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Low Rank Adaptive Filtering with the Fractional Fourier Transform for Severe, Non-stationary, Co-Channel Interference Suppression

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ABSTRACT

This technical note compares performance of the correlations subtraction architecture of the multistage Wiener filter (CSA-MWF) in the Fractional Fourier Transform (FrFT) domain, MMSE-MWF-FrFT, to the minimum mean-square error, MMSE-FrFT, algorithm and to the conventional MMSE-FFT algorithm in severe co-channel interference. Using four equal power binary phase shift keying (BPSK) signals and a short N = 4 sample training sequence, we show that the MMSE-MWF-FrFT algorithm is able to provide bit error rates (BERs) from 10^{-4} to 10^{-6} at just 10−12 dB E_b/N_0 where the other two algorithms fail.

Keywords: Adaptive Filtering, Fractional Fourier Transform, reduced rank, minimum mean-square error.

1. INTRODUCTION

The Fractional Fourier Transform (FrFT) enables us to translate a received signal to an axis in the time-frequency plane using rotational parameter ‘a’, where a signal-of-interest (SOI) and interference are more easily separable, by multiplying the sequence by a matrix $\mathbf{F}^a$ [1]. A minimum mean square error FrFT (MMSE-FrFT) solution for estimating an SOI in the no stationary interference and noise has been developed which does not rely on knowledge of the interference or noise [3]. When the environment is non-stationary, we must perform this estimation with few samples, i.e. before the statistics of the received signal change. MMSE-based algorithms, however, are known to require a large number of samples in no stationary scenarios, which results in estimation errors [2]. In addition, an MMSE-based solution requires inversion of a covariance matrix, which is a computational operation that limits its ability to perform in real-time [5]. In this technical note, we compare FrFT domain algorithms using the reduced rank multistage Wiener filter (MWF), termed MMSE-MWF-FrFT, solution [4] that is implemented with the efficient correlations subtraction architecture (CSA), to the full rank MMSE-FrFT solution [3] and the traditional solution that only uses the frequency domain, called MMSE-FFT solution, i.e. full rank solution with rotational parameter a = 1. Note that when a = 1 the FrFT reduces to the conventional Fast Fourier Transform (FFT).

An outline of the paper is as follows: Section 2 describes the co-channel adaptive filtering problem, now in the FrFT domain. Section 3 briefly summarizes the three algorithms. We define the simulation parameters, and we compare the three algorithms by example in Section 4. We conclude in Section 5.

2. PROBLEM FORMULATION

We model the SOI and interferers as binary phase shift keying (BPSK) signals, with N_1 bits per block and SPB (samples per bit), giving $N = N_1 SPB$ samples/block. In vector form, the N × 1 SOI is $\mathbf{x}(i)$. We denote the interferers as $\mathbf{x}_j(i)$, j = 1,2, …, J, and we have an additive white Gaussian noise (AWGN) signal $\mathbf{n}(i)$. Here, index i denotes the i^{th} block, where i = 1,2, …,M, and M is the total number of blocks that we process to compute a bit error rate (BER). The received signal $\mathbf{y}(i)$ is then

$$\mathbf{y}(i) = \mathbf{x}(i) + \sum_{j=1}^{J} \mathbf{x}_j(i) + \mathbf{n}(i). \quad (1)$$

The estimate of $\mathbf{x}(i)$, denoted $\hat{\mathbf{x}}(i)$, is obtained by computing [3]

$$\hat{\mathbf{x}}(i) = \mathbf{F}^{-a}\mathbf{G}\mathbf{F}^{a}\mathbf{y}(i), \tag{2}$$

where $\mathbf{F}^a$ and $\mathbf{F}^{-a}$ are the N × N FrFT and inverse FrFT matrices of order 'a', respectively, and $\mathbf{g} = \mathrm{diag}(\mathbf{G}) = (g_0, g_1, \ldots, g_{N-1})$ is an N × 1 set of filter coefficients that are chosen to minimize mean-square error (MSE)

$$J(\mathbf{g}) = \frac{1}{M}\sum_{i=1}^{M} ||\hat{\mathbf{x}}(i) - \mathbf{x}(i)||^2, \tag{3}$$

between the signal $\mathbf{x}$(i) and its estimate $\hat{\mathbf{x}}$(i). If we can determine both the best 'a' and the best $\mathbf{g}$, we can achieve significant interference suppression using the FrFT domain. Hence, the goal is to compute a set of optimum coefficients $\mathbf{g}_0$ that we can apply to Eq. (2) to estimate $\mathbf{x}$(i) from $\mathbf{y}$(i) in the presence of the interference term.

3. ADAPTIVE ALGORITHMS

MMSE-FrFT: This solution is given by [3]

$$\mathbf{g}_{0,MMSE-FrFT} = \frac{1}{2}\mathbf{Q}^{-1}\mathbf{b}, \tag{4}$$

where

$$\mathbf{Q} = \sum_{i=1}^{M} \mathbf{Q}(i), \tag{5}$$

$$\mathbf{Q}(i) = (\mathbf{F}^{-a}\mathbf{Z}(i))^H(\mathbf{F}^{-a}\mathbf{Z}(i)), \tag{6}$$

$$\mathbf{z}(i) = [z_0(i)\ z_1(i)\ \ldots\ z_{N-1}(i)]^T = diag(\mathbf{Z}(i)) = \mathbf{F}^a\mathbf{y}(i), \tag{7}$$

$$\mathbf{b} = \sum_{i=1}^{M} \mathbf{b}(i), \tag{8}$$

and

$$\mathbf{b}(i) = (-2Re(\mathbf{x}(i)^H\mathbf{F}^{-a}\mathbf{Z}(i)))^T. \tag{9}$$

We search over all 'a' using a training sequence $\mathbf{x}$(i) to find the minimum MSE.

MMSE-FFT: The FFT domain solution is easily obtained by setting a = 1 in the MMSE-FrFT solution, i.e.

$$\mathbf{g}_{0,MMSE-FFT} = \mathbf{g}_{0,MMSE-FrFT}|_{a=1}. \tag{10}$$

MMSE-MWF-FrFT: This solution, searching over all 'a' again using a known training sequence, is

$$\mathbf{g}_{0,MMSE-MWF-FrFT} = w_1\mathbf{h}_1 - w_1w_2\mathbf{h}_2 + \ldots \\ -(-1)^D w_1w_2\ldots w_D\mathbf{h}_D. \tag{11}$$

The scalar weights w_j and vectors $\mathbf{h}_j$ are given in Table 1, where

$$d_0(i) = (\mathbf{F}^a\mathbf{x}(i))^T, \tag{12}$$

and

$$\mathbf{x}_0(i) = \mathbf{Z}(i). \tag{13}$$

The parameter D is known as the rank of the filter, and rank reduction is achieved when D < N.

Table 1: Recursion Equations for the CSA-MWF

Initialization: $d_0(i)$ and $\mathbf{x}_0(i)$
Forward Recursion: For $j = 1, 2, ..., D$: $\mathbf{h}_j = \frac{\sum_{\Omega}\{d^*_{j-1}(i)\mathbf{x}_{j-1}(i)\}}{\|\sum_{\Omega}\{d^*_{j-1}(i)\mathbf{x}_{j-1}(i)\}\|}$ $d_j(i) = \mathbf{h}_j^H \mathbf{x}_{j-1}(i)$ $\mathbf{x}_j(i) = \mathbf{x}_{j-1}(i) - \mathbf{h}_j d_j(i)$
Backward Recursion: $\epsilon_D(i) = d_D(i)$ For $j = D, D-1, ..., 1$: $w_j = \frac{\sum_{\Omega}\{d^*_{j-1}(i)\epsilon_j(i)\}}{\sum_{\Omega}\{\lvert\epsilon_j(i)\rvert^2\}}$ $\epsilon_{j-1}(i) = d_{j-1}(i) - w_j^* \epsilon_j(i)$

4. EXAMPLE

We compare the performance of MMSE-FrFT, MMSE-FFT, and MMSE-MWF-FrFT filtering to show benefit of the latter. Once we obtain 'a', we compute the filter coefficients using Eqs. (4), (10), and (11), respectively, and then apply them to Eq. (2) to compute the bit estimates and hence obtain a BER. In all examples, the interference terms are also BPSK signals, each with a slightly different chirp or time-varying Gaussian pulse shape applied to it, to simulate a non-stationary environment. All signals have equal power, and there are J = 3 interferers.We let $N_1 = 2$, SPB = 2, so N = 4, and set D = 1. We set the amplitudes of all the signals to unity, so that the carrier-to-interference ratios (CIRs) between the desired signal and each of the J = 3 interferers are $CIR_j = 0$, for j = 1, 2, 3. Note that this is a stressing case because it is more difficult to separate signals when there is no power difference among them. We run enough trials, T, $T = 10^6$, to get a good BER measurement.

A plot of BER vs. E_b/N_0 is shown in Figure 1. Below E_b/N_0 = 6 dB, noise dominates and all techniques perform nearly the same, since these algorithms are designed for non-stationary interference and are not meant to suppress AWGN. Above E_b/N_0 = 6 dB, however, interference dominates, and the MMSE-MWF-FrFT algorithm performs well. This powerful technique is able to suppress multiple, equal power co-channel interferers, where all previous methods have failed. Hence, the technique could be used to pull signals out of a dense interference environment. MMSE-FrFT cannot handle the non-stationary environment, and MMSE-FFT cannot separate signals because they overlap in frequency. Future work entails applying the algorithm to real data in co-channel environments to determine how often training sequences are needed to estimate 'a'.

CONCLUSION

In this technical note, we demonstrate that the reduced rank MMSE-MWF-FrFT algorithm provides better interference suppression than the MMSE-FrFT algorithm of up to J = 3 near equal power, co-channel BPSK interferers in noise. Both FrFT algorithms perform better than FFT domain methods, which solely use the frequency domain. The MMSEMWF-FrFT is able to achieve low BER ($\lesssim 10^{-4}$) where the other two algorithms fail due to the non-stationary, co-channel interference. Ongoing research is to assess performance with more than J = 3 co-channel interferers and determine how performance can be improved.

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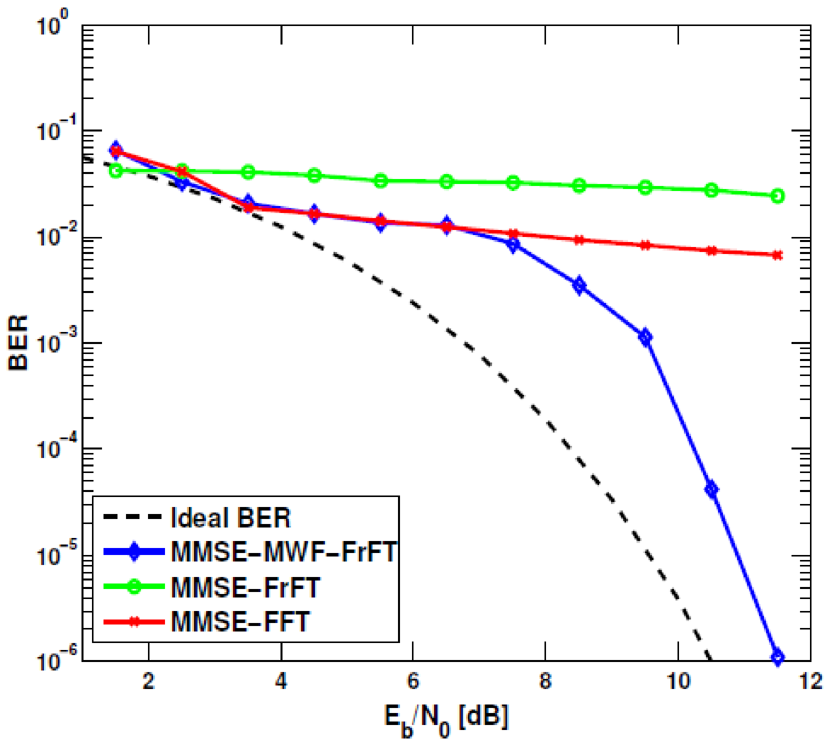

Fig. 1: E_b/N_0 [dB] vs. BER; Three Algorithms: MMSE-FrFT, MMSE-MWF-FrFT, and MMSE-FFT; J =3 ($CIR_{1,2,3}$ =0 dB)

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