

Processing of Emergent Phenomena in Complex Systems

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Abstract: In this paper are described some properties of emergent phenomena and situations that appear in complex systems. The basic factors for the understanding to emergent processes are: - the level of realization and the description of the emergent process, - sharp change of the system structure (in our case represented by the violence of a structural invariant), - a self-organizing phenomenon (that forms the emergent situation). In this paper we will deal only with the factor of the violence of structural invariants (not with self-organizing process) that we use for the detecting possibility of appearance of emergent situation in the complex system. In this paper is used only one type of structural invariant – Matroid and Matroid Bases (M, BM) investigating the influence of their violation to interacting elements (components) in so called basic group. A simple calculus for the emergent situation appearance computation is introduced. The application of the presented approach and computation method is demonstrated by simple emergent situations such as the change of the strategy in swarm colony, traffic jam and floods.

Keywords: emergent situations; structural invariants; matroid bases; Ramsey numbers.

1 Introduction

After an enormous effort of physicists to discover beginnings of nature phenomena (as some typical nature shapes) and especially of the origin of our Universe (as the essential and largest emergence) it was formulated a vision of mathematical formalisms for future with the following properties:

- To be more qualitative than quantitative.
- To contain some traces of emergent phenomena that lead to nature shapes [10].
- To allow the direct work with shapes as with qualitative entities, not with their descriptions (e.g., by analytical mathematical descriptions).

Author [1] titled such a discipline Morphomatics.

Though the idea of Morfomatics is an excellent one in some views excites the conflict with our scientific education that is based on principles of causality and continuity. The motivation of Morphomatics is not too distanced from Zadeh's "computation in words" however in different level of generality.

In this paper are introduced 5 working hypotheses. We introduce them here at the beginning of the paper rather for excitation of curiosity of the reader.

Hypothesis 1(H1): The emergent phenomenon is induced by a sharp change of complex system structure ("jump on the structure") that comes from outside of this system. ♦

Hypothesis 2(H2): In order to catch and correctly interpret an emergent phenomenon is needed an appropriate level of the complex system description and the synthesis of so called Synthesis Space – (see Section 3 and Fig. 3.1). This synthesis is in exclusive competence of a human observer (human problem solver).

Hypothesis 3(H3): In case that we accept H1, the eventuality of an emergent phenomenon appearance may be detected as a sharp violence of complex system structure (or the violence of Structural Invariants respectively). ♦

Note 1.1: In Sections 5 – 10 will be illustrated various examples of violence of Structural Invariants. In case that the observed complex system (or its properties) is (are) described as a Matroid and its Bases, one of such violence can be represented as an extension of some Basis.

Hypothesis 4(H4): One of possibilities how to represent detection of appearance of emergent situation by extension of a matroid basis is to anticipate the increase of number of interacting elements in so called basic group (compartment). ♦

Hypothesis 5(H5): In order to attain an emergent phenomenon (e.g., by an extension of a matroid basis) the complex system increases the number of elements (components, properties, processes) in basic group (compartment) by a minimum number of elements (components, properties, processes). ♦

Notes to organization of the paper.

In Section 2 there are introduced some works that are relevant to the topic of our paper. Various views to formulation of the essence of emergent situation are introduced in Section 3. Section 4 is devoted to problems of interpretation of novel (unknown) concepts and together with the base of formal description of systems is demonstrated one of basic emergent processes that leads to discovery of the content of a new concept. In Sections 5 and 6 there are explained problems of the level of the phenomena description associated with the violence of structures and structural invariants. Section 7 is devoted to the intuitive understanding of self-organizing processes. Section 8 is the kernel section and contains a calculus for the detection of appearance of emergent situation. In Section 9 is the calculus introduced in Section 8 applied for 3 phenomena that lead to emergent situations that appear on macro-structures of our world as – “the change of behavior of termites”, “floods” and “traffic jams”.

2 Some Related Works

Works [1], [11] and [24] have in the context of this paper rather methodological significance. Great interest in detecting emergent situations, understood in a general sense, has appeared and still exists in the field of fault diagnosis [13], [14], [15]. However, emergent situations are also of interest in other fields. There are important emergent situations in safety engineering and in control of complex systems [17], [18], [19], [20]. Essential knowledge sources about complex systems, emergence phenomena and complexity are concentrated in [21], [22], [23]. Very original and ambitious seems to be, e.g., the interpretation of complexity as a linguistic variable [22]. A great effort in specification of the essence of emergent phenomena was recognized in [24], [8] but also in [3], [7] and [25]. Especially – first two works, though very different in environments, investigated relations between the nature of emergent phenomena and the sign system in which the emergent phenomena are represented. Other interesting works, e.g., [26], [27], [29], [30] and [31] will be commented in the appropriate places of the paper.

3 The standard and the emergent situations

Though it seems to be rather subjective, the platform for the distinguishing between standard and emergent situations is associated with the sign tool used for the description of reality (considered as “outside of the observer”). Following the line of the evolution of structural modeling from its beginning we find that in most cases human described the world in co-ordinates \langle “Position”, “Motion”, \langle P, M \rangle . By the time and with the completion of \langle P, M \rangle description it was discovered that this description is the appropriate and sufficient for the most of standard phenomena in nature and society and that this way leads to very universal models and modeling technologies. On the other side there remained phenomena and situations that were considered as rare, abnormal, surprising (we will speak about all of them as about surprising situations) that could not be described by \langle P, M \rangle approach. They were too quick, too unexpected or invisible. In the following classification (A, B, C) we narrow the field of such situations to the main interest of this paper – emergent situations. These descriptions illustrate rather our experience with such situations however there are not definitions of emergent situations. The substantial feature of all these situations is that are induced by a sudden considerable change of the system structure (“jump on the structure”) without a direct association to the history of the considered system.

A. Situations that were surprising and in a certain context can be considered as surprising

Their causes and their output forms (outputs, shapes) are known. It is possible to recognize them and predict their appearance. Examples of processes and systems that generate such situations are, e.g.: Belousov-Zhabotinski reaction; environments for initiation of solitons; Oregonator; Brusselator. They all belong to the field of Synergetics. Such situations are not considered (in this paper) as Emergent Situations.

B. Situations that were surprising and are still surprising

Their causes are not known however their output forms are known. Such situations are considered (in this paper) as:

Emergent situations (EMSs). They have the following properties:

b1) The situation appears suddenly without explicit association to situations of the previous relevant “time-space” context of the system. (It means that the situation is induced by something that comes from outside of observation time-space related to the considered system.)

b2) The situation appears as a discrete concept meaning-full in observer mind, e.g., behavior (of the group of termites), object (photograph), fact (floods), shape (e.g., design), property (to be attractive).

b3) The global reason of such a situation appearance is a violation of the system structure (not a violation of the system function). In other words – the situation is induced by a jump in system structure.

b4) The detailed reasons and the internal causes of the appearance of the situation are not known (i.e., it is impossible to propose a complete model of situation evolution and prediction).
 (However the shape of the emergent situation has been recognized (post factum) - i.e., is known how such a situation looks.)

b5) The appearance of such a situation is possible to detect.
 Situations that belong to this class are, e.g.: change of strategy of behavior in a swarm colony; appearance of floods; appearance of rough waves; traffic jams.

C. Situations that will be surprising

Neither their causes nor their output forms are known. Such situations are considered (in this paper) as **Emergent situations** (EMSs). They have the following properties:

- c1) The situation appears suddenly without explicit association with situations of the previous relevant context in the system. (It means that the situation is induced by something that comes from outside of observation time-space related to the considered system.)
- c2) The situation is assumed and surmised as a discrete concept meaning-full in observer mind, e.g., behavior (of the group of termites), object (photograph), fact (floods), shape (e.g., design), property (to be attractive).
- c3) The global reason of such a situation appearance is a violation of the system structure (not a violation of the system function).
- c4) No model of such a situation is available before it first operate.
- c5) The appearance of such a situation is possible to detect.

Situations that belong to this class are, e.g.: Possible instabilities in Ecosystems; sharp growth of extents of large towns; appearance of artifacts in nano-structures; situations of discoveries in conceptual design; the violation of super-symmetries in quantum mechanics. A sign description of Emergent Phenomena (EPs) (if we accept that some description exists) does not arise from nothing, but it begins with some essences developed originally for $\langle P, M \rangle$ description. Only after then we recognize an EP in this environment though it comes from outside and from the first look it has nothing common with phenomena and objects in $\langle P, M \rangle$. In Fig. 3.1. we can see the interaction between 5 environments (one of them is a human operator denoted here as an Observer). In order to form a background for the explanation of our approach we introduce in further text a few special concepts and constructions. Basic concepts are: Model of Reality, Interpretation Space, Sign Model, Connection, Observer. From the level of understanding that is situated above the level of Model of Reality, Interpretation Space, Sign Model, Connection, Observer there is possible to distinguish in the formation of these concepts three phases: Initial perception, Externalization phase and Operation phase.

Initial perception. In this phase interacts Model of Reality (M_R) with something what is called here Interpretation Space. For this purpose the Observer (O) associates the remembered indicators (i.e., the seen, heard and surmised indicators) with their semantic content. He develops (consciously or unconsciously) the Interpretation Space (IS) within this process and he learns the correct interpretation of these indicators. He uses for it memories, interpreted impressions, signs but also some additional informations discovered after the process. Substantial in this process is that a very serious part of this collection is gradually released from direct contact with M_R and that it gets very early into the form of signs, signs formations and sign models. The objects M_R , IS and SM are developed in this phase as a part of the **Observer** and this process is symbolically represented (in Fig. 3.1) by the function ξ_1 and by the expression

$$\text{Sem} \circ \xi_1 = \varphi_1. \tag{3.1}$$

Externalization phase. In this phase is executed a separation of M_R , IS and SM from the **Observer** and goes a preliminary testing of the connection

$$\langle \text{Sem}, I \rangle = \text{Sem} \circ I = I \circ \text{Sem}, \tag{3.2}$$

where Sem is one of semiotic mappings from IS to SM , I is one of interpretation mappings from SM to IS , ξ_1 is a mapping from M_R to IS , φ_1 is a mapping from M_R to SM and “o” is a composition operation for mappings.

Operation phase. This phase goes in two levels:

- Operative level, when a part of the mappings in Fig.3.1, after receiving a suspicious signal finds what does it mean. A standard way for it is the composition of mappings $\varphi_1 \circ \xi_2 \circ I$. (The **Observer**, which owns a natural **IS** uses for it mapping ξ_1 .)
- Design level that enables for a human solver an experimental design in **SM** and then by φ_2 or (I and ξ_2) to discover the “face” of the design results and the conditions of their implementation in **M_R**.

The previous text implicitly points out that „a reasonable“interpretation process starts no soon than after the externalization phase of the sign model and better after the operation phase. Interpretation is now the process when to the signs (symbols) that we use for expressions in the sign model are assigned semantic contents in the interpretation space. (No adventures of the research – only motions in **SM**, **IS** by connection $\langle \text{Sem}, I \rangle$!!)

Nevertheless – for the interpretation of an unknown object (situation, state structure) we need an emergent process that forms

Synthesis Space in which is the interpretation realized.

In this paper we introduce some formal objects and operations for the support and induction of this emergent interpretation process. As one of such operations (for the case that the considered system is described by matroid and matroid bases – as it will be explained in section 6) is the induction of an emergent process by the extension of some matroid basis at least by one element (Bila, 2010), (Bila, 2013), (Bila, 2014).

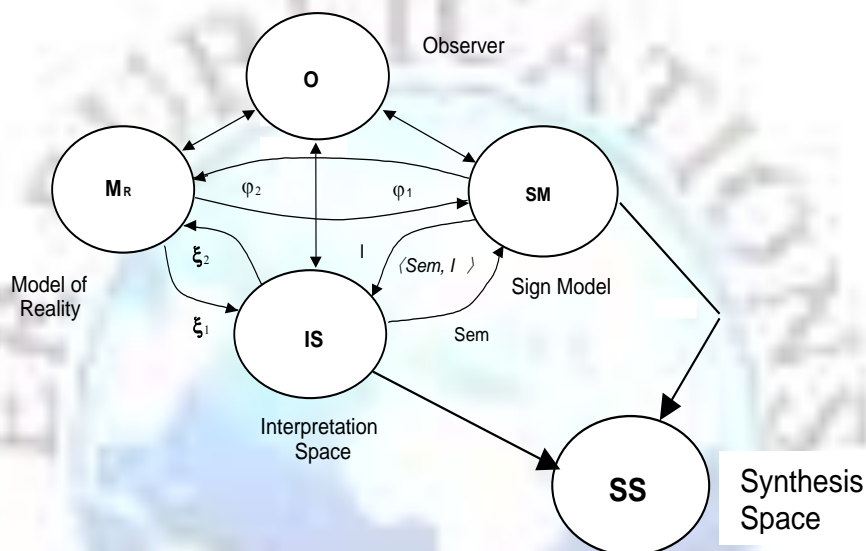


Fig. 3.1: Structural Modeling.

4 Interpretation weakness in the symbolical modeling is a consequence of an emergent process

The models that we use in this paper are structural models. Such models represent the world by means of two categories:

- substrate,
- system of bonds,

both associated with certain level of conceptualization. These categories are dissolved in our case into **M_R**, **IS**, **SM** and **SS** (after externalization phase).

4.1 Levels of the realization and the description of emergent phenomena

In our approach we have the only one level of the realization of emergent phenomena and it is Model of Reality (**M_R**). Model of Reality is not the same as „the Reality“ because it is a property of the **Observer**. The level of the description is associated with our perception and with transformations from Fig. 3.1. However neither **M_R** nor **IS** and **SM** do not provide a reliable environment for an emergent phenomenon that brings a new fact (or a solution of a problem) and is necessary to form (as a result of further emergence) the Space of the Synthesis (**SS**). The level of the description and the understanding of phenomena are very important and for each of them usually exists a special descriptive and interpretation apparatus. Various authors introduce various responses for the question “How many of such levels is and how they are ordered”. The author of work [26] introduces 8 levels (see table Tab.4.1), the author of work [27] names 40 descriptive levels (see table Tab.4.2). The substantial rule in our consideration about “modeling” of emergent situations is that we have to choose someone (it is associated with a process of knowledge acquisition) and to respect that in some descriptive levels some processes and objects from reality will be invisible. The level of the description involves the whole scheme in Fig.3.1, including the emergence of the Space of Synthesis (**SS**).

Note 4.1.1: Comments to Table 4.1:

The descriptive levels (rows) are ordered according to sequences of emergencies. E.g., the emergencies in level of Cell biology are consequences of phenomena in level Biochemistry.

The content of the right column does not correlate with content in the left column (e.g., Geology is not an analog of Cell biology).

The essences of systems are confused with the description of systems (e.g., in natural systems does not exist any "Material Science").

All these "peculiarities" have been undertaken from the source [26].

Note 4.1.2: Comments to Table 4.2:

Ch-quantity and mass are quantities recomputed for the diameter and the mass of a relevant ball.

Large moons of planets are called here lunets). For example Jupiter has lunets "Europa", "Io", "Callisto" and "Ganymedes".

4.2 Model of Reality (M_R)

We consider Model of Reality as a primary representation of a world fragment (in which we "are situated"). This representation is conditioned by our senses (may be amplified by artificial sensors and transducers (e.g., glasses), by the culture of our age ("what and how to recognize and to understand"), by our personal habits and activities. Reality in the society is unified by culture (including technology). This discussed (cognizable) "Reality" (in given context) is performed on the background of a deeper Reality that is considered as uncognizable. In other words – the discussed "Reality" is a model of a "real Reality" - M_R.

Examples: "forest", "ocean", "human brain", "theater stage", "crossroad", "computer network".

Table 4.1: Levels of the description

Number of the	Natural	Artificial systems
Level 8	Cosmology	Sociology/Economics/Pol
Level 7	Astronomy	Psychology
Level 6	Space, solar	Physiology
Level 5	Geology, Earth	Cell biology
Level 4	Material	Biochemistry
Level 3	Physical	Organic Chemistry
Level 2	Atomic Physics	Atomic Physics
Level 1	Particle Physics	Particle Physics

Table 4.2: Characteristics of some descriptive levels

The	Ch-quantity	Mass [kg]	Density	The field of a physical world
-20	1,5 E -19	1.58 E -37	1.12 E+19	The field of the weak
-16	4.86 E-15	1.664 E-	3.45 E14	Parameters of nuclei of atoms
-12	1.57 E-10	1.74 E -	1.06 E +10	Parameters of atoms
-10	2.83 E-8	5.66 E-15	5.92E+7	Parameters of viruses
-8	5.1 E-6	1.8 E-10	3.29 E+5	Eukariont cell
-4	1.6536 E-1	1.925 E-1	1.065 E+1	Parameters of man (embryo)
0	5.36 E+3	2.02 E+8	3.13 E-4	Gravitationally collapsed star
2	9.64 E+5	6.55 E+12	1.43 E-6	Large moons of planets

4.3 Interpretation space (IS)

Interpretation space is evolved gradually as the set of "records" that penetrate to observer mind from a "model of Reality" - M_R. Besides - it contains also our reflections of those "records", evaluations of those "records", their explanations, descriptions of their use, etc. Such clouds of receptions, observations, descriptions and knowledge are many for one object (or phenomenon). IS can be described as a stratified structure with correspondences between the strata. The existence of these correspondences (may be mappings) is well known from experience but their properties

are not known. And besides, it is important that our mind does not work with **IS** as with one whole but selects (according to working context) certain fragments of **IS** on which concentrates the attention. Summarizing – we imagine **IS** as a very large fragmental map of contents, that covers our world. Then we introduce **IS** as a multilevel structure

$$\mathbf{IS} = \langle \{^j\mathbf{IS}, {}^j\zeta\}, \{\gamma_{j,j+1}\}, j=1, \dots, k \rangle, \quad (4.1)$$

where $\{^j\mathbf{IS}\}$ is a set of strata of the Interpretation Space, ${}^j\zeta$ is a fragmental function ${}^j\zeta: {}^j\mathbf{IS} \rightarrow {}^j\text{Frag}$, where ${}^j\text{Frag} \subset {}^j\mathbf{IS}$ and $\{\gamma_{j,j+1}\}$ are mappings between strata ${}^j\mathbf{IS}$ and ${}^{j+1}\mathbf{IS}$. Further considerations about morphology of strata are superfluous.

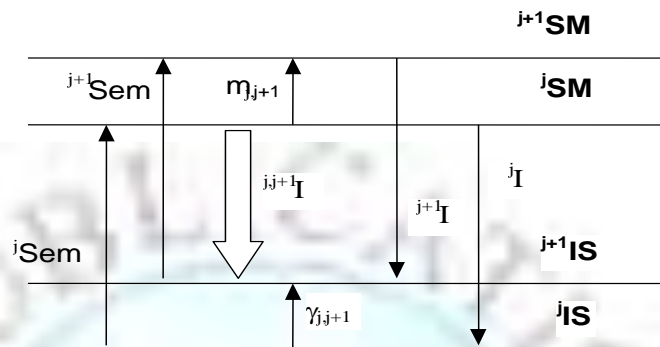


Fig.4.1 Connections.

4.4 Sign model (SM)

Sign Model is a multi-strata structure

$$\mathbf{SM} = \langle \{^j\mathbf{SM}\}, \{m_{j,j+1}\}, j=1, \dots, k \rangle, \quad (4.2)$$

where $\{^j\mathbf{SM}\}$ is the set of strata of the Sign Model and $\{m_{j,j+1}\}$ are transformations from stratum ${}^j\mathbf{SM}$ to stratum ${}^{j+1}\mathbf{SM}$.

The stratum ${}^j\mathbf{SM}$ may have various forms convenient for the construction of sign structures. One of such forms is the structure

$${}^j\mathbf{SM} = \langle \langle {}^jX, {}^jF, {}^jV \rangle, \langle {}^jG, {}^jW \rangle, \langle {}^j\mathbf{f}_1, \dots, {}^j\mathbf{f}_m \rangle, \langle {}^j\mathbf{v}_1, \dots, {}^j\mathbf{v}_m \rangle, {}^j\mathbf{I} \rangle, \quad (4.3)$$

where jX is the set of basic signs (symbols), jF denotes the set of symbols of operations (including constants as the operations of zero order), jV is the set of symbols of relations, jG is the symbol for a grammar of sign formations, jW is the set of words constructible by grammar jG . The rules of the symbolical manipulations written in a procedural form and effecting on the set of variables and constants are contained in the set ${}^j\mathbf{f}_1$, the operations effecting on elements from ${}^j\mathbf{f}_1$ are contained in the set ${}^j\mathbf{f}_2, \dots$ etc. The set ${}^j\mathbf{v}_1$ contains morphological relations on the set of sign formations jW , the set ${}^j\mathbf{v}_2$ contains morphological relations on the set ${}^j\mathbf{v}_1$, the set ${}^j\mathbf{v}_3$ contains morphological relations on the set ${}^j\mathbf{v}_2, \dots$, etc. ${}^j\mathbf{I}$ is the interpretation mapping in stratum j . ${}^j\mathbf{I}$ provides interpretation process from ${}^j\mathbf{SM}$ to ${}^j\mathbf{IS}$ and contains the tools for interpretation of the symbolical formations from jW into ${}^j\text{Frag}$, i.e.:

$${}^j\mathbf{I}: {}^jW \rightarrow {}^j\text{Frag}, \text{ where } {}^j\text{Frag} \subset {}^j\mathbf{IS}. \quad (4.4)$$

Note 4.3.1: Comparing with **IS** it is not necessary to be careful with fragmental function of **SM**. We know from experience that all-covering sign model does not exist. There are thousands of sign and especially symbolical models that need not mutual direct connections and on the contrary that can live purposefully in a greater system of sign models.

4.5 The relation between the interpretation space and the sign model

IS and **SM** are in a mutual development in the Initial perception phase (see Section 3). **IS** is still under the influence of the \mathbf{M}_R and adapts objects, phenomena, stories and events into internal representations. Similarly – **SM** is modified by activities of observing, experimenting and modeling Reality, or by activities of self development (observation, experiments, modeling and simulation). A rough image of these mutual effects and influences was illustrated in Fig.3.1.

The assigning of the internal representation of **IS** in stratum “j” to signs and sign formations in ${}^j\text{SM}$ is represented by mapping ${}^j\text{Sem}$ (called here “semiotic mapping”). Similarly - the assigning of signs and sign formation in ${}^j\text{SM}$ to semantic concepts and meanings in ${}^j\text{IS}$ is represented by mapping ${}^j\text{I}$ (called here “interpretation mapping”).
 Collection of mappings

$$\langle {}^j\text{Sem}, {}^j\text{I}, \gamma_{j,j+1}, m_{j,j+1} \rangle, \tag{4.5}$$

is called in this paper “ ${}^j\text{Sem} - {}^j\text{I}$ connection” and will be denoted $\langle {}^j\text{Sem}, {}^j\text{I} \rangle$. All “ ${}^j\text{Sem} - {}^j\text{I}$ connections” together (for all $j=1, \dots, k$) are denoted as $\langle \text{Sem}, \text{I} \rangle$.

$$\langle \text{Sem}, \text{I} \rangle = \langle \langle {}^j\text{Sem}, {}^j\text{I} \rangle \rangle, j=1, \dots, k, \tag{4.6}$$

Connections are defined in detail according to objects and phenomena in strata ${}^j\text{IS}$ resp. ${}^{j+1}\text{IS}$. The whole diagram related to connections is in Fig.4.1.

The diagram contains one empirical knowledge that is in our paper one of the principal: mapping ${}^j, {}^{j+1}\text{I}$ is **not attainable**, in general, by a constructive way using the known components $\gamma_{j,j+1}, m_{j,j+1}, {}^j\text{I}$ (i.e. does not hold “equation” (4.7))

$${}^j, {}^{j+1}\text{I} = \gamma_{j,j+1} \circ {}^j\text{I} = {}^{j+1}\text{I} \circ m_{j,j+1}. \tag{4.7}$$

Criticized non correctness of expression (4.7) implies two facts:

- If some unknown sign (sign structure) comes on stratum ${}^j\text{SM}$, there is impossible to discover its semantic content in stratum ${}^{j+1}\text{IS}$ by a constructive way.
- If some unknown sign (sign structure) comes on stratum ${}^j\text{SM}$, there is impossible to transform it in stratum ${}^{j+1}\text{SM}$ and then to discover semantic content of the transformed structure in stratum ${}^{j+1}\text{IS}$.

This **interpretation weakness** is a consequence of an emergent process, that comes from outside (and may be from “a local future”). Our formal and cognitive tools (including “mappings, connections and inferences”) provide the way from a history, from a past time, from past knowledge. The following simple example 4.3.1 demonstrates the problem of emergence of some novel object and its interpretation.

Example 4.3.1: Let us consider Fig. 4.2. In this case is considered a set of named objects (e.g., buildings of university campus) $A1, \dots, A4$ in stratum ${}^{j+1}\text{SM}$, their descriptions ($DA1, \dots, DA4$), their semantic contents - SemCont of $DA1, \dots, \text{SemCont}$ of $DA4$ and an unknown object AX with its description DAX . This artifact (sign formation, drawing, construction, sculpture) seems to be meaningful and we are interested in its functionality, semantic content and significance (SemCont of AX and SemCont of DAX).

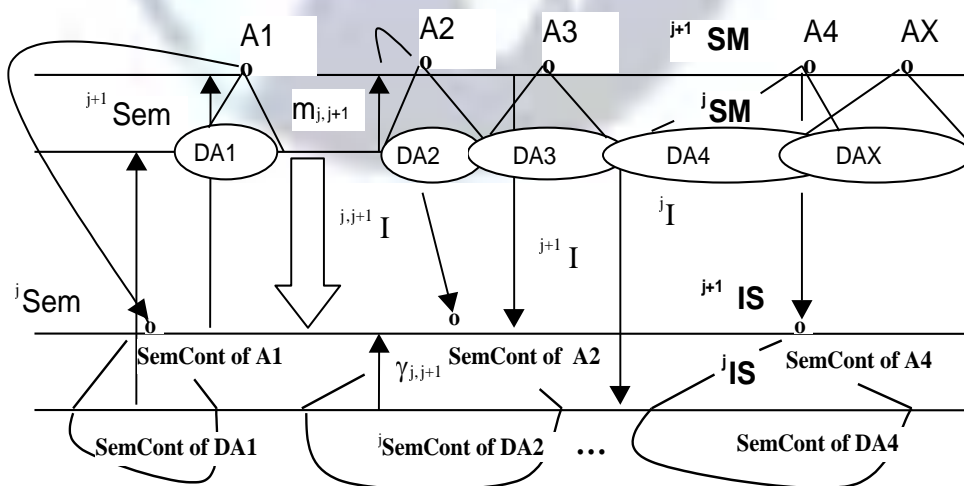


Fig. 4.2. The unavailable interpretation of class AX.

There are seen identifiers of objects (classes) $A1, \dots, A4, AX$ in stratum ${}^{j+1}\text{SM}$ and their descriptions ($DA1, \dots, DA4, DAX$) in stratum ${}^j\text{SM}$. (As an example – $A1, \dots, A4$ are buildings in university campus, e.g., rectorate, main building of faculty of civil engineering, water laboratory, etc. Their sign descriptions (e.g., as vectors of values of descriptive

parameters (height, color, shapes of windows, ...) are in stratum ${}^j\text{SM}$.) In stratum ${}^{j+1}\text{IS}$ are semantic contents of objects (classes) SemCont of A1, ..., SemCont of A4. These semantic contents are associated with mind of human observer, depend on his knowledge and there are not represented as sign or other morphological structures (“everyone knows what is and how functions the rectorate”, ..., “what is a water laboratory”, ...). Similarly – without morphological representation are considered semantic contents of the descriptions SemCont of DA1, ..., SemCont of DA4 (“everyone knows what is and what measures the height, what is the color”, ...).

The set of quadruplets of the type $\langle A_i, \text{DA}_i, \text{SemCont of } A_i, \text{SemCont of DA}_i \rangle$, for $i = 1, \dots, n$ (in our case is $n = 4$) is called Interpretation Reference Context (IRC).

$$\text{IRC} = \{ \langle A_i, \text{DA}_i, \text{SemCont of } A_i, \text{SemCont of DA}_i \rangle, \dots, \langle A_n, \text{DA}_n, \text{SemCont of } A_n, \text{SemCont of DA}_n \rangle \}. \quad (4.8)$$

Interpretation process is concentrated on object (class) AX and its sign description DAX. It is needed to discover its unknown semantic content in strata ${}^{j+1}\text{IS}$ and ${}^j\text{IS}$. (E.g., “AX is a grey 30 m high concrete column in front of the building of faculty of architecture”. What is it?) Interpretation in this case depends on the formation of the Space of Synthesis (SS – see Fig.3.1) that belongs exclusively into competence of human brain and mind. In other words – we are depended on the quality of the human solver brain and on the excitation of the brain by the proposed sign formations from IRC. Unfortunately we shift by this way to the field of psychology and semiology and this is not our goal. In the next section will be introduced another approach that is associated by the task of interpretation but leads to the detection of the appearance of emergent situation. In this task we begin from conditions of the appearance of emergent situation in the space of interacting elements from which we derive the changes in this space that induce an emergent phenomenon. The processes realized between strata ${}^j\text{SM}$ and ${}^j\text{IS}$ allow us to derive $({}^{j,j+1}\text{I})^{-1}$ – see Fig. 4.3. The demonstration of the proposed method will be done for emergent situations of the class (B) - the change of the strategy in swarm colony, traffic jam and floods.

Note 4.3.3: If we want to detect emergent situation from the class C, we will need to derive ${}^{j,j+1}\text{I}$, that is by the proposed method available however it is much more difficult.

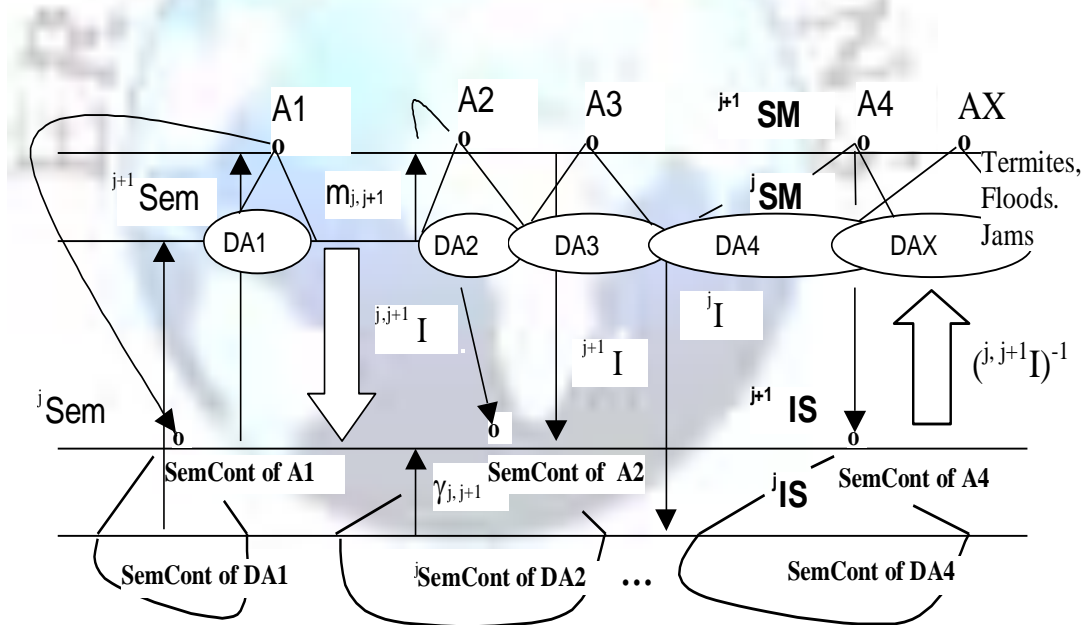


Fig.4.3. The detection of emergent situations

5 Violence of system structure – jumps in the structure

According to H1 (from Introduction) – “the emergent phenomenon is induced by a sharp change of complex system structure (“jump on the structure”) that comes from outside of this system“ and it influences the style of representation of emergent phenomenon and emergent situation.

Let us introduce a few examples of structures:

- Structure of a family house. (A topology of walls, of rooms, positions of windows, of doors, construction of the roof).

- Structure of molecule. (Coordinates of atoms, the bonds between atoms, types of bonds).
- Structure of a control circuit. (The type of controlled system, the type of inputs and outputs, the type of a controller, the type of feedback.)
- Structure of region. (Positions of towns, villages, characteristics of the landscape, positions of factories, type of the industry, roads, water channels. Bonds between introduced components.)
- Structure of matrix. (Rows and columns of the matrix, the bonds between them (dependence, linear independence).
- Matroid. (The carrier of the matroid, independent sets, bases.)

General types of violence of the structures:

- Disappearance or addition of elements or components,
- Disappearance or addition of bonds,
- Deformation of elements, components and bonds.

All introduced examples correspond to idea of the change of the structure. However – for the idea of an emergent phenomenon these changes must be induced from outside of the system and changes must be sharp in time. More – emergent phenomena are induced rather by additions (extensions) than reductions.

Nevertheless – neither this specification is not sufficient. As for symbolical structures as are matrices and matroids, sharp changes of structure are easily demonstrated, e.g., the change of the rank of matrix by the addition of an independent row or the addition of an independent element to matroid bases. However let us imagine the extension of a region by addition of some village. Does it induce emergent phenomenon? From our experience – rather not.

6 Structural Invariants and their violence

The structure of a complex system can be defined conventionally as something that for a certain sufficient time provides for the system a reliable platform for the evolution and realization of its functions. Structural invariants (SIs) are concentrated factors of the structure that are able to represent and demonstrate its changes. In [7], there were introduced and described some essential types of SIs, e.g., the pairs (Matrix, Rank of Matrix), (Matroid, Bases of Matroid), (Dulmage-Mendelsohn Decomposition, Tree Ordering), (Hasse Diagram, Set of Associated Rules) and (Set of Situations, Algebra of Transformations).

There have been tested the following violence of SIs that induce the appearance of Emergent Situation (EMS).

- (Matrix, Rank of Matrix) – the addition of linearly independent row or column.
- (Matroid, Bases of Matroid) – the addition of at least one element to basis.
- (Dulmage-Mendelsohn Decomposition, Tree Ordering) – violence of tree ordering.
- (Hasse Diagram, Set of Associated Rules) – violence of set of rules,
- (Set of Situations, Algebra of Transformations) - the deformation of group operation and as a consequence - construction a false element into the group carrier.

In the following sub-section we will deal only with one SI - (Matroid, Bases of Matroid) – (M, BM).

6.1. Structural Invariant (M, BM) and its violence

At first a few general facts:

Matroid has the following pleasant properties:

- It is possible to construct it for each set of elements (carrier of the system) when we have the relation of independence or when they are given independent sets.
- If we have relation of independence, there are investigated all elements (of the system) with regard to relation of independence.
- If the relation of independence exists it is easy (in most cases) to associate it with a semantic content (according to real conditions).
- Matroid is usually introduced as the following structure

$$M = \langle X, \text{IND}, \{N_1, N_2, \dots, N_n\} \rangle = \langle X, B \rangle, \quad (6.1)$$

where X is the ground set of elements (components), IND is a relation of independence, N_1, N_2, \dots, N_n are independent sets and B is a set of matroid bases. Matroid bases are maximum (according to cardinality) independence sets.

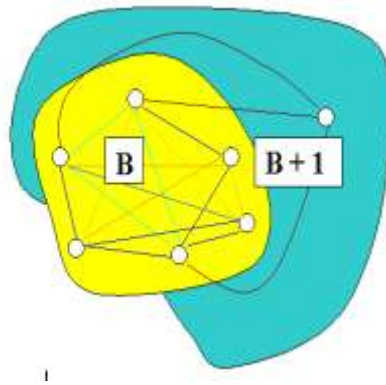


Fig. 6.1 Extension of a matroid basis by one element

The violence of this SI is considered (in this paper) as an extension of a matroid basis at least by one element (and it will be considered as an indicator of emergent situation appearance). In case that relation IND is considered as a binary relation it is possible to use the following consequences: The bases (\mathbf{B}) will be constructed as perfect sub-graphs (in perfect graph on X). The independent (\mathbf{B}) and dependent elements ($X \setminus \mathbf{B}$) in the perfect graph are easy constructed by coloring the edges by two colors and the formalism of Ramsey numbers - $R(\#\mathbf{B}, \#Y)$ is offered to be used. The extension of one of bases (\mathbf{B}) by one element is illustrated in Fig 6.1. The perfect graph in Fig.6.1 has six nodes and 15 (brown) edges. Coloring the edges by green and blue colors, there appears at least one perfect sub-graph with 3 nodes and 3 edges (basis \mathbf{B}) – for example the green one. For extension of \mathbf{B} by one element (into $\mathbf{B}+1$ with 4 nodes) we need to add at least 3 elements (that are invisible here). Historical note: Till now there are known only some Ramsey numbers (RNs), e.g.: $R(3, 3) = 6$, $R(3, 4) = 9$, $R(3, 5) = 14$, $R(3, 6) = 18$, ..., $R(3, 15) = [73, 78]$, ..., $R(4, 4) = 18$, $R(4, 11) = [96, 191]$, ..., $R(6, 10) = [177, 1171]$, ..., $R(10, 10) = [798, 23\ 556]$, ..., $R(19, 19) \geq 17\ 885$. (In computations in Section 10. will be used known table quantities from [30].)

Steps of the work with matroid:

- S1. The relation of independence is introduced.
- S2. The matroid is constructed.
- S3. The bases on the matroid are extracted.
- S4. The extension of some basis is executed.

Note 6.1.1: According to Hypothesis 5 (from the Introduction) the system selects as optimal those Ramsey numbers for which is needed to add the minimum elements for the extension of basis by one element.

Example 6.1.1: $\#X = 1600$. ($\#\mathbf{B} = 11$ for $\#X \geq 1597$) and for one element extension of Basis ($\#\mathbf{B} = 12$ for $\#X \geq 1637$) is needed to add 40 elements.

7. Self-organizing processes

Self-organizing process (SOP) has the following properties:

- It begins and operates on complex system without the influence of man or without the influence of another rational substance.
- Between the beginning and the end of SOP (and also during the run of SOP) is increased the organization of the complex system.
- The result of SOP is a sequence of situations that an educated human observer recognizes as purposeful, smart, meaningful, ..., or inharmonic, shabby, unorganized,, etc.

In this paper we will not deal with special SOPa but we introduce here some operators (physical or artificial) that contain SOPa:

- Chaos, (damage, formation of unidentified mixtures).

- Whirls (centroidal ordering - on the surface of water, in the Universe).
- Gravitation.
- The memory (as the ability to turn back to known facts and concepts).
- Self assembly of a chemical robot.
- The evolution of viruses.
- Synthesis of proteins.

Our method detects certain numbers of interacting elements (in a given level of the realization and the description) that – in case of arriving of an appropriate SOP – results in emergent situations (EMSs).

8. Approximation of EMPa in quantitative characteristics of complex system

For understanding to EMPa could be enough to apply the extension of BM by at least one element and then experimentally investigate if EMS comes or not.

In this section will be presented the way how to approximate the discrete phenomenon of EMS and the corresponding change (jump on the structure) by the motion of values of some measurable variables.

However – it will be seen here that neither extension of BM nor the external description of EMP is not universal and the success of our computations depends on the selection and the development of level of description.

8.1 The calculus for computation of approximations in descriptive levels

Speaking about EMPa we need a set of elements (components) in so called basic group (X) (compartment) for each level of the description that is associated with the elements of a matroid bases. On the other side we need the idea about the consequences of the emergent phenomenon representing them in some quantitative values. For these purposes was developed a special formalism (that is universal one and it can be used for every level of the description). The formalism has the following basic concepts:

The Power of the emergent phenomenon $\Delta H_p(\mathbf{B}+1)$ that has an intuitive meaning (e.g., damage of houses by floods) and corresponds in our case to the extension of a BM by one element (in some level of the description).

The estimation of $\Delta H_p(\mathbf{B}+1)$ is done by means of quantities of external variables x_i , $i = 1, \dots, n$ estimated for emergent (x_{iem}) and for nominal (x_{inom}) situations:

$$\Delta H_p(\mathbf{B}+1) = \left(\sum_{i=1,n} (\omega_i (x_{iem}/x_{inom})^2) \right)^{1/2}, \text{ for } i=1, \dots, n, \quad (8.1)$$

where ω_i are quotients of importance (computed by Saaty method [12]). The power of emergent phenomenon is computed (8.1) as a dimensionless real number.

This external estimation of the power is associated with the extension of BM by the following relations:

$$H_p(\mathbf{B}+1) = H_p(\mathbf{B}) + (u/c) H_{COM}(\mathbf{B}), \quad (8.2)$$

$$\Delta H_p(\mathbf{B}+1) = (u/c) H_{COM}(\mathbf{B}), \quad (8.3)$$

The variable $H_{COM}(\mathbf{B})$ from expression (8.2) represents the complexity of the basic group of the complex system. $H_{COM}(\mathbf{B})$ is considered here as a number of elements of matroid basis. The variable (u/c) represents the “intelligence” of SOP that will execute the emergent phenomenon.

The Complexity of the matroid basis extended by one element $H_{COM}(\mathbf{B}+1)$ is expressed as

$$H_{COM}(\mathbf{B}+1) = H_{COM}(\mathbf{B}) + u H_p(\mathbf{B}), \quad (8.4)$$

where u is the quotient of self-organization $u \in \langle 0, c \rangle$, c is the limit of self-organization (both depends on the emergent environment). $H_{COM}(\cdot)$ is approximated in our case by number of elements of basis, $\Delta H_p(\mathbf{B}+1)$ is the needed power of the phenomenon expressed in % (for example, contribution for 20 % is calculated as $(120/100) = 1.2$).

In order to use expression (8.4) for computation of the number of matroid basis is necessary to set up the quotient (u/c) . For the beginning set up we use Table.8.1 and in further is applied the iteration procedure (that will be explained after the moment).

Table 8.1: Quotient (u/c) for various Complex Systems

(u/c)	Classes of Complex Systems - the examples of Complex Systems
0.1	“Inanimate” natural systems – avalanches, floods, earthquake, tsunami.
0.2	Lower alive systems – bees, termites, ants.
0.3	Mixed natural and artificial systems – road and airplane transport systems, ecological systems.
0.4	Consciousness of man, symbolical functions, design.
0.5	Systems of the synthesis of the sign (symbolic) emergent spaces.
0.6	Advanced chemical systems able of transmutations or of the synthesis of chemical robots.
0.7	Complex systems coordinating billions of elements (components) – human brain, ocean, atmosphere.
0.8	Metaphorical systems – systems with sharp reduction of information, holographic systems.
0.9	Supervising intelligent systems not available for us since – e.g., consciousness of Universe.
1	Complex systems with super-intelligence.

Note 8.1.1: Table 8.1 has been formed on the base of empirical experience and by verification computations of examples. Nevertheless it is good to add small comments.

We are speaking about a complex system that contains in general many mutually interacting elements (components). Quotient (u/c) expresses the ability of this system to execute emergence, to realize emergent phenomenon and to achieve (eventually) emergent situation. In table Table 8.1 are introduced examples of complex systems that correspond to given quotient (u/c). Unfortunately is impossible to compare them each other because the environment and the conditions of the existence of complex systems, conditions of emergences and the levels of the description are different. As for example (u/c) = 0.3 is introduced for road and airplane systems, where are involved into consideration not only transport means but also weather, atmosphere, and so on. These systems have the lower affinity to emergences than, e.g., chemical systems which realize synthesis of chemical robots (self assembly – without a positive contribution of man activities, of course).

After the initial set up of (u/c) we verify the correctness of the relation between $\Delta H_p(\mathbf{B}+1)$ and (u/c). The quantity of (u/c) is computed by iteration procedure using the equation of information homogeneity (8.5). (Equation (8.5) was derived in problem solving field and though it seems to be distanced from our context, it is from many reason appropriate. For example – if we consider a trivial problem “to close the door”, does not matter if it is solved by the wind or by a man”).

$$I_{i1} * N_{i1} = I_{i2} * N_{i2} = I_{i3} * N_{i3} = \dots = I_{im} * N_{im} = I_s * 1, \quad (8.5)$$

where I_{ij} for $i, j=1, \dots, m$ are quantities of information needed for the solution of a problem by N_{ij} steps. I_s is a super quantity of information by which is the problem solved in one step.

$$(u/c) = (I_{ij} / I_s) = k * (1 / N_{ij}), \text{ for some “i,j” adequate to “a solved problem”}, \quad (8.6)$$

where k is a normalization constant, $k \in (0, \text{Upper bond})$. As a solved problem is considered in our context the induction of connectivity between the interacting elements from the set X (the ground set of elements of Matroid) and N_{ij} is the number of bonds between elements on X . The maximum of simple bonds between elements on X is

$$N_{ij} = (\#X) (\#X - 1) / 2. \quad (8.7)$$

The iteration procedure starts with the quantity $\Delta H_p(\mathbf{B}+1)$ computed by external variables and with some given quantity $(u/c)_0$. Then is computed the number of elements in the matroid basis $\#\mathbf{B}$ and from tables [30] is selected a lowest Ramsey number $R(\#\mathbf{B}, Y)$, (Y is an integer from the table [30]) and then is computed the number N_{ij} by (8.8):

$$N_{ij} = (R(\#\mathbf{B}, Y)) (R(\#\mathbf{B}, Y) - 1) / 2. \quad (8.8)$$

For computation $(u/c)_1$ is used normalization constant k

$$(u/c)_1 = (I_{ij} / I_s) = k * (1 / N_{ij}). \quad (8.9)$$

In case when the absolute normalized value of the difference is higher than 0.15

$$\delta = |((u/c)_1 - (u/c)_0) / (u/c)_0| \geq 0.15, \quad (8.10)$$

the procedure is repeated with higher $(u/c)_0$ (for the case $(u/c)_1 < (u/c)_0$) and v.v.

8.2 Computation with $H_{COM}(B)$, $H_P(B)$ and $\Delta H_P(B+1)$

We respect the following sequence of steps:

- C1. The quantity of $\Delta H_P(B+1)$ for a given emergent environment is estimated.
- C2. The quantity of (u/c) for a given emergent environment is set up.
- C3. The number of elements of Basis B is computed.

$$H_{COM}(B) = \xi((c/u) \Delta H_P(B+1)) = \#B, \quad (8.11)$$

where $\xi(x)$ is the nearest higher complete number (e.g., $\xi(2.5) = 3$).

- C4. The number ($\#X$) of interacted elements needed for an emergent situation appearance is computed.

Example 8.1.1: For $\Delta H_P(B+1) = 2$ and $(u/c) = 0.5$ is computed $\#B = 4$ and Ramsey numbers: 9, 18, 25, 41, 61, ..., 282. For the extension of the basis by one element ($\#(B+1) = 5$) we find Ramsey numbers: 14, 25, 49, 87, ..., 464. Optimal pairs of Ramsey numbers for the induction of an emergent situation are (9→14/5), (18→25/7), (25→25/0), (41→49/8), (61→87/26), etc.

Table 8.2: Table for computation of cardinality of basis B (#B)

$\Delta H_P(B+1)/(u/c)_0$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	10	5	4	3	x	x	x	x	x
1.5	15	8	5	4	3	x	x	x	x
2.0	20	10	7	5	4	4	x	x	x
2.5	25	13	9	7	5	4	4	x	x
3	30	15	10	8	6	5	5	4	x
3.5	35	13	12	9	7	6	5	5	x

Table 8.3: Table for computation of constant k

$\#B/(u/c)_0$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
3	x	x	x	6	7.5	x	x	x	x
4	x	x	10.8	14.4	18	21.6	25.2	28.8	x
5	x	18.2	27.3	x	45.5	54.6	63.7	72.8	x
6	x	x	x	x	76.5	91.8	x	x	x
7	x	x	75.9	101.2	126.5	x	x	x	x
8	x	75.6	x	151.2	x	x	x	x	x
9	x	x	189	252	x	x	x	x	x
10	90.3	180.6	270.9	x	x	x	x	x	x
12	x	x	513.3	x	x	x	x	x	x
13	x	469.2	x	x	x	x	x	x	x
15	304.2	608.4	x	x	x	x	x	x	x
20	605.0	x	x	x	x	x	x	x	x
26	1117.5	x	x	x	x	x	x	x	x

Note 8.2.1: In Table 8.3 are introduced only minimal values “k” for the case $Y = 3$ (Y from the expression (8.8)).

9. Appearance of emergent situations in simple cases

The emergent phenomenon in a complex system has in external view three phases (that Nature realizes in non living and in living systems):

- Formation of basic groups of interacting elements (formation of compartments).
- Impact with constraint (constraints),
- Self-organizing phenomenon.

While the first phases are not necessarily “visible”, the third phase realizes the external image of the emergent phenomenon. The illustration of these phases will be done on emergent phenomena that were introduced in work [13] - the change of behavior of termites in cases of external danger, floods and a traffic jams:

The change of behavior of termites in cases of external danger

One dimensional case. Termites are in the mutual interaction not only by tactile contacts but especially by the transfer of concentration of the activation pheromone.

- Formation of compartments will be illustrated (in sub section 9.1) by computation of possible Ramsey numbers.
- Constraint – a contact with enemy community, that induce the necessity of reformation of a part the own population into soldiers,
- Self-organizing phenomenon – spreading of concentration of activation pheromone by a domino effect in compartments.

Floods

Two dimensional case. Water carriers (streams, rivers, ponds) are in a mutual interaction by transfer and transport of accumulated water mass.

- Formation of compartments will be illustrated (in sub section 9.2) by computation of possible Ramsey numbers.
- Constrains – limited transfer and transport constraint in motion of accumulated water into following flow,
- Self-organizing phenomenon – spreading of impact with constraint in compartments by a domino effect against the direction of moving water mass in individual flows.

Traffic jams

Three dimensional case. Cars go one after the other and their mutual interaction is done by the motion of transport column.

- Formation of compartments will be illustrated (in sub section 9.3) by computation of possible Ramsey numbers.
- Constraint – e.g., the narrowing of road strips into one line, road accident, etc.
- Self-organizing phenomenon – spreading of impact with constraint in compartments by a domino effect against the direction of a column of cars motion.

9.1 Change of the strategy of behavior in a Swarm colony

The changes of termite behavior (e.g., the formation of attack patterns) are induced by the changes of termite pheromone concentration.

There exists a basic group of termites (compartment) that causes the highest increase of pheromone concentration and induces the spreading of this message for other termites. Using the calculus that had been explained in Section 8 we may compute the number of termites in the compartment and the number of termites that are needed to be added to this basic for the induction of the emergence behavior.

In this case is natural to use one external variable x_1 ... concentration of pheromone in basic group with quantities $x_{1nom} = 50\%$ and $x_{1em} = 75\%$. The quotient of importance $\omega_1 = 1$.

Applying expression (8.1) we compute

$$\Delta H_p(\mathbf{B}+1) = \omega_1 (x_{1em}/x_{1nom}) = 75/50 = 1.5.$$

For the first estimation of $(u/c)_0$ we set up $(u/c)_0 = 0.2$ (from Table 8.1).

Using expression (8.11) we compute the first approximation of cardinality of the basis $\#\mathbf{B}$

$$H_{COM}(\mathbf{B}) = \xi (\Delta H_p(\mathbf{B}+1) / (u/c)_0) = \xi (1.5 / 0.2) = 8 = \#\mathbf{B}.$$

For $\#\mathbf{B} = 8$ we find in tables [30] minimum Ramsey number $R(\#\mathbf{B}, Y) = R(8, 3) = 28$.

Using expression (8.8) we compute

$$N_{ij} = (R(\#\mathbf{B}, Y)) (R(\#\mathbf{B}, Y) - 1) / 2 = (R(8, 3) R(8,3) - 1) / 2 = 14 * 27 = 378.$$

From Table.8.3. we use normalization constant $k = 75$.

$$(u/c)_1 = (I_{ij} / I_s) = k * (1 / N_{ij}) = 75 / 378 = 0.199$$

that fulfils condition (8.10). And there is possible to work with $\#B = 8$.

For $\#B = 8$ there are RNs: 28, 56, 84, 127, 495, 1031, 1870.

For $\#(B+1) = 9$ (the extension by one element) there exist RNs: 36, 69, 121, 316, 1713, 565, 153, 780, 6588.

Optimal pairs of RNs for induction of emergent situation (according to H5 from section 1), are (28→36/8), (56→69/17), (127→153/26), (84→115/31), (495→565/70).

Note 9.1.1: In reality: So that, e.g., if it is discovered basic group of termites with 127 members which inclines to union with at least 26 another termites, then may be expected an appearance of some EMS.

9.2 Floods

Floods are induced not only by the rain and aggregated volumes of water but by a self-organizing process in which interact water carriers (brooks, streams, ponds, ...). Appearance of emergent situation (in this case - floods) depends on the initiation of "self-organizing" process and on the number of elements (brooks, streams, ponds, ...) in a basic group. "Self-organizing" process is hard to describe but is possible to compute numbers in basic groups (compartments).

Using the calculus that had been explained in Section 8 we may compute the number of water carriers in a basic group. In this case we use two external variables:

x_1 ... an average height of water levels in water carriers (rivers, brooks, streams, basins, ... that interact in the considered region),

x_2 ... number of buildings in danger of flooding, in the considered region.

There are considered the following nominal and emergent quantities:

$x_{1nom} = 3$ m and $x_{1em} = 8.1$ m. The quotient of importance $\omega_1 = 0.4$,

$x_{2nom} = 1000$ and $x_{2em} = 2500$. The quotient of importance $\omega_2 = 0.6$,

Applying expression (8.1) we compute

$$\Delta H_P(\mathbf{B}+1) = ((\omega_1(x_{1em}/x_{1nom}))^2 + (\omega_2(x_{2em}/x_{2nom}))^2)^{1/2} = ((0.4*2.7)^2 + (0.6*2.5)^2)^{1/2} = 1.85.$$

For the first estimation of $(u/c)_0$ we set up $(u/c)_0 = 0.09$ (from Table 8.1).

Using expression (8.11) we compute the first approximation of cardinality of the basis $\#B$

$$H_{COM}(\mathbf{B}) = \xi (\Delta H_P(\mathbf{B}+1) / (u/c)_0) = \xi (1.85) / 0.09 = 21 = \#B.$$

For $\#B = 21$ we find in tables [30] Ramsey numbers $R(\#B, Y)$: $R(21, 3)$, $R(21, 4)$, $R(21, 5)$, $R(21, 6)$, $R(21, 7)$, $R(21, 8)$: = {122, 242, (≤ 10626), (≤ 53130), 1214, 1328}. Using expression (8.8) we compute

$$N_{ij} = (R(\#B, Y)) (R(\#B, Y) - 1) / 2 = (R(21, 3) R(21, 3) - 1) / 2 = 61 * 122 = 7442.$$

From Table 8.3 we use for normalization constant $k = 664$ and

$$(u/c)_1 = (I_{ij} / I_s) = k * (1 / N_{ij}) = 664 / 7442 = 0.08922$$

that fulfils condition (8.10). And there is possible to work with $\#B = 21$.

For $\#(B+1) = 22$ (the extension at least by one element) there exist Ramsey numbers $R(\#(B+1), Y)$: $R(22, 3)$, $R(22, 4)$, $R(22, 5)$, $R(22, 6)$: {125, 282, 422, 1070}.

Optimal pairs of RNs for induction of emergent situation (according to H5 from section 1) are (122→125/3), (242→282/40).

Note 9.2.1: More real is the second pair of Ramsey numbers - (242→282/40).

Note 9.2.2: The computation of other optimal pairs of Ramsey numbers was complicated in this case by small knowledge of precise values of Ramsey numbers, e.g., $(R(21, 5) \leq 10626, R(21, 6) \leq 53130), R(22, 7) \leq 34\ 337\ 716$.

9.3 Traffic Jams

Traffic jam is an emergent result of interaction of many transport elements and factors (cars, traffic lights, structure of transport symbols, weather, hours in the day time, etc.) and mainly of an "self-organizing phenomenon" that forms the external performance of traffic jam. Here is considered only an emergent phenomenon that starts in a basic group of cars.

"Self-organizing" process is hard to describe but is possible to compute numbers of elements in a basic group.

In this case we use three external variables:

x_1 ... number of cars within 1[km],
 x_2 ... driver visibility in [m], (in computation is used quantity $(1/x_2)$),
 x_3 ... number of bottlenecks (e.g., places of some repairs where is allowed to use only one road strip),

There were considered the following nominal and emergent quantities:

$x_{1nom} = 28$ and $x_{1em} = 120$. The quotient of importance $\omega_1 = 0.3$,

$x_{2nom} = 100$ and $x_{2em} = 40$. The quotient of importance $\omega_2 = 0.3$,

$x_{3nom} = 2$ and $x_{3em} = 0.4$. The quotient of importance $\omega_3 = 0.4$,

Applying expression (8.1) we compute

$$\Delta H_P(\mathbf{B}+1) = ((\omega_1(x_{1em}/x_{1nom}))^2 + (\omega_2(x_{2em}/x_{2nom}))^2 + (\omega_3(x_{3em}/x_{3nom}))^2)^{1/2} = (0.09 * 18.36 + 0.09 * 6.25 + 0.16 * 4) = 1.7$$

For the first estimation of $(u/c)_0$ we set up $(u/c)_0 = 0.3$ (from Table 8.1).

Using expression (8.11) we compute the first approximation of cardinality of the basis # \mathbf{B}

$$H_{COM}(\mathbf{B}) = \xi (\Delta H_P(\mathbf{B}+1) / (u/c)_0) = \xi (1.7 / 0.3) = 6 = \#\mathbf{B}.$$

For # $\mathbf{B} = 6$ we find in tables [30] minimum Ramsey number $R(\#\mathbf{B}, Y) = R(6, 3) = \{18, 41, 87, 165, 298, 495\}$.

Using expression (8.8) we compute

$$N_{ij} = (R(\#\mathbf{B}, Y) (R(\#\mathbf{B}, Y) - 1) / 2 = (R(6, 3) R(6, 3) - 1) / 2 = 9 * 17 = 153.$$

From Table 8.3 we use for normalization constant $k = 46$ and

$$(u/c)_1 = (I_{ij} / I_S) = k * (1 / N_{ij}) = 46 / 153 = 0.3006$$

that fulfils condition (8.10). And there is possible to work with # $\mathbf{B} = 6$.

For # $\mathbf{B} = 6$ there are RNs: $\{18, 41, 87, 165, 298, 495\}$,

For # $(\mathbf{B}+1) = 7$ (the extension by one element) there exist RNs: $\{23, 61, 143, 298, 540 \dots\}$.

Optimal pairs of RNs for induction of emergent situation (according to H5 from section 1) are $(18 \rightarrow 23/5)$, $(41 \rightarrow 61/20)$, $(87 \rightarrow 143/56)$, $(165 \rightarrow 298/133)$, $(495 \rightarrow 540/45)$, $(298 \rightarrow 298/0)$.

Note 9.3.1: In this example is seen that the number of elements needed for the induction of EMS in the beginning pairs increases and then decreases even to zero $(298 \rightarrow 298/0)$. (These results have been indirectly supported in [10].)

Note 9.3.2: Detecting emergent situations is hard to provide their reproducibility. It means, e.g., that the sharp increase of number of vehicles under certain conditions and in a certain segment of highway from 165 to 298 km that induced a traffic jam today, it will cause it (under the same conditions and in the same segment) tomorrow. And besides – it is clear that our method and computations lead to qualitative results and they should not be the point for question of the type “How we recognize if the traffic situation is the jam or it is not yet the jam?”.

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Conclusions

The method presented in the paper computes “the distance of a possible appearance of an emergent phenomenon in a given complex system”. This computation has been performed in this paper only for emergent situation for which are known consequences and not causes (type B). Anticipating the fact that emergent phenomena are induced by sharp changes of complex system structure there have been investigated in the paper relevant types of such changes (because not each type of structure change leads to emergent phenomena). There has been demonstrated that processing of emergence and emergent phenomena needs to establish three following concepts:

- The level of the description (including the interpretation process).
- Formalism for the jump violence of the system structure (violence of structural invariants).
- The self-organizing process that itself realizes the emergence and emergent phenomenon.

A simple method that computes a possible appearance of emergence and emergent phenomenon for one special Structural Invariant (Matroid and Base of the matroid) has been introduced. The method introduced in sections 6 and 8 was explained in this paper on examples of macrostructures ($10^{-2} \div 10^2$ m) (termite colonies, floods and traffic jams). (The application of the method in the level of atoms (10^{-10} m) and in the nuclei of atoms (10^{-15} m) was proposed in [31].)

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