance.

Keywords: Gain scheduling, Hinfinity, LPV, MIMO System, Robust Control.

Nomenclature

$\theta_{h/v}$:is the azimuth/pitch angle of beam (horizontal/vertical plane)

$\omega_{h/v}$:is the rotational velocity of the tail/main rotor.

$u_{h/v}$:is the input voltage of the tail/main motor.

$J_{tr/mr}$: is the moment of inertia in motor tail/main propeller subsystem.

$k_{ah/v}\,\varphi_{h/v}$:is the torque constant of the tail/main motor.

J_v :is the moment of inertia about the horizontal axis.

r_{ms}, r_{ts} :are the radius of the main and tail shield.

$\Omega_{h/v}$:is the angular velocity around the vertical/horizontal axis.

m_m, m_t :are the masses of the main and the tail parts of the beam.

m_{cb}, l_{cb} :represent the mass of the counter-weight and the distance between the counter-weight and the joint.

m_{mr}, m_{tr} :are the masses of the main and the tail DC-motor with main and tail rotor.

m_b, l_b :are the mass and the length of the counter-weight beam.

m_{ms}, m_{ts} :are the masses of the main and tail shields.

Introduction

The prime control objective in this work is to balance the twin rotor system in coupled condition and to make the beam to follow the desired trajectory to achieve desired positions in 2DOF accurately and quickly. Many theoretical techniques were already proposed to solve these problems. The article [1] presents the techniques of the modal control, and in[2],[3] and [4], we find the Gaussian quadratic linear LQG/LTR control, the control by fuzzy logic and a non linear H∞ approach for handling the coupling taken as a disturbance that should be rejected. In [7], it is considered a sliding mode control by defining a sliding surface that allows dealing with cross-coupling inherent in the twin rotor dynamics. The concept of system linear with variable parameters in time (LPV) was introduced for the first time into [8], [9]. This kind of systems constitutes a class of model make it possible to describe processes which can be represented by linear models, but whose dynamic characteristics change according to the operating conditions [15]. In recent years the interest for gain scheduling methods has increased. Gain scheduling is a collection of methods that try to tackle the challenging problem of nonlinear control in a divide and conquer manner. This control design methodology allows designing an H∞ gain-scheduling controller that is able to adapt the control gains with the operating point using a scheduling function and scheduling variables [10]. This article is arranged as follows. In section 2 we present the formulation of the polytopic LPV model of the Twin rotor system. In section 3, a synthesis of the LPV/H∞ controller is detailed. Section 4, details the results obtained when applying the presented controller to the system to prove its effectiveness and performance. Finally, we will close the paper with some concluding remarks.

Twin Rotor MIMO System Model

An accurate dynamic model of the system, that take into account the high non-linearity and cross-coupling between its two axes, an accurate dynamic model of the system is thus required to achieve control objectives satisfactorily. Several work was realized of which the goal to develop and identify the Twin rotor system fig. 1 model [2], [11], and [12].

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A. Derivation of the Nonlinear Model

To develop the dynamic model we used a direct method based on the calculation of the forces acting on the body of the simulator [14].

The non-linear model results in a set of six non-linear differential equations [13].

$$\frac{di_{ah/v}}{dt} = -\frac{R_{ah/v}}{L_{ah/v}} i_{ah/v} - \frac{k_{ah/v}\,\varphi_{h/v}}{L_{ah/v}} \omega_{h/v} + \frac{k\,1/2}{L_{ah/v}} u_{h/v} \tag{1}$$

$$\frac{d\,\omega_{ah/v}}{dt} = -\frac{k_{ah/v}\,\varphi_{h/v}}{J_{tr/mr}} i_{ah/v} - \frac{B_{tr/mr}}{J_{tr/mr}} \omega_{h/v} - \frac{f_{1/4}\left(\omega_{h/v}\right)}{J_{tr/mr}} \tag{2}$$

$$\frac{d\,\Omega_h}{dt} = \frac{l_t f_2\left(\omega_h\right)\cos\theta_v - k_{oh}\Omega_h - f_3\left(\theta_h\right)}{D\cos^2\theta_v + E\sin^2\theta_v + F} + \frac{k_m \omega_v \sin\theta_v\,\Omega_v\left(D\cos^2\theta_v - E\sin^2\theta_v - F - 2E\cos^2\theta_v\right)}{\left(D\cos^2\theta_v + E\sin^2\theta_v + F\right)^2}$$
$$+\frac{k_m\cos\theta_v\left(k_{av}\,\varphi_v\,i_{av} - B_{mr}\,\omega_v - f_4\left(\omega_v\right)\right)}{\left(D\cos^2\theta_v + E\sin^2\theta_v + F\right)J_{mr}} \tag{3}$$

$$\frac{d\,\theta_h}{dt} = \Omega_h \tag{4}$$

$$\frac{d\,\Omega_v}{dt} = \frac{l_m f_5\left(\omega_v\right) + k_g\,\Omega_h\,\mathrm{f}_5\left(\omega_v\right)\cos\theta_v - k_{ov}\,\Omega_v}{J_v} + \frac{g\left(\left(A-B\right)\cos\theta_v - C\sin\theta_v\right) - 0.5\Omega_h{}^2 H\sin 2\theta_v}{J_v}$$
$$+\frac{k_t\left(k_{ah}\varphi_h i_{ah} - B_{tr}\,\omega_h - f_1\left(\omega_h\right)\right)}{J_v J_{tr}} \tag{5}$$

$$\frac{d\,\theta_v}{dt} = \Omega_v \tag{6}$$

The nonlinear functions f_i into account the frictions and coupling effects between horizontal/vertical dynamics, are defined as follows:

$$f_1\left(\omega_h\right) = sign\left(\omega_h\right) k_{th}\,\omega_h^2 \tag{7.1}$$

$$f_2\left(\omega_h\right) = \begin{cases} k_{fhp}\,\omega_h^2 & if \quad \omega_h \geq 0 \\ -k_{fhn}\,\omega_h^2 & if \quad \omega_h < 0 \end{cases} \tag{7.2}$$

$$f_3\left(\theta_h\right) = \begin{cases} k_{chp}\,\theta_h & if \quad \theta_h \geq 0 \\ k_{chn}\,\theta_h & if \quad \theta_h < 0 \end{cases} \tag{7.3}$$

$$f_4\left(\omega_v\right) = sign\left(\omega_v\right) k_{tv}\,\omega_v^2 \tag{7.4}$$

$$f_5\left(\omega_v\right) = \begin{cases} k_{fvp}\,\omega_v^2 & if \quad \omega_v \geq 0 \\ -k_{fvn}\,\omega_v^2 & if \quad \omega_v < 0 \end{cases} \tag{7.5}$$

$$f_6\left(\theta_v\right) = \begin{cases} k_{cvp}\left(\theta_v - \theta_v^0\right)^2 & if \quad \theta_v \geq \theta_v^0 \\ k_{cvn}\left(\theta_v - \theta_v^0\right)^2 & if \quad \theta_v < \theta_v^0 \end{cases} \tag{7.6}$$

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Where the input vector is $u = [u_h, u_v]^T$ and state vector is $x = [\omega_h, \Omega_h, \theta_h, \omega_v, \Omega_v, \theta_v]^T$

The constants of the non-linear model are defined as:

$$l_1 = \left(\frac{m_t}{2} + m_{tr} + m_{ts}\right) \mathrm{l_t} \tag{8.1}$$

$$l_2 = \left(\frac{m_m}{2} + m_{mr} + m_{ms}\right) \mathrm{l_m} \tag{8.2}$$

$$l_3 = \left(\frac{m_b}{2}\ \mathrm{l_b} + \mathrm{m_{cb}} l_{cb}\right) \tag{8.3}$$

$$l_4 = \frac{\mathrm{m_b}}{3} \mathrm{l_b^2} + \mathrm{m_{cb} l_{cb}^2} \tag{8.4}$$

$$l_5 = \left(\frac{\mathrm{m_m}}{3} + \mathrm{m_{mr}} + \mathrm{m_{ms}}\right) \mathrm{l_m^2} + \left(\frac{\mathrm{m_t}}{3} + \mathrm{m_{tr}} + \mathrm{m_{ts}}\right) \mathrm{l_t^2} \tag{8.5}$$

$$l_6 = \mathrm{m_{ms}}\ \mathrm{r_{ms}^2} + \frac{\mathrm{m_{ts}}}{2} \mathrm{r_{ts}^2} \tag{8.6}$$

$$l_7 = A l_t + B l_m + \frac{\mathrm{m_b}}{2} \mathrm{l}_b^2 + m_{cb} l_{cb}^2 \tag{8.7}$$

B. Derivation of the LPV Model

For the purpose to define the linear time varying model whose state-space matrices are fixed functions of some vector of varying parameters ψ_t . [13,16]

They can be described by time state-space equations of the form:

$$\begin{aligned} \dot{x}(t) &= A(\psi_t)x(t) + B(\psi_t)u(t) \\ y(t) &= C(\psi_t)x(t) + D(\psi_t)u(t) \end{aligned} \tag{9}$$

Where $x(t) \in \mathbb{R}^{n_x}$: The state vector. $u(t) \in \mathbb{R}^{m_u}$: The control inputs

$y(t) \in \mathbb{R}^{n_y}$: The sensor outputs. $(A, B, C, D) : \mathbb{R}^N \to \left(\mathbb{R}^{n\times n}, \mathbb{R}^{n\times m}, \mathbb{R}^{p\times n}, \mathbb{R}^{p\times m}\right)$

Matrices scheduled by ψ_t of varying parameters :

$$\psi_t = [a_{11} \quad a_{21} \quad a_{22} \quad a_{23} \quad a_{24} \quad a_{25} \quad a_{51} \quad a_{52} \quad a_{54} \quad a_{55} \quad a_{56} \quad b_{22}]$$

The model can be expressed in an LPV form by introducing the following set of time varying parameters

As follows:

$$\begin{bmatrix} \dot{\omega}_h(t) \\ \dot{\Omega}_h(t) \\ \dot{\theta}_h(t) \\ \dot{\omega}_v(t) \\ \dot{\Omega}_v(t) \\ \dot{\theta}_v(t) \end{bmatrix} = A(\psi_t) \begin{bmatrix} \omega_h(t) \\ \Omega_h(t) \\ \theta_h(t) \\ \omega_v(t) \\ \Omega_v(t) \\ \theta_v(t) \end{bmatrix} + \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22}(p_t) \\ 0 & 0 \\ 0 & b_{42} \\ b_{51} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_h(t) \\ u_v(t) \end{bmatrix} \tag{10}$$

Where:

$$A_{(\psi_t)} = \begin{bmatrix} a_{11}(p_t) & 0 & 0 & 0 & 0 & 0 \\ a_{21}(p_t) & a_{22}(p_t) & a_{23}(p_t) & a_{24}(p_t) & a_{25}(p_t) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{44}(p_t) & 0 & 0 \\ a_{51}(p_t) & a_{52}(p_t) & 0 & a_{54}(p_t) & a_{55}(p_t) & a_{56} \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{11}$$

With:

The elements a_{ij} of the matrix $A_{(\psi_t)}$ are:

$$a_{11}(p_t) = -\frac{k_{ah}^2 \varphi_h^2 / R_{ah} + B_{tr} + f_1(\omega_h)/\omega_h}{J_{tr}}$$

$$a_{22}(p_t) = -\frac{k_{0h}}{D\cos^2\theta_v + E\sin^2\theta_v + F}$$

$$a_{21}(p_t) = \frac{l_t \cos\theta_v f_2(\omega_h)/\omega_h}{D\cos^2\theta_v + E\sin^2\theta_v + F}$$

$$a_{23}(p_t) = -\frac{f_3(\theta_h)/\theta_h}{D\cos^2\theta_v + E\sin^2\theta_v + F}$$

$$a_{24}(p_t) = -\frac{k_m\left(B_{mr} + k_{av}^2 \varphi_v^2 / R_{av} + f_4(\omega_v)/\omega_v\right)\cos\theta_v}{\left(D\cos^2\theta_v + E\sin^2\theta_v + F\right)J_{mr}}$$

$$a_{44}(p_t) = -\frac{k_{av}^2 \varphi_v^2 / R_{av} + B_{mr} + f_4(\omega_v)/\omega_v}{J_{mr}}$$

$$a_{25}(p_t) = \frac{k_m \omega_v \sin\theta_v \left(D\cos^2\theta_v - E\sin^2\theta_v - F - 2E\cos^2\theta_v\right)}{\left(D\cos^2\theta_v + E\sin^2\theta_v + F\right)^2}$$

$$a_{51}(p_t) = -\frac{k_t\left(B_{tr} + f_1(\omega_h)/\omega_h + k_{ah}^2/R_{ah}\right)}{J_v J_{tr}}$$

$$a_{52}(p_t) = -\frac{k_g \cos\theta_v f_5(\omega_v)/\omega_v - 0.5\Omega_h H \sin 2\theta_v}{J_v}$$

$$a_{55} = -\frac{k_{0v}}{J_v}$$

$$a_{54}(p_t) = \frac{l_m f_5(\omega_v)/\omega_v}{J_v}$$

$$a_{56}(p_t) = \frac{g\left((A-B)\cos\theta_v - C\sin\theta_v\right)}{J_v \theta_v}$$

$$b_{11} = \frac{k_{ah}\varphi_h k_1}{J_{tr} R_{ah}}$$

$$b_{42} = \frac{k_{av}\varphi_v k_2}{J_{mr} R_{av}}$$

$$b_{22}(p_t) = \frac{k_m \cos\theta_v k_{av}\varphi_v k_2}{\left(D\cos^2\theta_v + E\sin^2\theta_v + F\right)R_{av} J_{mr}}$$

$$b_{51} = \frac{k_{ah}\varphi_h k_1 k_t}{J_v J_{tr} R_{ah}}$$

The scheduling variables in the model (10) are

$$p_t = \begin{bmatrix} \theta_h & \theta_v & \omega_h & \Omega_h & \omega_v & \Omega_v \end{bmatrix}^T \tag{12}$$

An LPV model for the system can be obtained by means of state transformation from (10) by considering:

$$x_1 = \begin{bmatrix} \theta_h & \theta_v \end{bmatrix}^T ; x_2 = \begin{bmatrix} \omega_h & \Omega_h & \omega_v & \Omega_v \end{bmatrix}^T \tag{13}$$

By defining new states and inputs, a new LPV model of the twin rotor simulator can be obtained:

$$\begin{bmatrix} \dot{\hat{\omega}}_h(t) \\ \dot{\Omega}_h(t) \\ \dot{\theta}_h(t) \\ \dot{\hat{\omega}}_v(t) \\ \dot{\Omega}_v(t) \\ \dot{\theta}_v(t) \end{bmatrix} = \hat{A}_{abs}(\psi_t) \begin{bmatrix} \hat{\omega}_h(t) \\ \Omega_h(t) \\ \theta_h(t) \\ \hat{\omega}_v(t) \\ \Omega_v(t) \\ \theta_v(t) \end{bmatrix} + \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22}(p_t) \\ 0 & 0 \\ 0 & b_{42} \\ b_{51} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{u}_h(t) \\ \hat{u}_v(t) \end{bmatrix} \tag{14}$$

$$\hat{A}_{abs(\psi_t)} = \begin{bmatrix} a_{11} & -\frac{d\omega_h^{eq}}{d\theta_h} & 0 & 0 & -\frac{d\omega_h^{eq}}{d\theta_v} & 0 \\ a_{21} & a_{22} & 0 & a_{24} & a_{25} & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{44} & -\frac{d\omega_v^{eq}}{d\theta_v} & 0 \\ a_{51} & a_{52} & 0 & a_{54} & a_{55} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (15)$$

Where

$$\hat{\omega}_{h/v}(t) = \omega_{h/v}(t) - \omega_{h/v}^{eq}(t), \hat{u}_{h/v}(t) = u_{h/v}(t) - u_{h/v}^{eq}(t) \quad (16)$$

$$\omega_h^{eq} = \frac{f_3(\theta_h)}{l_t \cos(\theta_v) f_2(\omega_h)}$$
$$\omega_h^{eq} = \frac{g\,\omega_v\left[C\sin(\theta_v) + (B-A)\cos(\theta_v)\right]}{l_m f_5(\omega_v)} \quad (17)$$

$$u_{h/v}^{eq}(t) = \frac{k_{ah/v}\varphi_{h/v}}{k_{1/2}} + \frac{R_{ah/v}(B_{tr/mr} + f_{1/4}(\omega_{h/v})/\omega_{h/v})/(k_{ah/v}\varphi_{h/v})}{k_{1/2}}\omega_{h/v}^{eq} \quad (18)$$

$$\frac{d\omega_h^{eq}}{d\theta_h} = \begin{cases} \dfrac{k_{chn}\omega_h}{l_t\cos\theta_v f_2(\omega_h)} \; if\ \theta_h \ge 0 \\ \dfrac{k_{chp}\omega_h}{l_t\cos\theta_v f_2(\omega_h)} \; if\ \theta_h < 0 \end{cases} \quad (19)$$

$$\frac{d\omega_h^{eq}}{d\theta_h} = \frac{f_3(\theta_h)\omega_h\sin\theta_v}{l_t f_2(\omega_h)\cos^2\theta_v} \quad (20)$$

$$\frac{d\omega_v^{eq}}{d\theta_v} = \frac{g\,\omega_v\left[C\cos\theta_v + (A-B)\sin\theta_v\right]}{l_m f_5(\omega_v)} \quad (21)$$

To design a controller for an LPV system (9) is to approximate it by a polytopic LPV system corresponds to those whose parameter vector varies within a polytope. The state space matrices range in a polytope of matrices defined as the convex hull of a finite number of matrices N. Each polytope vertex corresponds to a particular value of scheduling variable:

$$\begin{pmatrix} A(\psi_k) & B(\psi_k) \\ C(\psi_k) & D(\psi_k) \end{pmatrix} \in Co\left\{\begin{pmatrix} A_j & B_j \\ C_j & D_j \end{pmatrix}, j = 1..N\right\} = \sum_{j=1}^{N}\alpha_k^j(\psi_k)\begin{pmatrix} A_j & B_j \\ C_j & D_j \end{pmatrix} \quad (22)$$

With

$$\alpha_k^j(\psi_k) \ge 0 \text{ And } \sum_{j=1}^{N}\alpha_k^j(\psi_k) = 1$$

Where j each model is called a vertex system, this type of LPV systems is polytopic.

The polytopic approximation consider on the limitation of every parameter LPV by an interval “bounding box approach” and its conduct to an interval LPV model, the convex hull that contains them is obtained using the quick hull algorithm. [17].

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The polytopic LPV system (22) can be expressed as follows:

$$x(k+1)=\sum_{j=1}^{N}\alpha_k^j(\psi_k)\left[A_j x(k)+B_j u(k)\right] \tag{23}$$

$$y(k)=\sum_{j=1}^{N}\alpha_k^j(\psi_k)C_j x(k) \tag{24}$$

Where $x(k)\in\mathbb{R}^{n_x}$ The state vector, $u(k)\in\mathbb{R}^{n_u}$ The control inputs, $y(k)\in\mathbb{R}^{y_u}$ The outputs.

$(A_j,B_j,C_j):\mathbb{R}^N\rightarrow\left(\mathbb{R}^{n_x\times n_x},\mathbb{R}^{n_x\times n_u},\mathbb{R}^{n_y\times n_x}\right)$ Are time invariant matrices defined for j model.

The polytopic system is scheduled with functions:

$\alpha_k^j(\psi_k),\forall j\in[1,\ldots,N]$ Which are situated in a convex:

$$\Omega=\left\{\alpha_k^j(\psi_k)\in\mathbb{R}^N,\alpha_k^j(\psi_k)=\left[\alpha_k^1(\psi_k),..,\alpha_k^N(\psi_k)\right]^T \quad \alpha_k^j(\psi_k)\geq 0,\forall j,\sum_{j=1}^{N}\alpha_k^j(\psi_k)=1\right\} \tag{25}$$

Controller Design

In this section, we seek to find a parameter dependent controller that in a closed loop with the LPV system (14) attains the norm bound for all possible trajectories of the scheduling variable specified by the bounds of the vertices.

A. LMI Constraints

Parameterized LMIs are convex, but of infinite dimension as they may be represented by a range of LMIs in the parameters.Therefore, a scheme for reducing the infinite to a finite problem is necessary for treatable computation of the parameterized LMIs. [15,13]. As in LTI H∞ design is also used in H∞ LPV synthesis. This will result in system (9),

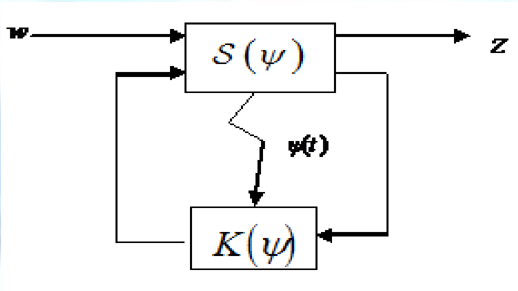

Figure 1. **closed loop design**

The LPV system (9) has a quadratic H∞ performance γ if and only if:

The system is quadratically stable on Θ, and the L2 gain of the input/output is bounded by γ.
That is: $\|y\|_2<\gamma\|u\|_2$ Along all possible parameter trajectories $\psi(t)$ in Θ.

For all $\psi(t)$òΘ , there exists a single symmetric positive definite matrix X solution of the Linear Matrix Inequality (LMI):

$$\begin{pmatrix} A(\psi)^T X+XA(\psi) & XB(\psi) & C(\psi)^T \\ B(\psi)^T X & -\gamma I & D(\psi)^T \\ C(\psi) & D(\psi) & -\gamma I \end{pmatrix}<0 \tag{26}$$

specifying that if θ is frozen, the quadratic H∞ performance γ is reduced to the standard H∞ norm, secondly if the plant has polytopic representation becomes for all i=1,2,...,N there exists a single symmetric positive definite matrix X solution of the Linear Matrix Inequality

$$\begin{pmatrix} A_i^T X+XA_i & XB(\psi) & C_i^T \\ B_i^T X & -\gamma I & D_i^T \\ C_i & D_i & -\gamma I \end{pmatrix}<0 \tag{27}$$

The LMI must holds at each summit of the convex polytope.

B. **Controller formulas**

The simplest form to find an LPV controller guarantees robust stability of the closed loop system which, over all trajectories $\psi(t) \in \Theta$

- Stabilizes quadratically the closed loop system depends on the parameter T_{zw}.
- Minimizes the H∞ quadratic performance γ
- Places the closed loop poles in the left half plane Re(s) < -α (α>0).

The polytopic structure for controller is:

$$K(\psi) = \sum_{i=1}^{i=N} \alpha_i(\psi)_i K_i ; \alpha_i(\psi) \geq 0; \sum_{i=1}^{i=N} \alpha_i(\psi) = 1 \tag{28}$$

With

$$K_i = \begin{pmatrix} A_{ci} & B_{ci} \\ C_{ci} & 0 \end{pmatrix}$$

The closed loop system is:

$$S_e : \begin{cases} \dot{X}(t) = A_e(\psi) X(t) + B_e(\psi) w \\ Z(t) = C_e(\psi) X(t) \end{cases} \tag{29}$$

With

$$A_e(\psi) = \begin{pmatrix} A(\psi) & B_2(\psi) C_c(\psi) \\ B_c(\psi) C_2(\psi) & A_c(\psi) \end{pmatrix} \tag{30.1}$$

$$B_e(\psi) = \begin{pmatrix} B_1(\psi) \\ B_c(\psi) D_{21}(\psi) \end{pmatrix} \tag{30.2}$$

$$C_e(\psi) = \begin{pmatrix} C_1(\psi) & D_{12}(\psi) C_c(\psi) \end{pmatrix} \tag{30.3}$$

To perform the quadratic approach linearity is necessary, two options are available.

The first is to limit the class of LPV systems assuming that the matrices of input and output are invariant: [1], [18]:

$$B_2(\psi) = B_2, C_2(\psi) = C_2 \tag{31}$$

The second is looking for a controller with a linear time varying dynamic matrix and input and output constant matrices. [16]

Following a theorem that gives a sufficient condition of quadratic stabilizability with H∞ performance γ when ψ is time varying:

If exist a symmetric positive definite matrices $X, Y \in \Re^{n\times n}$ and matrices $L \in \Re^{m\times n}$, , $F \in \Re^{n\times q}$ and $\forall i = 1,2,...N$ $M_i \in \Re^{n\times n}$ solutions of the matrix inequalities:

$$\begin{cases} \begin{pmatrix} Y & 1 \\ 1 & X \end{pmatrix} > 0 & (32) \\ \begin{pmatrix} H_i & Z_i + M_i & B_1 & L^T D_{12}^T + yC_1^T \\ (*) & G_i & XB_1 + FD_{21} & C_1^T \\ (*) & (*) & -\gamma 1 & 0 \\ (*) & (*) & (*) & -\gamma 1 \end{pmatrix} < 0 & (33) \end{cases}$$

$H_i = A_i Y + Y_i A_i^T + B_{2i} L + L^T B_{2i}^T$
$G_i = X A_i + A_i^T X + F C_{2i} + C_{2i}^T F^T$
$Z_i = A_i + Y A_i^T X + L^T B_{2i}^T X + Y C_{2i}^T F^T$
$(\bullet)$ stands for the symmetric transpose

Then the system (14) is quadratically stabilizable in the form (28) and: $\left\| T_{zw(s)} \right\|_\infty < \gamma \quad \forall \psi \in \Theta$

The closed loop system is stable and $\left\| T_{zw(s)} \right\|_\infty < \gamma$ if and only if there exists a positive definite symmetric matrix $X_e \in \Re^{2n \times 2n}$ such that:

$$\begin{pmatrix} A_{ei}^T X_e + X_e A_{ei} & X_e B_{ei} & C_{ei}^T \\ B_{2i}^T X_e & -\gamma 1 & 0 \\ C_{ei} & 0 & -\gamma 1 \end{pmatrix} < 0$$
$$\forall i = 1, \ldots\ldots, N \tag{34}$$

X_e and X_e^{-1} partitioned depending on :

$$X_e = \begin{bmatrix} X & U \\ U^T & \hat{X} \end{bmatrix}$$
$$X_e^{-1} = \begin{bmatrix} Y & V \\ V^T & \hat{Y} \end{bmatrix}$$
$$Y = \left(X - U \hat{X}^{-1} U^T \right)^{-1} > X^{-1}$$

This results the first inequality (33). To prove (33) the regular matrix T_e can be set as follows:

$$T_e = \begin{bmatrix} Y & 1 & 0 & 0 \\ V^T & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{35}$$

We can obtain (33) by pre and post multiplying (34) by T_e and T_e^T respectively and doing the variable changes

$$L = C_C V^T, F = U B_C, M_i = V A_{ci}^T U^T$$
$$\forall i = 1, 2 \ldots\ldots N$$

It is evident that (33) is bilinear in terms of (Y,X,L,F). In order to obtain linear matrix inequality LMI, we can do the variable change $\hat{M}_i = Z_i + M_i$ this result to the following convex problem with linear constraints:

$$\min \gamma$$
$$\begin{cases} \begin{pmatrix} Y & 1 \\ 1 & X \end{pmatrix} > 0 & (36) \\ \begin{pmatrix} H_i & M_i & B_1 & L^T D_{12}^T + Y C_1^T \\ (*) & G_i & X B_1 + F D_{21} & C_1^T \\ (*) & (*) & -\gamma 1 & 0 \\ (*) & (*) & (*) & -\gamma 1 \end{pmatrix} < 0 & (37) \end{cases}$$

International Journal of Enhanced Research in Science Technology & Engineering, ISSN: 2319-7463
Vol. 3 Issue 11, November-2014, pp: (257-267), Impact Factor: 1.252, Available online at: www.erpublications.com

After the matrices X, Y, L, F and M_i were found, the controller realization matrices (Aci, Bc, Cc) relative to the i^th summit can be easily derived from the expression of L, F and M_i.

The controller transfer function is:

$$\begin{aligned} H_i(s) &= C_c\left(s1 - A_{ci}\right)^{-1} B_c = L\left(V^T\right)^{-1}\left(s1 - U^{-1}M_i^T\left(V^T\right)^{-1}\right)^{-1} U^{-1}F \\ &= L\left(sUV^T - M^T\right)F \end{aligned} \tag{38}$$

With $XY + UV^T = 1$ it becomes: $H_i(s) = L\left(s(1 - XY) - M_i^T\right)F$

This corresponds to the state space realization

$$\begin{cases} \dot{x}_c = A_{ci}x_c + B_c y \\ u = C_c x \end{cases} \tag{39}$$

$$A_{ci} = (1 - XY)^{-1} M_i^T ; B_c = (1 - XY)^{-1} F ; C_c = L$$

Gain scheduling provides a reasonable compromise between performance and robustness, yet at the expense of higher complexity and delicate stability in the switching zone.

The system (9) is quadratically stabilizable by output feedback and the poles of the system lies in the left half plane $\mathrm{Re}(s) < -\alpha(\alpha > 0) \quad \forall\psi \in \Theta$

If there a symmetric definite positive matrix X_e such that:

$$A_e X_e + X_e A_e^T + 2\alpha X_e < 0$$

By pre and post multiplying by adequate matrices as done with (34), we obtain the new constraint expressed in the LMI form: [16,19]

$$\begin{pmatrix} H_i & (\bullet) \\ M_i^T + Z_i^T & G_i \end{pmatrix} + 2\alpha\begin{pmatrix} Y & (\bullet) \\ 1 & X \end{pmatrix} < 0 \tag{40}$$

Simulation Results

The H∞ Linear Parameters Variant (HLPV) is applied to the LPV model of the Twin rotor platform obtained as shown in section 2, to control the elevation and azimuth trajectory.

By using the LMI Toolbox (MATLAB), we get the controller as in (38), the designed control should be robust and handles the coupling very efficiently, the H∞ performance obtained is $\gamma = 2.6650$.

The following results obtained by delivering a step of 0.45rad for elevation, for azimuth we applied a step of 0.6rad.

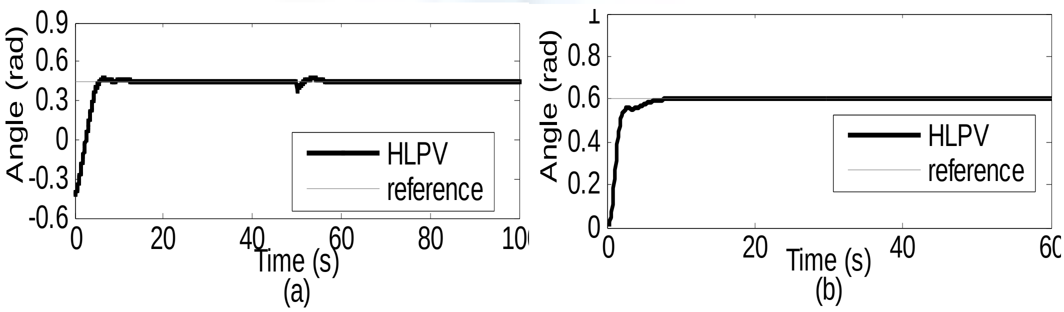

Figure 2. Step elevation and Azimuth simulations of closed loop performance, (a) elevation response (b) Azimuth response initial values $\theta h(0) = 0$ and $\theta v(0) = -0.48$ and a disturbance to the control input u(t) at t = 50s.

Fig. (2 a, b) show the elevation and azimuth behavior with the above mentioned controller, respectively. It is easy to see that the closed-loop nonlinear system is stable and has zero steady-state error even in the presence of disturbance with the proposed robust LPV/H∞ controller also improves the transient behavior.

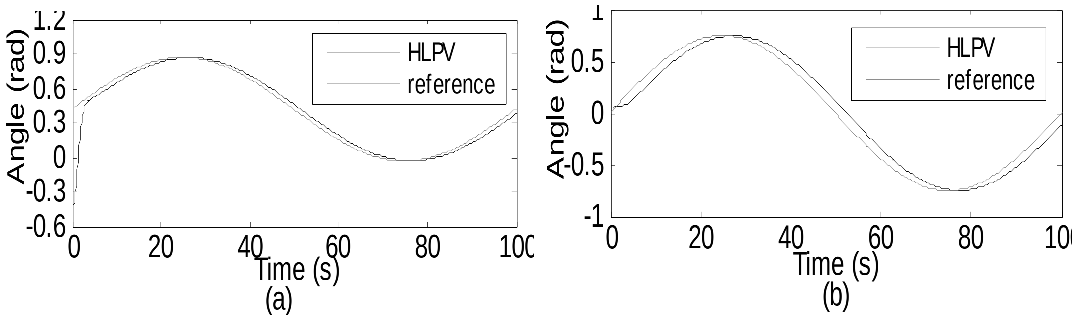

Figure 3. Sinusoidal simulations of closed loop performance(a) elevation response (b) Azimuth response. initial values θh(0) = 0 and θv(0) = -0.48.

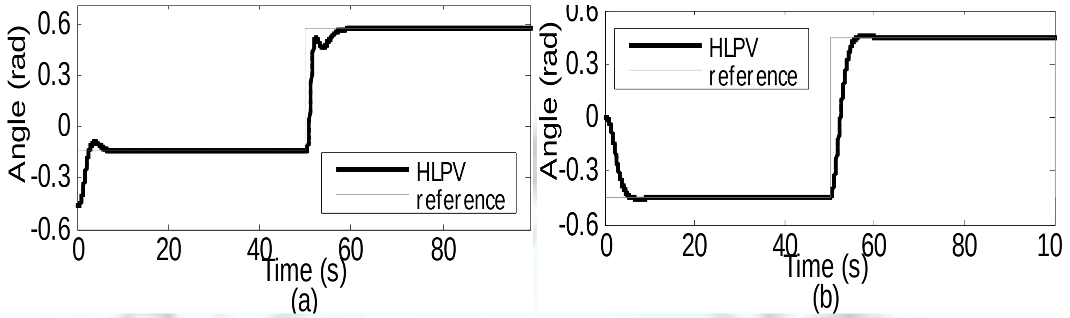

Figure 4. Square elevation and Azimuth simulations of closed loop performance, (a) elevation (b) Azimuth. initial values $\theta_h(0)$ = 0 and $\theta_v(0)$ = -0.48.

It should be noted that the controller presented here stabilize the system to a change of reference fig. 4, improves the performance for tracking signals in elevation and azimuth fig. 3, and improve system performance and robustness in the presence of the disturbance and time-varying uncertainty fig.2.The system output performance has been significantly improved.

LPV Gain Scheduling techniques, such as the one used here, offers a guarantee of the closed loop performance. However, in comparison with more traditional methods, the LPV methods are less intuitive to design and most often result in a more complex controller structure.

Conclusion

In this paper an Hinfinity gain scheduled has been applied to design the control of a non-linear model of the twin rotor MIMO system transformed into an LPV system and approximated in a polytopic way, the stability of the system is based on the resolution of an LMI problem under structural constraints. Simulation results not only confirmed the effectiveness of the proposed control but also showed that it has strong robustness, fast convergence, and good flexibility and can more effectively deal with disturbances, nonlinearity, and uncertainties of dynamics.

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