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Complexity Reduction and Improvement in Convergence Characteristics of Fir Filter Using Adaptive Algorithm

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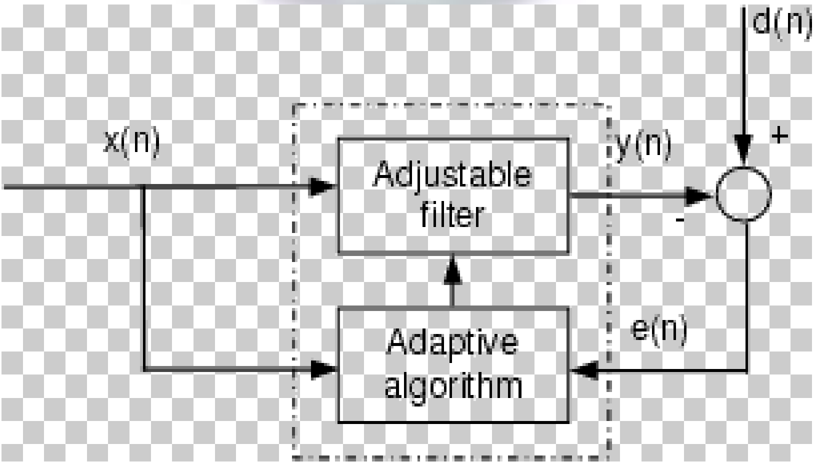
Abstract: Linear filtering is required in a variety of application. A filter will be optimal only if it is designed with some knowledge about the input data. If this information is not known, adaptive filters are used. These filters are adaptable to the changing environment. Adaptive filters finds its application in various fields like adaptive noise cancelling, line enhancing, frequency tracking, channel equalisation etc. Adaptive filtering involves two basic operations filtering and adaptation algorithms. First is filtering process in which output signal is generated from input signal using digital filter. Second is adaptation process which consists of adaptive algorithm which adjusts the coefficient of filter to minimize a desired cost function. There are two basic adaptive algorithms which are used in adaptive filtering least mean square algorithm and normalised least mean square algorithm. Least-mean-square (LMS) adaptive algorithm the most popular and widely used. Another most popular mean of FIR filtering technique is to utilize NLMS algorithm but as the length of the filter increases, number of filter coefficient increases so design of filter become complex in NLMS design but by using MMax – NLMS algorithms design of filter become easy but convergence characteristics occur at later stage take too long time for computation for processing of signal. In this work proposal of improving the convergence characteristics is made which doesn't affect the performance of design without compromising the tap-selection process of the MMax- NLMS algorithms.

Introduction

Adaptive Filter

Here we are studying a class of adaptive filters [7] that employ a gradient descent optimization procedure. [6] Implementation of this procedure requires knowledge of the input signal statistics, which are almost always unknown for real-world problems. Instead, an approximate version of the gradient descent procedure can be applied to adjust the adaptive filter coefficients using only the measured signals. Such algorithms are collectively known as stochastic gradient algorithms. Adaptive filters are using method of gradient descent algorithm [6] in which a concept of cost function is defined. The gradient descent procedure can be used to find the minimum of this function.



Present Work

Problem Formulation

So in order of improving the convergence characteristics without compromising the tap selection [8] a new algorithm is used called MMax NLMS algorithm. In NLMS algorithm all the filter coefficients are updated at each of iteration so computational complexity is high so by using MMax NLMS algorithm [8] we try to reduce the computational complexity. It is found that as the number of filter coefficient updated per iteration in a partial adaptive filter is reduced [3], the computational complexity is also reduced [8] but at the expense of some losses in performance thus we can conclude from above that complexity of NLMS algorithm can be reduced by using MMax NLMS algorithm but convergence characteristics reduces and loss in performance occur. The aim of our work is to improve these convergence characteristics of adaptive filter. It is found that the convergence performance of MMax-NLMS depends upon the step size μ(n). [3] Depending upon this a purposed algorithm of variable step size will be presented to achieve high rate of convergence with lower computational complexity compare to NLMS.

MMAX NLMS Algorithm

In the MMax-NLMS algorithm [1] , only those taps corresponding to the M largest magnitude tap-inputs are selected for updating at each iteration with $1 \leq M \leq L$ It is found that as the number of filter coefficient updated per iteration in a partial adaptive filter is reduced , the computational complexity is also reduced but at the expense of some losses in performance It is found that the convergence performance of MMax-NLMS is depend upon the step size μ(n) [3].

The output at the nth iteration, $V(n) = u^T(n)h(n)$ where $u(n) = [u(n),u(n-1)\ldots\ldots u(n-L+1)]^T$ is the tap vector while the unknown response $h(n) = [h_0(n),\ldots\ldots\ldots\ldots, h_{L-1}(n)]^T$ is of length L. An adaptive filter $\hat{h}(n) = [\widehat{h_0}(n),\ldots\ldots\ldots\ldots\widehat{h_{L-1}}(n)]^T$ which assume to be of be of equal length to the unknown system **h**(n), is used to estimate **h**(n) by adaptively minimizing a priori error signal e(n) using $\hat{v}(n)$ defined by

$$e(n) = u^T(n)h(n) - \hat{v}(n) + g(n) \qquad (4.1)$$

$$\hat{v}(n) = u^T(n)\hat{h}(n-1) \qquad (4.2)$$

With g(n) being the measurement noise.
Defining the sub-selected tap-input vector as

$$\hat{u}(n) = Q(n)u(n) \quad (4.3)$$

Where $Q(n) = \mathrm{diag}\{q(n)\}$ is a $L \times L$ tap selection matrix and $Q(n) = [q_0(n),\ldots\ldots\ldots\ldots, q_{L-1}(n)]^T$ element $q_j(n)$ for j = 0 ,1 , ……… L-1 is given by,

$$q_j(n) = \begin{cases} 1 & |u(n-j)| \in \{M \text{ Maximum of } |u(n)|\} \\ 0 & \text{otherwise} \end{cases} \qquad (4.4)$$

Where $|u(n)| = [\,|u(n)|,\ldots\ldots.|u(n-L+1)|\,]^T$ Defining ||.|| as the squared l2-norm.
The MMAX-NLMS tap-update equation is then

$$\hat{h}(n) = \hat{h}(n-1) + \frac{\mu Q(n)u(n)e(n)}{||u(n)||^2 + C} \quad (4.5)$$

Where, C is the regularization parameter. Defining $\mathbf{I}_{L\times L}$ as the L x L identity matrix, it is noted that if $\mathbf{Q}(n) = \mathbf{I}_{L\times L}$ i.e with M = L, the update equation in (Eq. 5) is equivalent to the NLMS algorithm. Similar to the NLMS algorithm, the step-size μ in (Eq. 5) controls the ability of MMax-NLMS to track the unknown system which is reflected by its rate of convergence. To select the maxima of |**u**(n)| in (Eq. 4), MMax-NLMS employs the SORTLINE algorithm [6] which requires $2\log_2 L$ sorting operations per iteration. The computational complexity in terms of multiplications for MMax-NLMS is O(L+M) compared to O(2L) for NLMS. Thus we can conclude that MMAX NLMS algorithm reduce the computational complexity when compared with NLMS algorithms.

The Proposed MMaxNLMSvss Algorithm

Following the approach of [6], differentiating (Eq. 10) with respect to μ and setting the result to zero,

$$\varphi\left\{\frac{\mu(n)\|\tilde{\mathbf{u}}(n)\|^2 e^2(n)}{\left[\|\mathbf{u}(n)\|^2\right]}\right\}=\varphi\left\{\grave{o}^T(n-1)\tilde{u}(n)\left[\|u(n)\|^2\right]^{-1}e(n)\right\}$$

giving the variable step-size

$$\mu(n)=\mu_{max}\times\frac{\grave{o}^T(n-1)\tilde{\mathbf{u}}(n)\left[\|\mathbf{u}(n)\|^2\right]^{-1}\mathbf{u}^T(n)\grave{o}(n-1)\|\mathbf{u}(n)\|^2}{\|\tilde{\mathbf{u}}(n)\|^2\grave{o}^T(n-1)\mathbf{u}(n)\left[\|\mathbf{u}(n)\|^2\right]^{-1}\mathbf{u}^T(n)\grave{o}(n-1)+\sigma_g^2 M(n)}$$

Where $0<\mu_{max}\le 1$ limits the maximum of $\mu(n)$ and from [6]

$$\mathbf{M}(\mathbf{n})=\frac{\|\tilde{\mathbf{u}}(\mathbf{n})\|^2}{\|\mathbf{u}(\mathbf{n})\|^2} \qquad (4.10)$$

is the ratio between energies of the sub-selected tap-input vector $\tilde{u}(n)$ and the complete tap-input vector $\mathbf{u}(n)$, while $\sigma_g^2 = \varphi\{g^2(n)\}$. To simplify the numerator of $\mu(n)$ further, considering $\tilde{\mathbf{u}}(n)\mathbf{u}^T(n)=\tilde{\mathbf{u}}(n)\tilde{\mathbf{u}}^T(n)$

$$\mu(n)=\mu_{max}\times\frac{\grave{o}^T(n-1)\tilde{\mathbf{u}}(n)\left[\|\mathbf{u}(n)\|^2\right]^{-1}\tilde{\mathbf{u}}^T(n)\grave{o}(n-1)\|\mathbf{u}(n)\|^2}{\|\tilde{\mathbf{u}}(n)\|^2\grave{o}^T(n-1)\mathbf{u}(n)\left[\|\mathbf{u}(n)\|^2\right]^{-1}\mathbf{u}^T(n)\grave{o}(n-1)+\sigma_g^2 M(n)}$$

$\mu(n)$ can be further simplified by letting,

$$\tilde{\mathbf{P}}(n)=\tilde{\mathbf{u}}(n)\left[\mathbf{u}^T(n)\mathbf{u}(n)\right]^{-1}\tilde{\mathbf{u}}^T(n)\grave{o}(n-1)$$
$$\mathbf{P}(n)=\mathbf{u}(n)\left[\mathbf{u}^T(n)\mathbf{u}(n)\right]^{-1}\mathbf{u}^T(n)\grave{o}(n-1) \qquad (4.11)$$

from which it is then shown that [9]

$$\|\mathbf{P}(n)\|^2=\grave{o}^T(n-1)\mathbf{u}(n)\left[\|\mathbf{u}(n)\|^2\right]^{-1}\mathbf{u}^T(n)\grave{o}(n-1)$$

Following the approach in [1], and defining $0 \ll \alpha < 1$ as the smoothing parameter, $\tilde{P}(n)$ and $P(n)$ are estimated iteratively by

$$\tilde{\mathbf{P}}(n)=\alpha\tilde{\mathbf{P}}(n-1)+(1-\alpha)\tilde{\mathbf{u}}(n)\left[\mathbf{u}^T(n)\mathbf{u}(n)\right]^{-1}e_a(n)$$
$$\mathbf{P}(n)=\alpha\mathbf{P}(n-1)+(1-\alpha)\mathbf{u}(n)\left[\mathbf{u}^T(n)\mathbf{u}(n)\right]^{-1}e(n) \qquad (4.12)$$

where $e(n) = u^T(n)\grave{o}(n-1)$ in eq.13 , the error ea(n) due to active filter coefficients $\tilde{u}(n)$ in (Eq. 13) is given as

$$e_a(n)=\tilde{\mathbf{u}}^T(n)\grave{o}(n-1)=\tilde{\mathbf{u}}^T(n)\left[h(n)-\hat{h}(n-1)\right] \qquad (4.13)$$

It is important to note that since $\tilde{u}^T(n)h(n)h(n)$ is unknown, ea(n) is to be approximated. Defining $\bar{\mathbf{Q}}(n)=\mathbf{I}_{L\times L}-\mathbf{Q}(n)$ [9] as the tap-selection matrix which selects the inactive taps, we can express $e_i(n) = [\bar{Q}(n)u(n)]^T\grave{o}(n-1)$ as the error contribution due to the inactive filter coefficients such that the total error e(n) = ea(n) + ei(n). As explained in [**6**], for $0.5L \le M < L$, the degradation in M(n) due to tap-selection is negligible. This is because, for M large enough, elements are small and hence the errors ei(n) are small, as is the general motivation for MMax tap-selection [**1**]. Approximating $ea(n) \approx e(n)$ in (Eq. 13) gives

$$\tilde{\mathbf{P}}(n)\approx\alpha\tilde{\mathbf{P}}(n-1)+(1-\alpha)\tilde{\mathbf{u}}(n)\left[\mathbf{u}^T(n)\mathbf{u}(n)\right]^{-1}e(n) \qquad (4.14)$$

The variable step-size is then given as

Where $c = M^2(n)\sigma_g^2$ Since σ_g^2 is unknown, it is shown that approximating C by a small constant, typically 0.0001. The computation of (Eq. 16) and (Eq. 18) each requires M additions. In order to reduce computation even further, and since for M large enough the elements in $\bar{Q}(n)u(n)$ are small, approximating

$$\|\mathbf{P}(\mathbf{n})\|^2=\|\tilde{\mathbf{P}}(\mathbf{n})\|^2$$

Gives

$$\mu(n)=\mu_{max}\frac{\|\tilde{P}(n)\|^2}{M^2(n)\|\tilde{P}(n)\|^2+C} \qquad (4.16)$$

When $\mathbf{Q}(n) = \mathbf{I}_{L\times L}$, i.e., M = L, MMAX-NLMS is equivalent to the NLMS algorithm and from (Eq. 11), M(n) = 1 and $\|\tilde{P}(n)\|^2 = \|P(n)\|^2$. As a consequence, the variable step-size $\mu(n)$ in (Eq. 16) is consistent with that presented in [1] for M = L.

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Simulation and Result

Speed of Convergence

Filter coefficient approach to their optimum value is called speed of convergence. Speed of convergence depend upon two factors as we have seen in analysis of LMS and NLMS algorithms

- Filter length (L)
- Step size (μ(n))

and as the length of the filter increases speed of convergence decreases and misadjustment increases which gives important conclusion here about the speed of convergence.

- filter length should be chosen as short as possible but long enough to choose the system and also if the step size increases speed of convergence increases but misadjustment also increases so one more important conclusion can be drawn here regarding speed of convergence.
- So a small step size should be chosen.

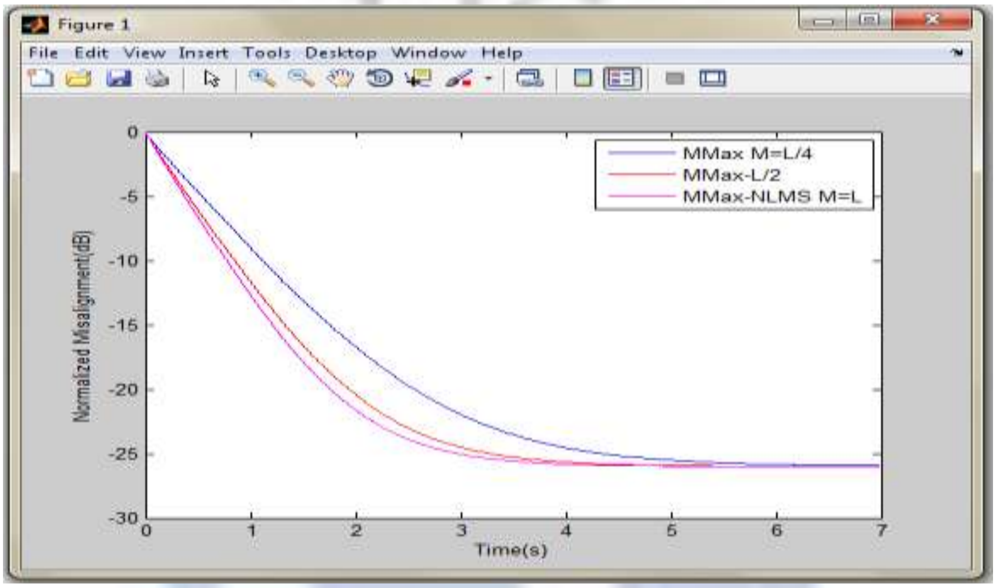

Figure 1: Normalised Misalignment curve for different M.

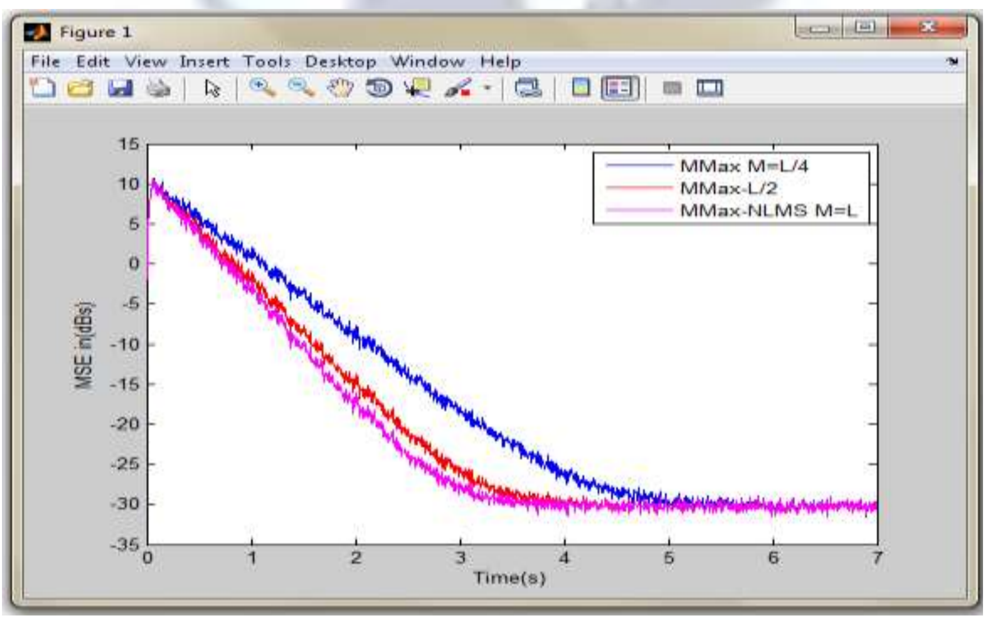

Figure 2: Convergence curves of MMax- NLMS for different M.

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Simulation Result

It is clear from the figure that convergence performance decrease as M = L/4. Since the aim of this thesis is to reduce the degradation of convergence performance due to partial updating of the filter coefficients. For improving the convergence characteristics a new type of algorithms with variable step size is purposed called MMax – NLMSvss. By using variable step size approach improvement in convergence characteristics is obtained. The performance of MMaxNLMSvss algorithm in terms of normalised misalignment is determined and defined in equation 1 using WGN input. Length of the of the filter coefficient chosen is 128,value of C = 0.0001 and $\alpha = 0.15$ are taken WGN is added to achieve SNR of 15dB. The value of $\mu_{max} = 1$ is taken for MMAX NLMSvss while step size μ for the NLMS algorithm is adjusted so as to achieve the same steady state performance for all simulations.

Fig shows the improvement in convergence performance of MMAX NLMSvss over MMAX NLMS for the case of M = L/4.

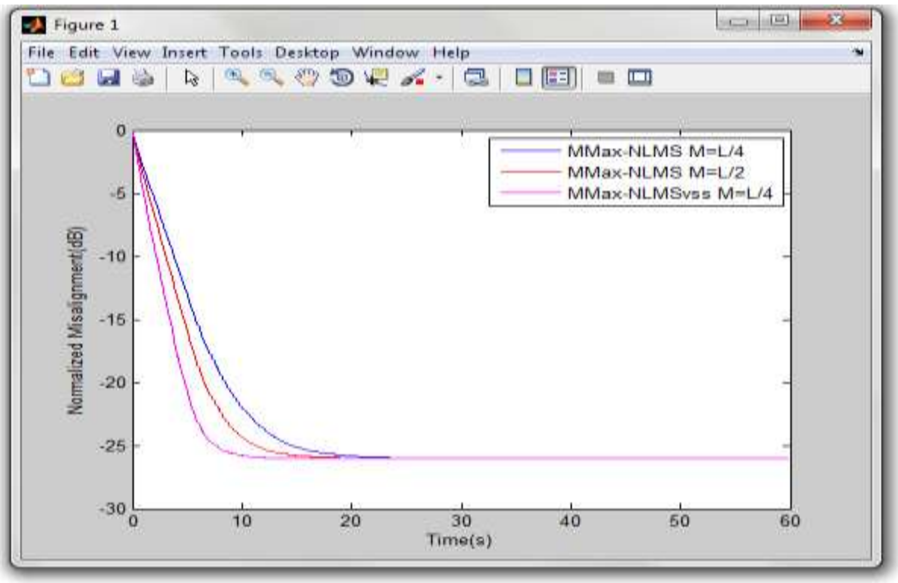

Figure 3: Normalised misalignment curve for improvement in convergence performance of MMax-NLMSvss over MMax-NLMS for different M

Comparison curve of convergence performance of MMax-NLMSvss with NLMS and MMax-NLMS is

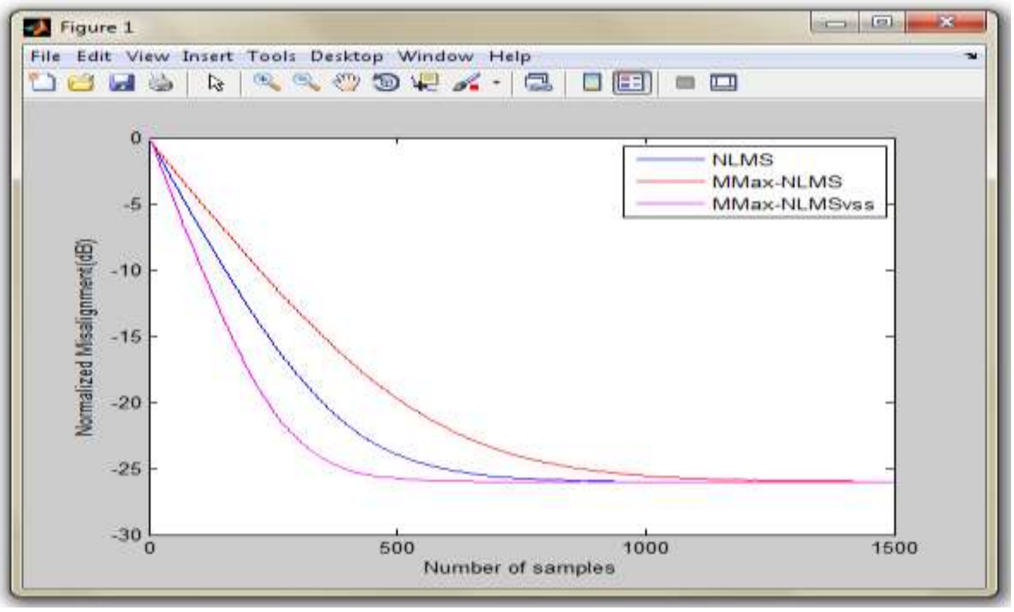

Figure 4 : Normalised misalignment curve for comparison of convergence performance of MMax-NLMSvss with NLMS and MMax-NLMS

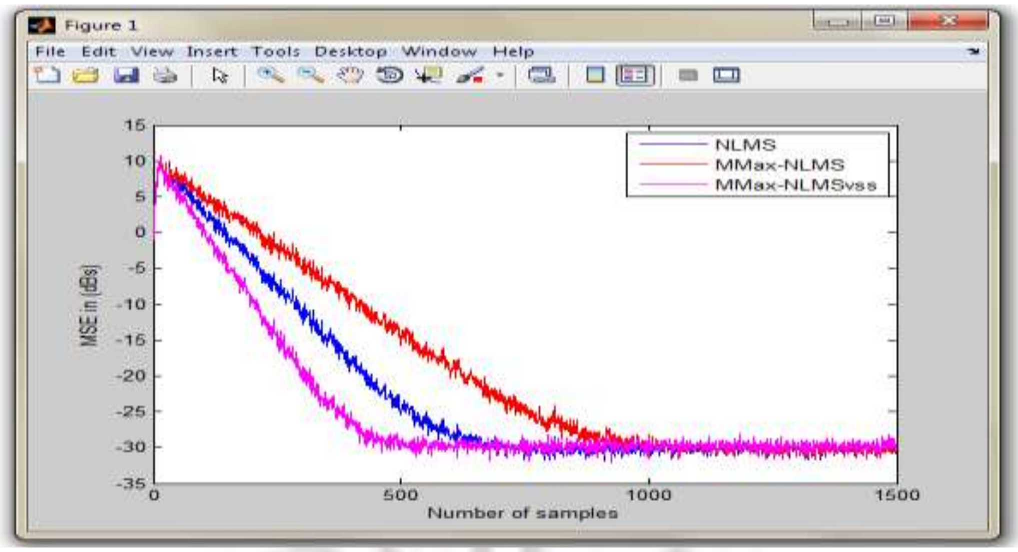

Figure 5: Mean square error curve for comparison of convergence performance of MMax-NLMSvss with NLMS and MMax-NLMS

The step-size of NLMS has been adjusted in order to achieve the same steady-state normalized misalignment. This corresponds to $\mu = 0.1$. More importantly, the proposed MMax-NLMSvss algorithm outperforms NLMS even with lower complexity when chosen for L = 128. This improvement in normalized misalignment of 3 dB (together with reduction of 25% in terms of multiplications) over NLMS is due to variable step-size for MMax-NLMSvss. The MMax-NLMSvss achieves the same convergence performance as the NLMSvss when M = L. In order to illustrate the benefits of the proposed algorithm, M= 256 taken for both MMax- NLMS and MMax-NLMSvss. This gives a 25% savings in multiplications per iteration for MMax-NLMSvss over NLMS. As can be seen, even with this computational savings, the proposed MMax-NLMSvss algorithm achieves an improvement of 1.5 dB in terms of normalized misalignment over NLMS.

Conclusion and Future Work

Performance of adaptive filter depend upon the algorithm being used many algorithms for the improvement of convergence characteristics of adaptive filter are discussed in this thesis and results are obtained using MATLAB simulations. Complexity of NLMS algorithm is reduced using MMax NLMS algorithm but at expense of degradation of performance of filter which is further improved by using MMaxNLMSvss algorithm with variable step size technique. By analysing the mean square deviation of MMax NLMS we can derive a partial update MMax NLMS algorithm with a variable step size during adaptation for improvement of convergence characteristics.

Performance of MMaxNLMSvss in term of normalised misalignment and mean square deviation is determined using WGN as input .and simulation results are shown is figure 1.17 , 1.18 , 1.19 , 1.20 .

From the simulation result we can conclude that the proposed MMaxNLMSvss algorithm outperform NLMS algorithm even with lower complexity when M = L/4 and shows improvement in normalised misalignment of 1.5 dB over NLMS algorithm.

Scope of Future Work

- These techniques can be implemented for networks using diffusion mode of communication which involves heavy computational complexity.
- These techniques have the possibility for non-linear cases like artificial neural networks.
- Networks using probabilistic diffusion mode of communication can implemented by this techniques.
- Communication complexity can be further reduced by transmitting differential estimation.

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