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FDTD Modelling of Lorentzian DNG metamaterials using Approximate-Decoupling Method Based on the Unconditionally-Stable Crank–Nicolson Scheme

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Abstract: An implicit finite-difference time-domain (FDTD) method using the approximate decoupling method based on the unconditionally-stable Crank-Nicolson scheme has been used to study a special class of artificially engineered materials having negative permittivity and permeability, called metamaterials. The 2-d propagation of the EM waves has been analyzed. The Convolution Perfectly Matched Layer (CPML) boundary condition has been used to truncate the computational space. Within the CN-FDTD formulation, the auxiliary differential equation (ADE) method has been used for treating the Lorentz media by making use of auxiliary variables. The process is easy, reliable and also causal in nature thus making it proficient. It uses fair approximations to explicate the model. The properties of metamaterial conform to their speculations of negative refractive index and energy absorption and enhancement property with the aid of graphs engineered by matlab simulation. The results achieved elucidate the validity and effectiveness of this method in designing (DNG) metamaterials.

Keywords: DNG, DPS materials, FDTD, Crank-Nicolson (CN), Approximate decoupling (CNAD), ADE-formulation, CPML Boundary Condition.

I. INTRODUCTION

Mediums having simultaneously negative permittivity and permeability, introduced by Veselago in 1968 [1], also known as Left Handed Metamaterials (LHMs) was paid great attention to the researchers in last decade. The LHMs provide some unfamiliar and noteworthy phenomenon like negative refraction, reversal of Doppler Effect and Cerenkov radiation etc. In the present context a DNG layer has been considered which is placed between two DPS layers. The DNG layer is a lossyDrude model having its electric permittivity and magnetic permeability negative while the DPS layer has both these properties positive. Ziolkowski and Heyam analyzed electromagnetic propagation for such mediums with FDTD method [10] implementing the medium with lossyDrude models for negative electric permittivity, magnetic permeability. Mohammed M. Bait Suwailam and Zhizhang (David) Chen investigated the electromagnetic behavior in DNG layer of metamaterials by Finite-Difference-Time-Domain (FDTD) method with the aid of the Z-transform in 2004 [7]. In this paper, an alternative formulation method is being implemented for the simulation of the DNG layer.

The Finite Difference Time Domain (FDTD) method [2,9] is an effective and robust technique being widely used for modeling electromagnetic wave interaction with various frequency dispersive and non- dispersive materials lucratively. Yee's scheme for FDTD is explicit and conditionally stable. Thus there is always a time step size limit given by the Courant–Friedrich–Levy limit or Courant limit. When the time step size exceeds the Courant limit, the scheme becomes unstable. The time-step size limit depends on the optimum mesh size. Some domains, such as low frequency study; the time step limit severely over-samples the signal, and can make the execution time extortionately long. To eliminate this limit, an unconditionally stable method is required. The Crank-Nicolson (CN) scheme, [3],[4],[5] is an established implicit algorithm in computational domain. The approximate decoupling method [4] based on the CN scheme has been used to formulate the computational space.

Moreover the stable and effective boundary condition namely the Convolution Perfectly Matched Layer (CPML) boundary condition has been used to truncate the computational space which allows negligible reflection of the outgoing electric and magnetic fields to take place from the boundaries.

II. FORMULATION

A. *Discretized Maxwell's Equations and FDTD*

The time-dependent Maxwell's curl equations that are required and needed to be solved are:

$$\frac{\partial D}{\partial t} = \nabla \times H \quad (1)$$

$$\frac{\partial B}{\partial t} = -\nabla \times E \quad (2)$$

Taking the central difference approximations, for both the time and space co-ordinates, we get the FDTD formulations.

$$\frac{D_x^{n+\frac{1}{2}}(k) - D_x^{n-\frac{1}{2}}(k)}{\Delta t} = \frac{H_y^n(k-\frac{1}{2}) - H_y^n(k+\frac{1}{2})}{\delta x} \tag{3}$$

$$\frac{B_y^{n+1}(k+\frac{1}{2}) - H_y^n(k+\frac{1}{2})}{\Delta t} = \frac{E_x^{n+\frac{1}{2}}(k) - E_x^{n+\frac{1}{2}}(k)}{\delta x} \tag{4}$$

where δx is the space increment and Δt the time step.
The equations shown above allow updating of both the electric and magnetic field in the DPS region; however the formulation changes in the DNG region.

$$D = \varepsilon(\omega) \times E \tag{5}$$

$$B = \mu(\omega) \times H \tag{6}$$

μ and ε are such chosen so that at a certain frequency, these parameters become negative, hence modeling the layer DNG.

B. CN SCHEME

The CN scheme, [3], [5] averages the right-hand-sides of the discretized Maxwell's Equations at time step n and time step $n+1$. Using a 2-D Yee's mesh, the CN scheme can be formulated as shown in (7a)–(7c), where Δt is the time step size; $a_1 = \Delta t/_{2\varepsilon}$, $a_2 = \Delta t/_{2\mu}$; ε and μ are the permittivity and the permeability of the material respectively. Δx and Δy are the spatial meshing sizes along and axes; and i and j are the integer-number indices of the computational cells; and is the time step index. For the sake of simplicity, it is assumed that the medium is linear, isotropic, non-dispersive, and lossless.

$$Ex^{n+1}\left(i+\frac{1}{2},j\right) = Ex^n\left(i+\frac{1}{2},j\right) + \frac{a_1}{\Delta y}\left(Hz^{n+1}\left(i+\frac{1}{2},j+\frac{1}{2}\right) - Hz^{n+1}\left(i+\frac{1}{2},j-\frac{1}{2}\right) + Hz^n\left(i+\frac{1}{2},j+\frac{1}{2}\right) - Hz^n\left(i+\frac{1}{2},j-\frac{1}{2}\right)\right) \tag{7a}$$

$$Ey^{n+1}\left(i,j+\frac{1}{2}\right) = Ey^n\left(i,j+\frac{1}{2}\right) - \frac{a_1}{\Delta x}\left(Hz^{n+1}\left(i+\frac{1}{2},j+\frac{1}{2}\right) - Hz^{n+1}\left(i-\frac{1}{2},j+\frac{1}{2}\right) + Hz^n\left(i+\frac{1}{2},j+\frac{1}{2}\right) - Hz^n\left(i-\frac{1}{2},j+\frac{1}{2}\right)\right) \tag{7b}$$

$$Hz^{n+1}\left(i+\frac{1}{2},j+\frac{1}{2}\right) = Hz^n\left(i+\frac{1}{2},j+\frac{1}{2}\right) + \frac{a_2}{\Delta y}\left(Ex^{n+1}\left(i+\frac{1}{2},j+1\right) - Ex^{n+1}\left(i+\frac{1}{2},j\right) + Ex^n\left(i+\frac{1}{2},j+1\right) - Ex^n\left(i+\frac{1}{2},j\right)\right) - \frac{a_2}{\Delta x}\left(Ey^{n+1}\left(i+1,j+\frac{1}{2}\right) - Ey^{n+1}\left(i,j+\frac{1}{2}\right) + Ey^n\left(i+1,j+\frac{1}{2}\right) - Ey^n\left(i,j+\frac{1}{2}\right)\right) \tag{7c}$$

The three electromagnetic field components in (7), as shown above, are coupled. To use (7) directly as a computational method, a large sparse matrix must be solved at each time step, which is much more expensive than Yee's FDTD, and not practical for many real problems. In order to solve (7) efficiently, the field components must be decoupled.
There are quite a good number of algorithms available in the literature for the purpose. We have focussed our attention precisely on the Approximate Decoupling Method for decoupling the above coupled equations.

C. APPROXIMATE DECOUPLING METHOD

After the first step simple elimination of Hz^{n+1} from equations (7a) and (7b) using equation (7c), we are left with two implicit equations with coupled electric field components.
For an efficient solution of Ex^{n+1} and Ey^{n+1}, they have to be decoupled further. So, we resort to the approximation method [4] for the complete decoupling.
If Ey^{n+1} is approximated with Ey^n, then the decoupling is realized. This method is termed the "approximate-decoupling method" (CNAD) since it drops some higher order terms from the CN scheme. The update equation for the principal component Ex^{n+1} then becomes:

$$(1+2{b_x}^2)Ex^{n+1}\left(i+\frac{1}{2},j\right) - {b_x}^2\left(Ex^{n+1}\left(i+\frac{1}{2},j-1\right) + Ex^{n+1}\left(i+\frac{1}{2},j+1\right)\right) = (1-2{b_x}^2)Ex^n\left(i+\frac{1}{2},j\right) + {b_x}^2\left(Ex^n\left(i+\frac{1}{2},j-1\right) + Ex^n\left(i+\frac{1}{2},j+1\right)\right) - 2b_xb_y\left(Ey^n\left(i+1,j+\frac{1}{2}\right) - Ey^n\left(i,j+\frac{1}{2}\right) - Ey^n\left(i+1,j-\frac{1}{2}\right) + Ey^n\left(i,j-\frac{1}{2}\right)\right) + 2a_1\left(Hz^n\left(1+\frac{1}{2},j+\frac{1}{2}\right) - Hz^n\left(i+\frac{1}{2},j-\frac{1}{2}\right)\right) \tag{8a}$$

Updating Ex^{n+1} using (8a) only requires solving a tridiagonal matrix, which is quite efficient. With this approximate solution for Ex^{n+1}, Ey^{n+1} can be found as:

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$$\left(1+2{b_y}^2\right)Ey^{n+1}\left(i,j+\tfrac{1}{2}\right)-{b_y}^2\left(Ey^{n+1}\left(i-1,j+\tfrac{1}{2}\right)+Ey^{n+1}\left(i+1,j+\tfrac{1}{2}\right)\right)=\left(1-2{b_y}^2\right)Ey^n\left(i,j+\tfrac{1}{2}\right)+{b_y}^2\left(Ey^n\left(i-1,j+\tfrac{1}{2}\right)+Ey^n\left(i+1,j+\tfrac{1}{2}\right)\right)-b_xb_y\left(Ex^n\left(i+\tfrac{1}{2},j+1\right)-Ex^n\left(i+\tfrac{1}{2},j\right)-Ex^n\left(i-\tfrac{1}{2},j+1\right)+Ex^n\left(i-\tfrac{1}{2},j\right)\right)-b_xb_y\left(Ex^{n+1}\left(i+\tfrac{1}{2},j+1\right)-Ex^{n+1}\left(i+\tfrac{1}{2},j\right)-Ex^{n+1}\left(i-\tfrac{1}{2},j+1\right)+Ex^{n+1}\left(i-\tfrac{1}{2},j\right)\right)-2a_1\left(Hz^n\left(1+\tfrac{1}{2},j+\tfrac{1}{2}\right)-Hz^n\left(i-\tfrac{1}{2},j+\tfrac{1}{2}\right)\right) \quad (8b)$$

Where $b_x=\frac{c\Delta t}{2\Delta x}$ $b_y=\frac{c\Delta t}{2\Delta y}$. After this, Hz^{n+1} can easily be found by using equations (8a) and (8b) in equation (7c).

D. THE METAMATERIAL REGION

The equations governing the formulation of the metamaterial region are given, as in [1], [6]

$$\varepsilon(\omega)=\varepsilon_0\,(1+\frac{\omega_{pe}^2}{\omega_{0e}^2-\omega^2+j\tau_e\omega}) \quad (9)$$

$$\mu(\omega)=\mu_0\,(1+\frac{\omega_{pe}^2}{\omega_{0e}^2-\omega^2+j\tau_m\omega}) \quad (10)$$

These parameters become negative at frequencies below ω_{peak} which crops up at distances between 45 and 70 as discerned from the Matlab program. This is the region of the DNG slab.

Now, for calculation of fields in this layer, we have used inverse Fourier transform to change the s-domain equations into time domain auxiliary differential equations, which are further reduced to recursive formulations by the central difference approximation method and hence the fields are updated by time marching algorithm. A new parameter S(ω) is introduced in the formulation.

$$D(\omega)=\ \varepsilon_0\,E(\omega)+S(\omega) \quad (11)$$

Where $S(\omega)=\frac{\varepsilon_0\,\omega_{pe}^2}{\omega_{0e}^2-\omega^2+j\tau_e\omega}E(\omega)$ (12)

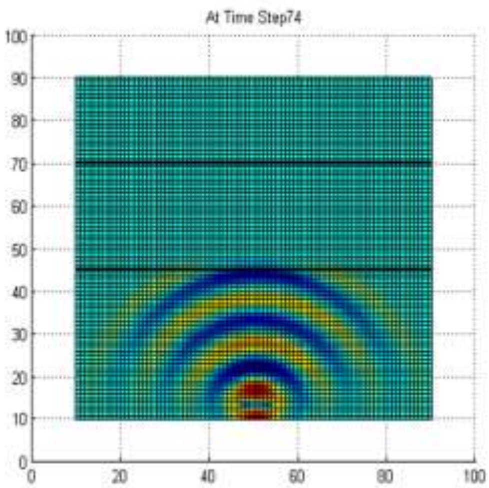
Or $\omega_{0e}^2S(\omega)-\omega^2S(\omega)+j\tau_e\omega S(\omega)=\varepsilon_0\omega_{pe}^2E(\omega)$ By using the conversion $j\omega S(\omega)\leftrightarrow\frac{(S^n-S^{n-2})}{2\Delta t}$ and $(j\omega)^2S(\omega)\leftrightarrow\frac{(S^n-2S^{n-1}+S^{n-2})}{\Delta t^2}$, and by denoting $S(\omega)$ as S^{n-1}, by recursion in time domain we finally get the updated equations,

$$\omega_{0e}^2S^{n-1}+\frac{\tau_e(S_n-S_{n-2})}{2\Delta t}+\omega_{0e}^2S^{n-1}+\frac{\tau_e(S_n-S_{n-2})}{2\Delta t}+\frac{S_n-2S_{n-1}+S_{n-2}}{\Delta t^2}=\omega_{pe}^2\varepsilon_0E^{n-1} \quad (13)$$

Arranging like terms (those with same co-efficients or order), we get,

$$S_n\left(\frac{\tau_e}{2\Delta t}+\frac{1}{\Delta t^2}\right)+S_{n-1}\left(\omega_{0e}^2-\frac{2}{\Delta t^2}\right)+S_{n-2}\left(\frac{1}{\Delta t^2}-\frac{\tau_e}{2\Delta t}\right)=\omega_{pe}^2\varepsilon_0E_{n-1}$$

Or, $S^n=\left(\frac{\frac{2}{\Delta t^2}-\omega_{0e}^2}{\frac{\tau_e}{2\Delta t}+\frac{1}{\Delta t^2}}\right)S^{n-1}+\left(\frac{\frac{\tau_e}{2\Delta t}-\frac{1}{\Delta t^2}}{\frac{\tau_e}{2\Delta t}+\frac{1}{\Delta t^2}}\right)S^{n-2}+\left(\frac{\omega_{pe}^2\varepsilon_0}{\frac{\tau_e}{2\Delta t}+\frac{1}{\Delta t^2}}\right)E^{n-1}$ (14)



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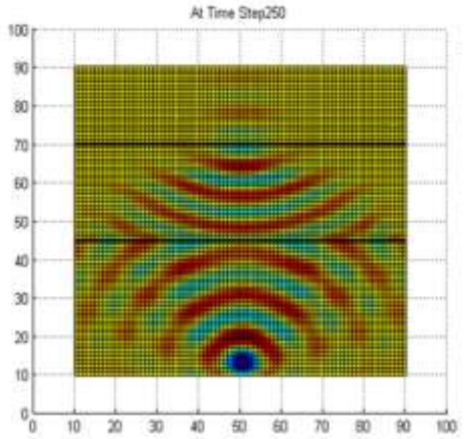

Figure 2.Wave propagating in DNG Layer at early time intervals.

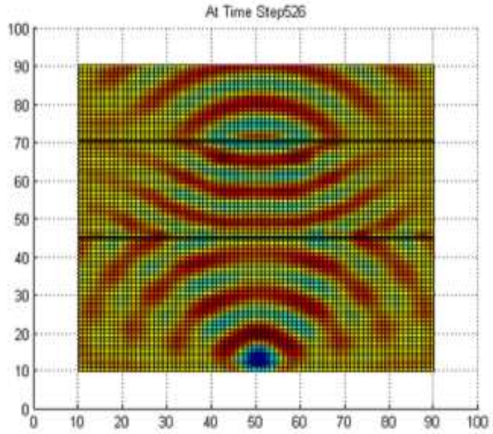

Figure 3.Wave propagating in DNG Layer at late time intervals.

Thus, we obtain S^n by this formulation.

Similarly converting (9) into time domain by IFT method we get,

$$E^n = \frac{(D^n - S^n)}{\varepsilon_0} \tag{15}$$

Finally, by the above formula we obtain the electric field in the time domain for the DNG slab region.

These equations used in this formulation compounded with the approximate decoupling CN-FDTD equations contribute in updating the fields in the DNG slab. However, many other formulation procedures can be applied successfully to achieve the electromagnetic fields such as the Z transform [7].

III. NUMERICAL RESULTS

In this section the numerical results of the above formulation are represented. The case under study is two dimensional. The DNG slab is inserted between the two DPS layers as proposed before and as shown in fig. (1). In the DNG layer, the following data are considered.

$$f_{oe} = f_{om=} \ 0.1591 \ GHz \tag{16}$$

$$f_{pe} = f_{pm=} \ 1.1027 \ GHz \tag{17}$$

$$\tau_e = \tau_m = 0 \tag{18}$$

The adversarial condition arises when $\omega = \omega_{peak} = 5 \times 10^9$ rad/s. The problem space is considered to be of 100×100 cells. is chosen as 0.037, $\Delta t = \delta x/2c$ and $c = \frac{1}{\sqrt{\varepsilon_0 \times \mu_0}} = 3 \times 10^8$m/s. The DNG slab is interjected for 25 cells in between two DPS slabs which are here taken as free space. CPML boundary condition is successfully applied at the four boundaries to restrict outgoing Electric fields to reflect back in to the problem space. As mentioned earlier CPML boundary conditions are very efficient in the 2-D space truncation and thus they exclude any reflected field components. The Sinusoidal pulse is launched 11 cells from the bottom boundary that is in the DPS 1 layer with its central frequency equal to ω_{peak} as shown in fig.1. The wave propagates forward in the DPS1 layer, but once it enters the DNG layered metamaterial region, it responds in a manner contrary to the normal DPS layers as shown in fig. 2. It appears as if it propagates in the regressive direction. However the energy flows in the positive direction. Actually, the parameters being negative, the pointing vector is antiparallel to the phase velocity vector. The wave amplitude is much more elevated in the DNG layer than the DPS layer. Once, the wave exits the DNG slab and infiltrates the DPS2 layer, the wave amplitude again starts to lag gradually and behaves like normal free space as shown in fig. 3.

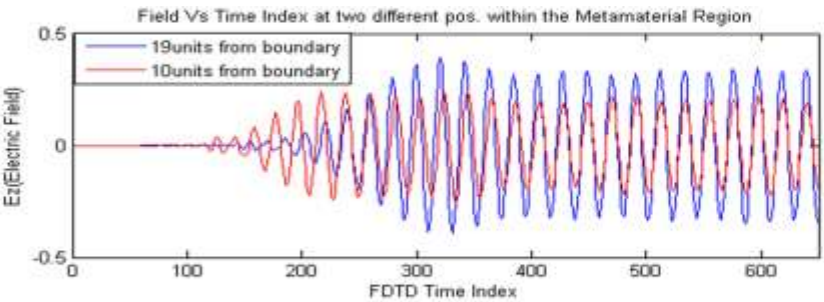

Figure 4. Time histories of the electric field recorded at two specific points within the DNG medium

After long time we find the wave amplitudes in the metamaterial much higher than that of the normal free space. So, the material thus enhances the energy or intensity of the wave in the region, thereby absorbing more energy from the surrounding mediums. Fig. 4shows the time histories of the electric field recorded at two specific points within the DNG medium. The early time response of the field at the 1st position occurs before the one at the 2ndpoint. This shows that causality in the direction of the wave propagation is preserved in the DNG medium. The second point leads the one at the first point in the late time. This is the result of negative index in the DNG slab.

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CONCLUSION/RESULTS

The electric and magnetic fields are found out for DNG metamaterials as well as DPS materials by means of approximate decoupling method which is an implicit method based on the unconditionally stable CN-FDTD scheme. The formulation procedure used to model the Lorentz medium is ADE method followed by recursive formulation in time domain. This formulation is effective and easy to implement as shown in above sections. The properties of a metamaterial are proven using this formulation and graphs shown below. Causality is maintained in the metamaterial. The effectiveness of the Negative Refractive Index has been complied with the help of the simulations and graphs as presented above. The DNG medium accumulates energy from the surrounding medium and hence preserves the energy.

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