

# CMA blind equalization for OFDM based Multi-hop Cooperative Systems

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**Abstract:** There has been emerging study on OFDM integrated multihop cooperative system and this paper focuses on investigation of such systems with blind equalization techniques. Relaying capable OFDM network incorporated with Constant Modulus Algorithm addresses towards solution of problems regarding delay and large values of MSE in multihop systems utilizing conventional equalization. The optimization is achieved through fast convergence at remarkable low values of Mean Square Error and results proved that the proposed algorithm provides robustness against fast fading and non-correlated noise among the nodes. Overall system presents an optimal design for CMA channel estimator for OFDM AF Relay system and performance of derived expressions are verified through simulation results.

**Keywords:** OFDM, MSE, CMA, STBC, SNR.

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## I. INTRODUCTION

Demand of dynamically adaptable equalizer has become very common today to get noise free detection as wireless networks are getting dynamic towards bandwidth and performance demands. Among various types of this noise removing block, blind equalizers having filters which avoids the use of training signal are preferred in most cases due to many reasons including efficiency declining overheads, failure in fast fading phenomena and scenarios where this training string is unknown to new-fangled receivers. Moreover, modern WLAN systems discourage long frame structures mostly resided by overhead burden instead of data blocks. Systems employing multiple antennas become impracticable when dynamic resource management is to be performed or links and individual nodes are size limited for diversity gain. Multihop relaying gives solution to the problems caused by increasing the density of intermediate stations for high data rates in wireless networks by communicating the information through multi-node transmission instead of single hop. Intermediate nodes perform well under fading phenomena and intense level of noise if channel state information is available at their end which affects the accuracy of the whole system. OFDM based multihop systems make use of high data rate support of multicarrier physical layer scheme accepted worldwide by many wireless standards including IEEE802.11 and WiMax [1].

In OFDM, multiplication with orthogonal subcarriers is done and then decision making process is performed to recover the transmitted data with better results even without using overburden transmission of pilot signals for adaptive receiver. Pilot aided equalization techniques either inserts pilot symbols to all subcarriers (block-type) or to some specific subcarriers (comb-type) and then channel frequency response is obtained on these training symbols. Channel estimation is then performed by interpolation on all subcarriers.

The major contribution includes proposal of an optimized approach of fast convergence and reduction of MSE of CMA relays in an OFDM based multihop cooperative network. Moreover, the goal is to investigate different properties of normalized CMA relay via simulations and to show that the proposed technique provides robustness and improvement in performance. Many today's applications demand high data rates over multihop networks including real time video camera surveillance and others which is possible through multicarrier modulation techniques which demands channel state information to be available at each intermediate node for frequency-flat channels. The paper is organized in the following order. The OFDM based multihop cooperative system architecture is explained in the section II and mathematically expressed in section III. The channel model with normalized CMA relays is introduced in section IV followed by the simulation setup details in section V. Finally, the results are presented in section VI with conclusions in section VII.

## II. PROBLEM STATEMENT

We emphasize that non constant-modulus periodic pre-coding will incur a loss in spectral efficiency. The reason being for a given average transmit power, the minimum distance between symbols decreases and in order to achieve a given bit error rate, a higher average transmit power is required. Therefore, even though the transmission rate is kept constant, there is a loss in spectral efficiency. In this sense, it is desirable to keep the amplitude variation in the pre-coding

sequences as small as possible. Finally, because of the time-varying transmit SNR, the power amplifiers in the system need a higher dynamic range, which increases hardware complexity.

### III. SYSTEM ARCHITECTURE

Infrastructure for today's mobile communication demands dynamic approaches in providing power and bandwidth. Arranging intermediate nodes in a spatially dispersed manner within the network is preferred over using multiple receivers as they provide diversity gain through their cooperation. Communication among these nodes could be simply relaying the incoming information after amplifying the signal under standard IEEE802.16 or relay nodes may decode-and-forward the input. In the latter case, each hop is equipped with N relays and forwarding process is optimized for minimizing the cooperation among them by selecting the relay giving best performance. This process gives full spatial diversity by selective relaying as compared to N-fold maximum gain by conventional decode-and-forward network. However full gain diversity is achieved only if each subcarrier selects an individual relay at each hop, causing traversing of subcarriers over different passageway, instead of forwarding whole OFDM data pack [2]. The supposed system consists of Amplify and forward relays. For N subcarriers, transmitted signal for kth subcarrier and ith symbol can be represented as

$$x(i, n) = \text{IFFT}_N[X(i, k)] = \frac{1}{N} \sum_{k=0}^{N-1} X(i, k) e^{j2\pi nk/N} \quad (1)$$

The received signal at the first hop becomes

$$Y(i, k) = X(i, k)H(i, k) + W(i, k) \quad (2)$$

The signal  $y_{i,k}$  is extracted from  $Y_{i,k}$  for channel estimation. The correction is applied and signal is forwarded to the next hop. And so on. At Destination node, after channel estimation, maximum likelihood detection is performed. The estimated frequency signal, for s number of constellation points, is given as

$$\tilde{X}(i, k) \arg \min |Y(i, k) - \tilde{H}(i, k)S|^2 \quad (3)$$

The channel response in time and frequency domain (N point FFT) is given as

$$h(i, n) = \sum_{l=0}^{L-1} h_l(i) \delta(n - T_l) \quad (4)$$

$$H(i, n) = \text{IFFT}_N[h(i, k)] = \frac{1}{N} \sum_{k=0}^{N-1} h(i, k) e^{-j2\pi nk/N} \quad (5)$$

In an OFDM system, data is transmitted in symbols  $\mathbf{X}_i$  of length N each. The symbol undergo an IFFT operation to produce the time domain symbol  $\mathbf{x}_i$  given as

$$\mathbf{x}_i = \sqrt{N} \mathbf{Q} \mathbf{X}_i \quad (6)$$

Where Q is the N×N IFFT matrix. Channel h of length L+1 with CP, results in an super-symbol of  $\bar{\mathbf{x}}_i$  producing the output sequence of  $\bar{\mathbf{y}}_k$ . The channel response can be assumed of the following order.

$$H = \begin{pmatrix} H_{0,0} & \cdots & H_{0,M-1} \\ \vdots & \ddots & \vdots \\ H_{N-1,0} & \cdots & H_{N-1,M-1} \end{pmatrix}$$

Here  $H_{M,N}$  is defined as

$$H_{M,N} = \frac{1}{N} \sum_{n=0}^{N-1} H_n^k e^{-j2\pi(M-k)n/N} \quad (7)$$

OFDM operation creates an input matrix of considerably large dimensions and using conventional techniques of channel inversion for channel estimation produces huge amount of calculations at a node. To avoid this, proposed design uses FFT to avoid the inverse operation. Second approach used for this design is to make use of useful information in OFDM symbol tail end i.e. cyclic prefix for statistical analysis in blind channel estimation.

### IV. CMA FOR OFDM MULTIHOP RELAY NETWORK

Cost function of CMA as presented in [10] is given as

$$J(n) = E \left[ \left( |y(n)|^p - R_p \right)^2 \right]_{\text{with}} R_p = E \left[ |u(n)|^{2p} \right] / E \left[ |u(n)|^p \right] \quad (8)$$

The Tap weight vector  $w(n)$  for tap input vector  $u(n)$  is updated with step size  $\mu$  in accord with the stochastic gradient algorithm generally expressed as

$$w(n+1) = w(n) + \mu u(n)e^*(n) \quad (9)$$

Using the stochastic gradient algorithm as the update of coefficients, the equalizer can be expressed as

$$\begin{aligned} C(n+1) &= C(n) - \mu \hat{\nabla} J(n) \\ &= C(n) - \mu e(n) Y^*(n) \end{aligned} \quad (10)$$

Where  $C(n)$  is the inverse of  $H(n)$ ,  $\mu$  is the step size,  $Y(n)$  is the equalizer input vector. Error equation becomes

$$e(n) = y(n)(R_p - |y(n)|^2) \quad (11)$$

Putting both error equations side by side with same amount of error we have generated reference signal as

$$d(n) = y(n)(R_2 - |y(n)|^2) + y(n) \quad (12)$$

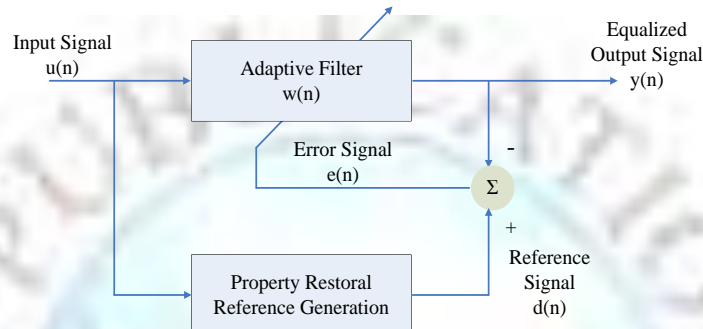


Figure 1: CMA Equalizer Structure

In the algorithm, the step size  $\mu$  should be carefully selected. A small step size will lead to slow convergence rate, whereas a large step will result into large residual error or even worse, algorithm divergence. Under weak conditions, the CMA performance function is shown to characterize the ISI sufficiently, where its stochastic minimization can be performed with no knowledge of the transmitted data. However, the algorithm performance after the initial convergence is not satisfactory, due to the large residual variance of the error signal. For non-constant modulus signals, the error variance could be intolerable. The channel autocorrelation in frequency domain is

$$\begin{aligned} R_{HH}(m, n) &= E[H(i, m)H^*(i, n)] \\ &= E\left[\sum_{k=0}^{N-1} h(i, k)e^{-j2\pi km/N} \cdot h^*(i, k)e^{-j2\pi km/N}\right] \end{aligned} \quad (13)$$

The eigen value matrix  $[L_0, L_1, \dots, L_{n-1}]$  for  $R_{hh}$  is

$$\lambda_k = \sum_{n=0}^{N-1} R_{HH}(0, n)e^{-j2\pi nk/N}, k = 0, 1, \dots, N-1 \quad (14)$$

Average power for N OFDM symbols for each tap is

$$P_{CMA}(k) = \frac{1}{N} \sum_{i=0}^{N-1} |\tilde{h}_{cma}(i, k)|^2, k = 0, 1, \dots, N-1 \quad (15)$$

The channel estimation presented in this paper modifies the simple linear precoding estimation as presented in [4] of statistical class. It precodes the OFDM symbol and exploits the induced correlation properties of the receiver to make the estimation. The algorithm transforms the  $i^{\text{th}}$  OFDM block of N information symbols  $d_{i,k}$ , where k is the subcarrier index which can be presented as T here. The precoding for channel estimation in time domain is given as

$$s_{i,k} = \frac{1}{\sqrt{1+|A|^2}} (d_{i,k} + (-1)^k A d_{i,T}) \quad (16)$$

where  $\sigma_q^2$  is the variance of  $q_{i,k}$ . Therefore the channel can be estimated from the correlation as

$$\hat{H}(k) = \frac{\sqrt{1+|A|^2}}{\sigma_q^2 A} z_k \quad (17)$$

Consider the correlation of the signals on the Kth and Tth subcarriers,

$$Z_{k,R} = E[y_{i,k} y_{i,T}^*] \quad (18)$$

Based on  $z_{k,T}$ , an estimate of channel  $H(k)$  can be obtained as:

$$\hat{H}(k) = \begin{cases} \frac{1+|A|^2}{(-1)^k A + (-1)^{K+T} |A|^2} Z_{K,T}, & k=0,1,\dots,N-1 \\ Z_{K,T} & K=T \end{cases} \quad (19)$$

The channel estimate can be further improved, if the length of the channel response  $h(l)$ ,  $l=0,1,\dots,L-1$ , where  $L < N$ . The latter length constraint can be enforced by performing IDFT on  $H(k)$ , setting to zero the last  $N-L$  samples of the IDFT output, and then performing an  $N$ -point DFT on the result. This procedure is referred to as denoising. A potential problem with the estimate might arise when the  $T$ -th carrier is in deep fade, in which case  $Z_{k,T}$  is close to zero for all  $k$ 's. However, it is interesting to note that, at the receiver, any subcarrier can play the same role as the  $T$ -th one. Let  $R$  be an integer in  $[0,1,\dots,N-1]$  with  $R \neq T$ .

$$Z_{k,R} = E[y_{i,k} y_{i,R}^*]$$

The channel response  $H(k)$  can be estimated as

$$\hat{H}(k) = \begin{cases} \frac{1+|A|^2}{(-1)^k A + (-1)^{K+T} |A|^2} Z_{K,T}, & k=0,1,\dots,N-1 \\ Z_{K,T} & K=R \\ \frac{1+|A|^2}{(-1)^k A^* + (-1)^{K+R} |A|^2} Z_{K,R}, & k=T \end{cases} \quad (20)$$

A criterion for selecting  $R$  can still be implemented at the receiver as

$$R_0 = \arg \max_R \sum_{K=0}^{N-1} |Z_{k,R}|^2 \quad (21)$$

This step would require the estimation of the entire correlation matrix of the received blocks, thus introducing a small increase in complexity. CMA provides amplitude response of the channel but terminates the need of phase information; however phase can be estimated by using a phase tracking method of multiplying the decision input with conjugate of the decision output. Phase tracking equation can be obtained by adding a step size to update the recursive equation as

$$\hat{\phi}(n+1) = \hat{\phi}(n) - \mu, \text{Im} \left\{ \hat{X}^*(n) Z(n) \exp(-j\hat{\phi}(n)) \right\} \quad (22)$$

## V. MULTIHOP SELECTIVE OFDM RELAYING

In this paper, a selection procedure between AF and DF is placed at every relay on the basis of received signal integrity which switches to either protocol after checking the input for CRC. Consider a  $N$ -Carrier OFDM based selective cooperative network with  $L$  relays at  $M$  number of Hops so that at hop  $i$ , where  $i=1,2,\dots,M-1$ , the relay  $j$ , where  $j=1,2,\dots,L-1$ , which performs best in terms of combined SNR  $\rho_{l,n,i}$  is first-rated for forwarding, i.e.  $\max_l \sum_{n=1}^N \rho_{l,n,i}$ , where  $n=1,2,\dots,N-1$  are subcarriers of OFDM technique. In the first time slot, Source node  $S$  put out the information signal to all  $L$  relay terminals. Since the channel between them is a simple SISO, it is separately estimated by adaptive filter at each relay. OFDM transforms the frequency selective fading channel to number of parallel flat frequency channels. Instead of forwarding OFDM block as a whole, the relay at a specific hop having highest quality subcarrier signal is responsible for broadcasting data at this particular subcarrier. The destination node  $D$  should have the knowledge of frequency response of  $L$  number of links to balance the effect of ISI and other multipath fading effects.

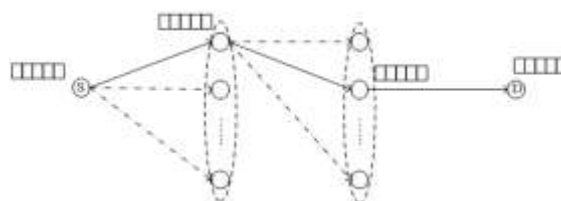


Figure 2: Selective OFDM relaying in 3-hop network

The end-to-end outage of system is given as

$$p_{out}^{OFDM} = 1 - \sum_{i=1}^M (1 - p_{out,i}^{OFDM}) = 1 - (1 - p_{out,M}^{OFDM})^{M-1} (1 - p_{out,M}^{OFDM}) \quad (23)$$

Where  $p_{out,a}^{OFDM}$  is the output probability of hop 1,2,...,M-1, given as

$$p_{out,a}^{OFDM} = 1 - \int_0^\infty \dots \int_0^\infty (1 - p_s(\epsilon | \rho_s)) \cdot f(\rho_1, \rho_2, \dots, \rho_N) d\rho_1 \dots d\rho_N \quad (24)$$

So L-Fold diversity gain is attained. In 3-Hop network, each having mutually independent channel characteristics which means that optimal combiner at the receiving nodes will give lowest output SNR, since there will be no correlation of noise among the cooperative nodes. For this type of set of branches, the optimal combiner is a maximal ratio combiner [6]. The received SNR at the destination node D is  $SNR_{s,D}$ . Consider first case when only amplify and forward protocol relay without diversity exists between source S and destination D terminals. SNR at D is given as

$$SNR_{s,D} = \frac{1}{\frac{N_0}{\epsilon_S |a_{S,D}|^2}} \quad (25)$$

SNR at D when n number of hops exists in the channel is

$$SNR_{D-1,D} = \frac{|a_{D-1}|^2}{E|\beta_{D-1}|^2 + \frac{N_{D-1,D}}{\epsilon_{D-1}|a_{D-1,D}|^2}}, \quad (26) \gamma_{D-1,D} = 1/SNR_{D-1,D}$$

If we substitute this matrix inversion operation, high level of complexity will be reduced at hops. For simplicity, STBC is used for encoding in multihop system. Space Time Block Code encoded cooperative system can be expressed in vector form For N number of OFDM subcarriers, input data  $x(n)$  encoded through STBC of index s and channel frequency response are represented as

$$X_{2s+i} = \text{diag}[X_{2s+i}(0), X_{2s+i}(1), \dots, X_{2s+i}(N-1)] H_{2s+i}^j = [H_{2s+i}^j(0), H_{2s+i}^j(1), \dots, H_{2s+i}^j(N-1)]^T \quad (27)$$

Received signal  $Y_{2s+i}$  contains convolution of the above two vectors as addition of both systems i.e. BS to RS and BS to DS.

## VI. SIMULATIONS AND RESULTS

A typical Rayleigh fading channel is considered as a static channel with the impulse response of

$$h_k = N(0, \frac{\sigma_k^2}{2}) + j * N(0, \frac{\sigma_k^2}{2}) \quad (28)$$

The performance of the proposed method is compared to that of [5] with channel length of L=6. Each tap was modeled as a Rayleigh Random variable aided by Gaussian noise expressed as sum of complex valued low-pass responses which makes i.i.d (independent and identically distributed) time dependent channel coefficients. Amplitude of channel coefficients follows Rayleigh distribution while channel taps decays exponentially. 3000 blocks of OFDM were simulated. Some system specifications are listed in Table 1.

Table 1. System Specifications

Parameter/ Feature	OFDM system
Bandwidth	20Mhz
No of Data Subcarriers	48
Subcarrier Frequency Spacing	0.3125Mhz
IFFT/FFT period	3.2μs
Symbol Interval	4μs
Modulation	4 QAM

The superiority of the proposed CMA channel estimator over other blind equalization methods is evaluated through MATLAB simulations. In order to illustrate the different features of proposed model of CMA for MIMO and Multihop networks, broad simulations are performed. Consider a 48-Carrier based OFDM multihop network with L relays each at M hops. The outage is plotted against PSNR per hop for selective OFDM multihop network with multiple values of L when M=3, showing slight increase in gain with increase of number of relays at hops.

### A. Constellation Diagrams for RCMA Relay

The interference cancelling ability of the system is demonstrated in the below figure through spectral plots of interference coupling signal and the retransmitted signal.

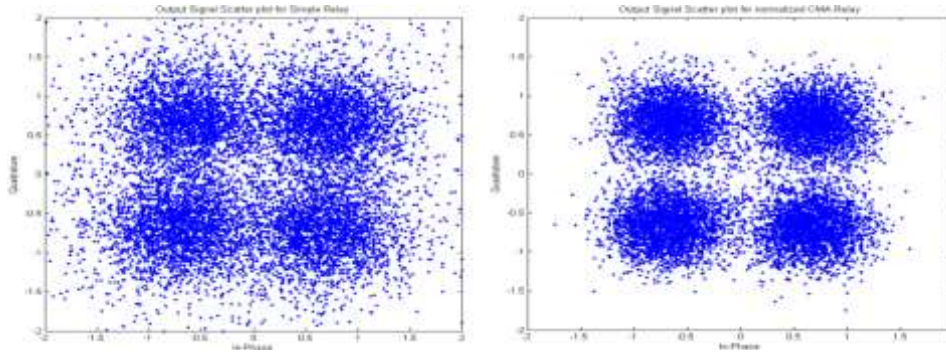


Figure 3: Constellation/Scatter plots for input and output of equalizer

### B. NMSE for Matched SNR

The MSE of the proposed equalization algorithm is given as

$$\begin{aligned} \phi_{MSE}(SNR, i) &= E \left[ \frac{1}{N} \sum_{k=0}^{N-1} |\tilde{H}_{Relay}(i, k) - H_{Tx}(i, k)|^2 \right] \\ &= E \left[ \frac{1}{N} \sum_{k=0}^{N-1} |\gamma(l) \tilde{h}_{Relay}(i, l) - H_{Tx}(i, l)|^2 \right] \end{aligned} \quad (29)$$

It can also be calculated as

$$NMSE = \frac{\sum_{k=0}^{N-1} |\hat{H}_0(k) - H(k)|^2}{\sum_{k=0}^{N-1} |H(k)|^2} \quad (30)$$

It can be observed that NMSE of the proposed RCMA equalizer with subcarrier correlation is much less than LMMSE algorithm presented in [7] over the SNR range from 0dB to 25dB.

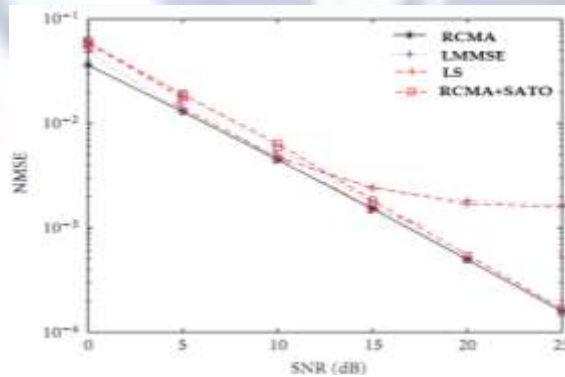


Figure 4: NMSE of RCMA with LMMSE and LS in Rayleigh Fading Channel

### C. Bit Error Rate (BER) Comparison:

Fig. 9 shows the BER of conventional CMA, Decision Directed Modified CMA (presented in [2]) and Regularized CMA. Here linear interpolation is adopted to attain the channel frequency response at all the OFDM subcarriers after the responses obtained by LMMSE and LS estimators. After that, maximum likelihood detection is used to obtain the estimated signal. It is clearly shown that BER waterfall curve falls very quickly for RCMA as compared to others over 0dB to 35dB. The RCMA is about 7-10dB better than DD-MCMA and CMA for OFDM based Multihop cooperative systems. Propagation delay varies due to the relaying process which is normalized through the use of Cyclic Prefix CP in OFDM symbols.

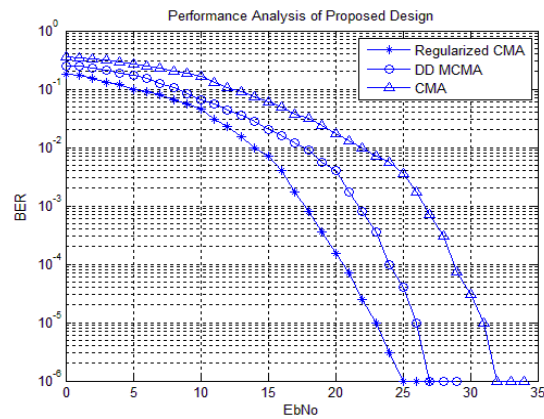


Figure 5: BER comparison between Proposed Design and Decision Directed MCMA

## VII. CONCLUSION

In this research, the network performance in terms of BER and NMSE was investigated for an improved model of regularized CMA blind adaptive equalization in OFDM based MMR. A selective multihop relying protocol is defined for AF multihop relays based on subcarrier equalizer and results showed their efficiency and optimized performance in Rayleigh fading channels with complex Gaussian noise. Overall complexity of the system is reduced by using inverse FFT operation instead of whole inverse correlation matrix lemma and time delay in forwarding is reduced by RCMA fast convergence property. In last, simulations show that design offers lowest MSE and improved detection for OFDM multihop cooperative systems.

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