

# Circuit Analysis Using Matlab/Simulink®

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**Abstract:** Circuits having various combinations of R, L and C are the building blocks for many important areas of study in electrical engineering and also used extensively in communication systems for design and development of filters and oscillators. Hence, effective simulation (or prediction) of such systems is imperative. This paper explores the ability of MATLAB/Simulink® to achieve this feat with relative ease- either by writing MATLAB code commands or via Simulink® for linear Initial Value Problems (IVPs). The applicability of the MATLAB/Simulink® also finds use in the integrated electrical drives.

**Keywords:** MATLAB/Simulink®, RLC Circuits, Circuit Response Analysis, ODE Solutions

## 1. INTRODUCTION

An LC circuit, also called a resonant circuit, tank circuit, or tuned circuit, consists of an inductor, represented by the letter L, and a capacitor, represented by the letter C. When connected together, they can act as an electrical resonator, an electrical analogue of a tuning fork, storing energy oscillating at the circuit's resonant frequency. A major application of tuned circuits can be seen in television and Radio sets. Many different signals reach the antenna of a radio receiver at the same time. Within the radio or TV receiver, the actual "selecting" of the desired signal and the rejecting of the unwanted signals are accomplished by the use of tuned circuits. Another combination forms the class of filters. Filters are technical realizations of given system functions, which affect the spectral characteristics of an input signal in the main (Frequency selection). In the context of electro-technology the realizations with electrical networks interest as analog and digital circuits. Matlab/Simulink® can play a significantly important role in the pre-implementation stages of such circuits by providing a platform for modeling real world systems with appreciable accuracy.

## 2. ELECTRICAL ENGINEERING APPLICATION

### 2.1. Series RL circuit

The circuit consists of a resistance R and an inductor coil L connected in series. Fig.1. below gives a pictorial view of the typical circuit arrangement in consideration.

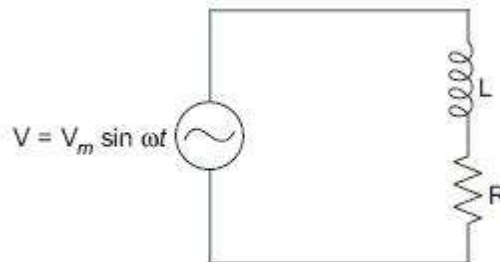


Fig.1. Typical RL series circuit

The first order ordinary differential equation<sup>2</sup> that describes a simple series electrical circuit with a resistor, inductor and sinusoidal voltage source is as follows:

$$\frac{di}{dt} + \frac{1}{\tau_{RL}} i = \frac{1}{L} v_s$$

$$\tau_{RL} = \frac{L}{R}$$

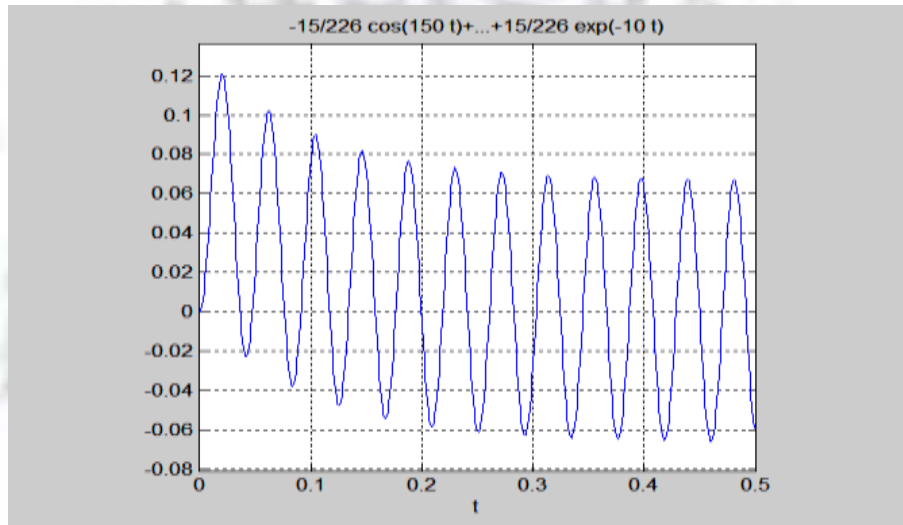
For this example, the inductance,  $L$  is 1 Henry and the resistance  $R$  is 10  $\Omega$ . The voltage source is sinusoidal with a peak voltage equal to 10 volts and an angular velocity of 150 rad/sec. The initial conditions have been assumed zero, i.e.  $i(0)=0$ .

The MATLAB statement written to solve (1)

symbolically is as follows:

```
c=dsolve('Dc=-10*c+10*sin(150*t)', 'c(0)=0');
```

```
ezplot(c, [0 0.5]), grid
```



**Fig. 2. Graphical result of the RL circuit dynamics**

```
dsolve('eq1,eq2,...','cond1,cond2,...','v')
```

Symbolically solves the ordinary differential equation(s) specified by (1), (2), using  $v$  as the independent variable and the boundary and/or initial condition(s) specified by  $cond1, cond2$ . The default independent variable is  $t$ . The letter 'D' denotes differentiation with respect to the independent variable; with the primary default, this is  $d/dx$ . A 'D' followed by a digit denotes repeated differentiation. For example,  $D2$  is  $d^2/dx^2$ . Any character immediately following a differentiation operator is a dependent variable. For example,  $D3y$  denotes the third derivative of  $y(x)$  or  $y(t)$ . Initial/boundary conditions are specified with equations like  $y(a) = b$  or  $Dy(a) = b$ , where  $y$  is a dependent variable and  $a$  and  $b$  are constants. If the number of initial conditions specified is less than the number of dependent variables, the resulting solutions will contain the arbitrary constants  $C1, C2, \dots$ . The output from the  $dsolve$  function is the symbolic solution to the differential equation. Next, the differential equation describing the RL electrical circuit is modeled using Simulink®. Simulink® is a block diagram environment for multi-domain simulation and Model-Based Design. It supports system-level design, simulation, automatic code generation and continuous test & verification of embedded systems. With Simulink®, the differential equation is described using blocks from Simulink® library. The complete Simulink® model for the electrical circuit is depicted in Fig. 3. Note that a time clock is added to the Simulink® model to enable the exportation of the simulation time to Matlab workspace for accessibility to plot simulation time against the current.

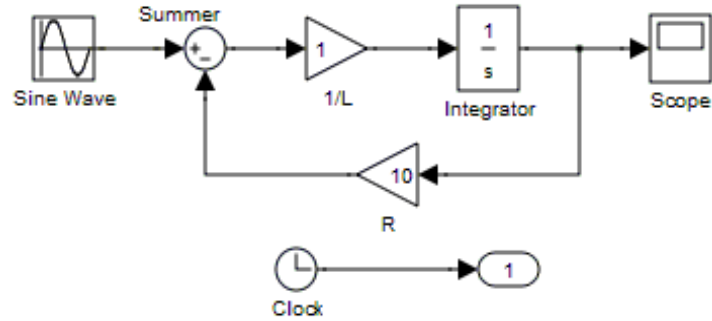


Fig. 3. Simulink® model of the RL series circuit

The results obtained from the Simulink® model of the RL circuit are presented below:

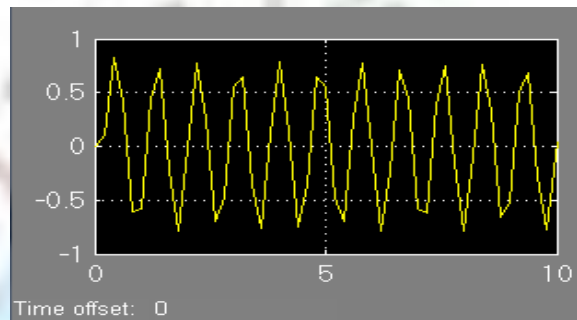


Fig. 4. Obtained result from Simulink® model of RL circuit

## 2.2. Series RC circuit

The circuit consists of a resistance R and an capacitor C connected in series. Fig. 4 below gives a pictorial view of the typical circuit arrangement in consideration.

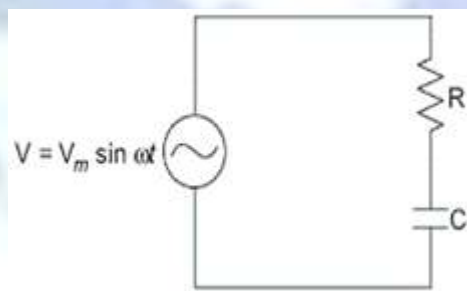


Fig.5. Typical RC series circuit

The first order ordinary differential equation that describes a simple series electrical circuit with a resistor, inductor and sinusoidal voltage source is as follows:

$$\tau_{RC} \frac{dv_C}{dt} + v_C = v_s$$

$$\tau_{RC} = RC$$

For this example3, the capacitance C is 4mF and the resistance R is 10 Ω. The voltage source is sinusoidal with a peak voltage equal to 85 volts and an angular velocity of 150 rad/sec. The initial charge on the capacitor has been assumed to be -0.05C, i.e.

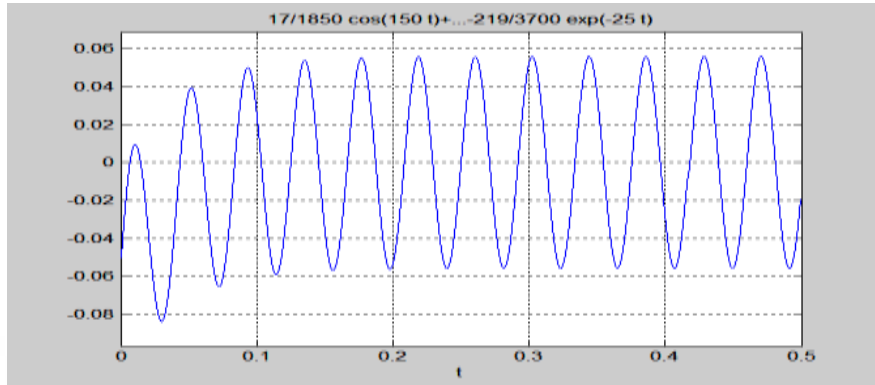
$$q(0) = -0.05C.$$

The MATLAB statement written to solve (3) symbolically (i.e., obtaining exact solution) is as follows:

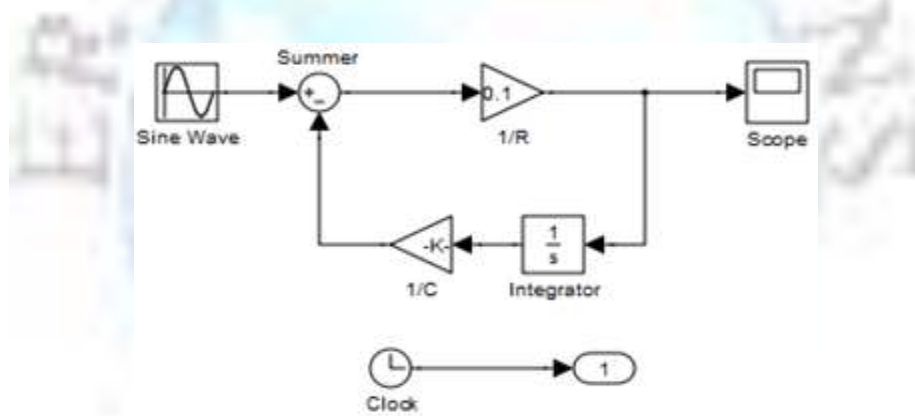
```
q=dsolve('Dq=-25*q+8.5*cos(150*t)', 'q(0)=-0.05');
```

```
i=diff(q);
```

```
ezplot(q, [0 0.5]), grid
```

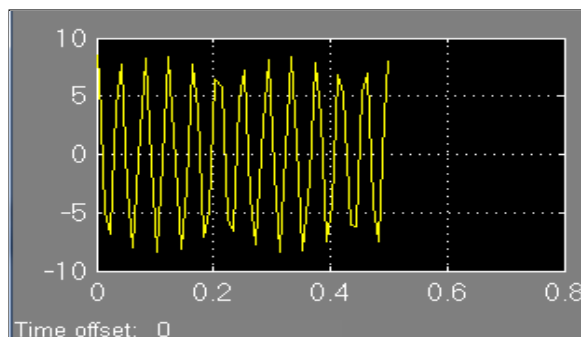


**Fig. 6. Graphical representation of RC circuit dynamics Next, the differential equation describing the RC electrical circuit is modeled using Simulink®.**



**Fig. 7. Simulink® model of the RC series circuit**

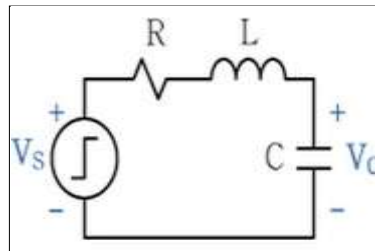
The following results are obtained when the said simulation is run:



**Fig. 8. Obtained result from Simulink® model of RC circuit**

### 2.3. Series RLC circuit

The circuit consists of a resistor, inductor and capacitor, all connected in series. The initial conditions are assumed to be zero. Fig. 9 below gives a pictorial view of the typical circuit arrangement in consideration



**Fig. 9. Typical RLC series circuit**

The first order ordinary differential equation that describes a simple RLC series<sup>4</sup> electrical circuit with a resistor, inductor, capacitance and sinusoidal voltage source is as follows:

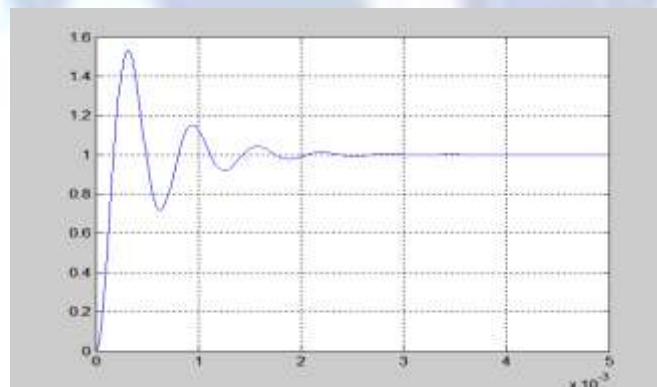
$$L \frac{d^2 q}{dt^2} + R \frac{dq}{dt} + \frac{q(t)}{C} = e(t),$$

where  $i(t) = \frac{dq}{dt}$  and  $e(t) = E_0 \cos(\omega t)$

For this example, the inductance is 1mH, the capacitance C is 1mF and the resistance R is 40 Ω. The voltage source supplies a step input of 1V.

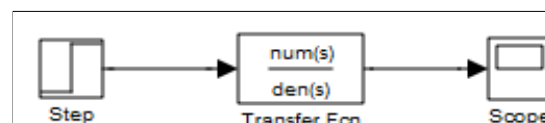
The MATLAB statement written to solve (5) symbolically (i.e., obtaining exact solution) is as follows:

```
w=10000;
t=0:0.00001:0.005;
vc=1-(cos(w*t)+.2*sin(w*t)).*exp(-2000*t);
plot(t, vc, 'blue'), grid
```



**Fig. 10. Showing response of RLC series circuit**

Next, the differential equation describing the RLC series electrical circuit is modeled using Simulink®.



**Fig. 11. Simulink® model of the RLC series circuit**

$$\frac{V_C(s)}{V_S(s)} = \frac{1}{LCs^2 + RCs + 1}$$

Fig. 12. Transfer function of series RLC circuit

### 3. MECHANICAL ENGINEERING APPLICATION

Given the following rotating mechanical system with gears<sup>5</sup>:

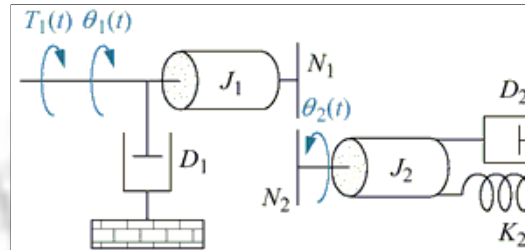


Fig. 13. Rotating mechanical system with gears

We will not explore the solution in its entirety. But we are giving a brief idea about how such a problem could be approached.

The Laplace transformed model of the above setup when modeled in Simulink®:

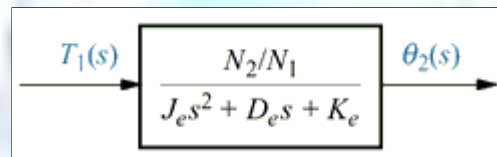


Fig. 14. Transfer function of mechanical system

$$D_e = D_1 \left( \frac{N_2}{N_1} \right)^2 + D_2 \quad (6)$$

$$J_e = J_1 \left( \frac{N_2}{N_1} \right)^2 + J_2 \quad (7)$$

$$K_e = K_2 \quad (8)$$

### CONCLUSION

In this paper, the authors presented three circuits with two methods each, using MATLAB and Simulink® to solve Ordinary Differential Equations (ODEs) of RL & RC filters and RLC Tuning Circuits. Example problem sets for each of two engineering disciplines have been provided. With the RL, RC and RLC circuits, we were able to show that using Matlab to analyze these circuits saves time and provides accurate results with minimal prerequisite knowledge of programming. And by using Simulink®, we were able to analyze the circuit using its transfer function representation. Thus, identification of the transfer function of the system is the very first step in obtaining its solution. We solved the circuits, using the MATLAB symbolic solver (dsolve) for analytical solutions, using the block diagram programming language-Simulink®. With less effort, these techniques allow scientists to solve engineering problems with an excellent graphical picture of the result. The purpose of comparing simulation results with analytical one is to ensure that the numerical approximation gives an acceptable result using the analytical one as a benchmark.

### REFERENCES

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