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Physical Constraints Nonlinear Position Control of a PMSM

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ABSTRACT

A nonlinear control has been developed and applied to realize a globally stable movement of a permanent magnet synchronous motor (PMSM). Physical constraints imposed by the PMSM manufacturer such as the current, power and speed are simultaneously imposed on the PMSM dynamics. In order to preserve global stability of the non linear control in presence of the physical constraints, an artificial intelligence algorithm is developed. Based on the supervision of the machine state, this algorithm selects the non linear control so no violation of the physical constraints is an allowed. Simulation results of the PMSM dynamics equipped with the non linear control shows the effectiveness of the proposed algorithm.

Keywords: Permanent magnet Synchronous machine (PMSM); constraint; nonlinear control; current limitation; stability study.

1. INTRODUCTION

There have been several improvements in the field of machine control and their internal architecture, particularly for the passage from servomotors of DC machines to servomotors of PMSM [7]. The technology used in the autopilot motor is based on PMSM with position, speed and torque control. Many authors consider the DC motor model in order to simplify the PMSM dynamic model [10], [12]. The MSPM technology is widely used in systems to carry out very fast and precise tasks as Cartesian robots and articulated robots, which also request offline control [11]. Such control system, as it minimize the time laps of the robot task, pushes the motors to their maximum physical constraints leading to undesired behavior. For the PMSM control certain authors [1], [5], [10] based their studies on stator current control, to reach the desired position. Others authors [15] use the intelligence methods such as artificial neural network for their control strategies.

In this paper, the synthesis of an optimal nonlinear control system for PMSM is proposed. This control strategy guaranties the global stability of the PMSM movement. Since manufacturer physical constraints are imposed on the PMSM, stability can be altered. In order to maintain global stability of the PMSM even in presence physical constraints, an artificial intelligence algorithm is developed. The proposed non linear control is capable to realize movement in minimal time period [13], [16].

2. NONLINEAR MODELLING OF A PMSM

The model of PMSM's can be written in state equation as follows [2], [4], [6] :

$$\begin{cases} \dfrac{di_d}{dt} = -\dfrac{R_s}{L_d} i_d + p\dfrac{L_q}{L_d} i_q \dot{\theta} + \dfrac{1}{L_d} U_d \\ \dfrac{di_q}{dt} = -\dfrac{R_s}{L_q} i_q - p\dfrac{L_d}{L_q} i_d \dot{\theta} - p\dfrac{\phi_f}{L_q}\dot{\theta} + \dfrac{1}{L_q} U_q \\ \ddot{\theta} = -\dfrac{f}{J}\dot{\theta} + p\dfrac{\left(L_d - L_q\right)}{J} i_d i_q + p\dfrac{\phi_f}{J} i_q - \dfrac{C_r}{J} \\ \dot{\theta} = \Omega \end{cases} \tag{1}$$

where R_s denotes the stator's resistance, θ the angular position, Ω the angular velocity, f the friction's coefficient, J the rotor's inertia, ϕ_f the rotor's magnets flow, p the number of pole's pairs, i_d the stator current along the axis d, i_q the stator current along the axis q, U_d the stator voltage along the axis d, U_q the stator voltage along the axis q, L_d the stator inductance along the axis d, L_q the stator inductance along the axis q, C_r the resistive charge torque and C_e the electromotive torque.

The model is composed by two subsystems. The first is an electrical subsystem driven by the only input U : the stator's voltage. The second is a mechanical subsystem driven by the electrical part. We propose the following state vector of the PMSM :

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{2}$$

Where

$x_1 = \begin{bmatrix} i_d \\ i_q \end{bmatrix}$: State of the electrical subsystem; $x_2 = \begin{bmatrix} \dot{\theta} \\ \theta \end{bmatrix}$: State of the mechanical subsystem;

The PMSM model can be written in a state equation, as follows:

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} f_1(x) \\ f_2(x) \end{bmatrix} + \begin{bmatrix} g_1 \\ \varnothing \end{bmatrix} U \tag{3}$$

Where:

$$f_1(x) = \begin{bmatrix} -\frac{R_s}{L_d} i_d + p\frac{L_q}{L_d} i_q \dot{\theta} \\ -\frac{R_s}{L_q} i_q - p\frac{L_d}{L_q} i_d \dot{\theta} - p\frac{\phi_f}{L_q}\dot{\theta} \end{bmatrix} \;;\; f_2(x) = \begin{bmatrix} -\frac{f}{J}\dot{\theta} + p\frac{(L_d - L_q)}{J} i_d i_q + p\frac{\phi_f}{J} i_q - \frac{C_r}{J} \\ \dot{\theta} \end{bmatrix}; g_1 = \begin{bmatrix} \frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_q} \end{bmatrix};$$

The PMSM input is the voltage applied to the stator $U = \begin{bmatrix} U_d & U_q \end{bmatrix}^T$;

We can write the mechanical subsystem state equation as follow:

$$\dot{x}_2 = f_2(x) = Ax_2 + F(x_1) - C \tag{4}$$

With:

$$A = \begin{bmatrix} -\frac{f}{J} & 0 \\ 1 & 0 \end{bmatrix} ; C = \begin{bmatrix} \frac{C_r}{J} \\ 0 \end{bmatrix} ; F(x_1) = \begin{bmatrix} p\frac{(L_d - L_q)}{J} i_d i_q + p\frac{\phi_f}{J} i_q \\ 0 \end{bmatrix}$$

We notice that the mechanical subsystem is not driven by a direct input.

The torque provided by the electrical motor can be defined as:

$$C_e = p(L_d - L_q) i_d i_q + p\phi_f i_q \tag{5}$$

3. NONLINEAR CONTROL OF A PMSM WITHOUT PHYSICAL CONSTRAINTS

A non linear control of the PMSM is developed in this section. Since from modeling the PMSM system, it is clear that the input U affects only the electrical subsystem. The mechanical subsystem x_2 is driven by the state of electrical subsystem x_1. Our strategy, to derive the non linear control, is first by selecting a reference mechanical position x_{2ref} which is used to define the electrical current reference x_{1ref} [3], [7], [8], [9]. The control system chain is chosen as in figure 1.

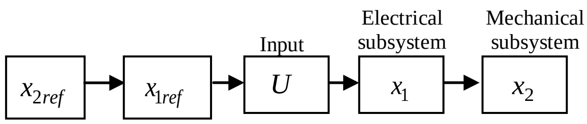

Figure 1. Control's chain of the PMSM.

Where the desired electric state is $x_{1ref} = \begin{bmatrix} i_{dref} & i_{qref} \end{bmatrix}^T$, And the mechanical desired state is $x_{2ref} = \begin{bmatrix} \dot{\theta}_{ref} & \theta_{ref} \end{bmatrix}^T$.

A. Electrical subsystem control

Consider the following model reference for computing the control of the electrical subsystem :

$$\begin{cases} \dot{x}_1 + k_1\left(x_1 - x_{1ref}\right) = 0 \\ \forall\, k_1 = \begin{bmatrix} k_{11} & 0 \\ 0 & k_{11} \end{bmatrix} > 0 \end{cases} \tag{6}$$

This model is globally stable since k_1 is positive. The final state of the reference model is x_{1ref} .

By substituting (6) and (3), the control of the subsystem is:

$$U = l\left(x, x_{1ref}\right) = g_1^{-1}\left[-f_1\left(x\right) - k_1\left(x_1 - x_{1ref}\right)\right] \tag{7}$$

Note that x_{1ref} will be chosen based on the final desired state of the mechanical subsystem x_{2ref} .

B. Mechanical subsystem control

We consider the following reference model for the mecanical subsystem:

$$\dddot{\theta} + k_{21}\ddot{\theta} + k_{22}\dot{\theta} + k_{23}\left(\theta - \theta_{ref}\right) = 0 \tag{8}$$

θ_{ref} is the final stationary position of the reference model when it is stable.

When the differentiel equation (8) has triple pole, it can be writen as:

$$\dddot{\theta} - 3\lambda\ddot{\theta} + 3\lambda^2\dot{\theta} - \lambda^3\left(\theta - \theta_{ref}\right) = 0 \tag{9}$$

When the triple pole λ is real and negative, the reference model is globally stable and θ is always be inferior to θ_{ref} .

In state form equation (9) may be writen as follow :

$$\ddot{x}_2 + \Psi_1\dot{x}_2 + \Psi_2\left(x_2 - x_{2ref}\right) = 0 \tag{10}$$

Here, the definited matrices Ψ_1 and Ψ_2 are given by the equation (11) :

$$\begin{cases} \Psi_1 = \begin{bmatrix} -3\lambda & 3\lambda^2 \\ -1 & 0 \end{bmatrix} \\ \Psi_2 = \begin{bmatrix} 0 & -\lambda^3 \\ 0 & 0 \end{bmatrix} \end{cases} \tag{11}$$

The time derivative of the mechanical equation (4) is:

$$\ddot{x}_2 = \dot{f}_2\left(x_1, x_2\right) = A\dot{x}_2 + \dot{F}\left(x_1\right) - \dot{C} \tag{12}$$

When the load C_r is constant, (13) becomes :

$$\ddot{x}_2 = \dot{f}_2\left(x_1, x_2\right) = A\dot{x}_2 + \frac{\partial F\left(x_1\right)}{\partial x_1}\dot{x}_1 \tag{13}$$

From (4), (6), (10) and (13), we can write :

$$\frac{\partial F\left(x_1\right)}{\partial x_1}k_1\left(x_1 - x_{1ref}\right) = h\left(x_{1,}x_2, x_{2ref}\right) = \left(A + \Psi_1\right)\left(Ax_2 + F\left(x_1\right) - C\right) + \Psi_2\left(x_2 - x_{2ref}\right) \tag{14}$$

With : $\frac{\partial F\left(x_1\right)}{\partial x_1} = \begin{bmatrix} p\left(L_d - L_q\right)i_q & p\left(L_d - L_q\right)i_d + p\phi_f \\ 0 & 0 \end{bmatrix}$: This matrix is singular.

This implies the following equation:

$$i_{qref} = \frac{-J}{k_1 p\left(L_d - L_q\right)i_d + k_1 p f_f}\left[\left(-3\lambda + \frac{f}{J}\right)\ddot{\theta} + 3\lambda^2\dot{\theta} - \lambda^3\left(\theta - \theta_{ref}\right)\right] - \frac{\left(L_d - L_q\right)i_q}{\left(L_d - L_q\right)i_d + f_f}\left(i_{dref} - i_d\right) + i_q \tag{15}$$

The reference current solutions (i_{dref}, i_{qref}) of equation (15) bring the system to the reference position θ_{ref} . We chose to set $i_{dref} = 0$ for simplicity of implementation [1]. We note that $i_{dref} = 0$ is not the optimal value for energy consumption [1], [16]. In practice i_d is always oscillating around i_{dref} [1], thus approaching the PMSM behavior to a DC motor.

The block diagram of the resulting nonlinear control of a PMSM scheme is shown in figure 2.

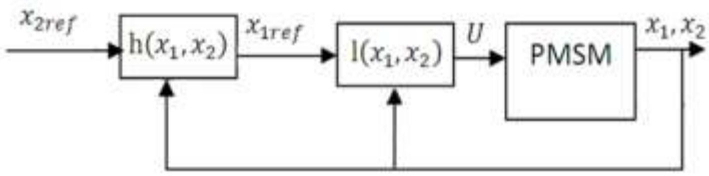

Figure 2. Nonlinear control with physical de constraints of a PMSM

Two simulations were realized for the PMSM equipped with the non linear control of equation (7). The selected triple pole is set at $\lambda_0 = -10$. The reference desired position x_{2ref} is set respectively at the following positions:

$$\left(x_{2ref}\right)_{simulation1} = \begin{bmatrix} 0 \\ 1Rad \end{bmatrix} \qquad \left(x_{2ref}\right)_{simulation2} = \begin{bmatrix} 0 \\ 100Rad \end{bmatrix}$$

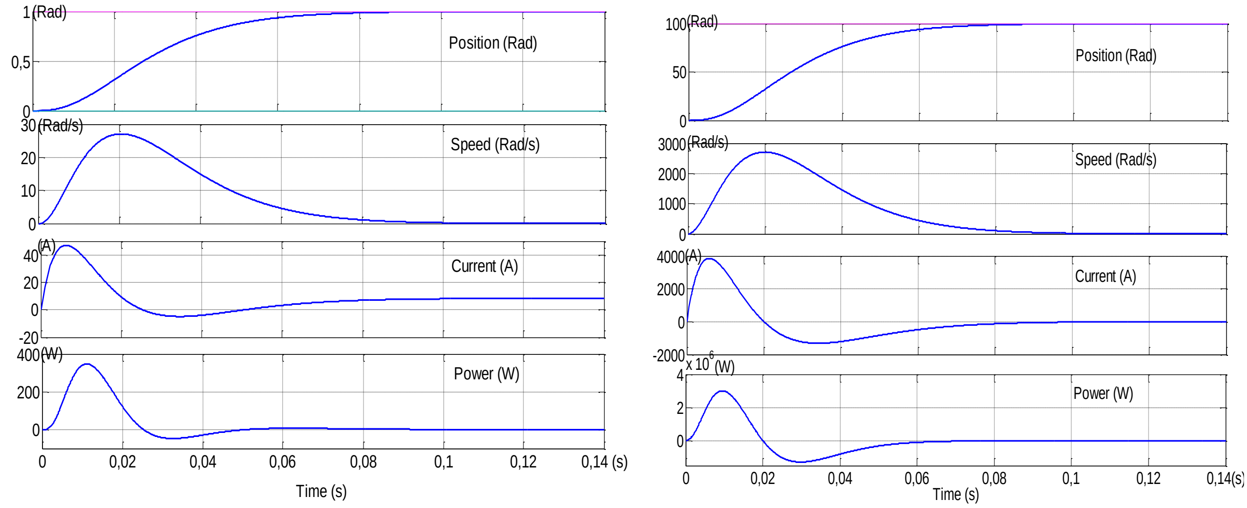

Figure 3. PMSM CSCL control with a reference position of 1rad and 100rad with a load of 4Nm

The results of the two simulations show that the control law stabilizes the system at the desired reference values with the same time response. Only during the second simulation the current, power and speed motor were beyond the physical limits. This is due to the difference in setting $\left(x_{2ref} = \begin{bmatrix} 0 & 1Rad \end{bmatrix}^T\right)_{Simulation1}$ and $\left(x_{2ref} = \begin{bmatrix} 0 & 100Rad \end{bmatrix}^T\right)_{Simulation2}$ for the first and second simulation.

In fact since the time response is equal for the two simulations, the energy requested in the second simulation is much important then the first simulation, therefore demanding more current, power and speed.

Similar observation can be made if the same position reference were used in the two simulations but with different value of the triple pole ($\lambda_0 = -10$ and $\lambda_0 = -100$).

4. CONSTRAINT NONLINEAR CONTROL

In this section, a non linear control is developed not only to reach the position control $\theta_2 \rightarrow \theta_{2ref}$ with minimum time response but also not violate the physicals constraints. The constraints defined by the PMSM manufacturer are generally composed by three maximums limits:

- Current constraint
- Power constraint
- Speed constraint

Figure 4 represents the manufacturer constraints diagram where the limit's of the three constraints are defined ($\dot{\theta} = \Omega$).

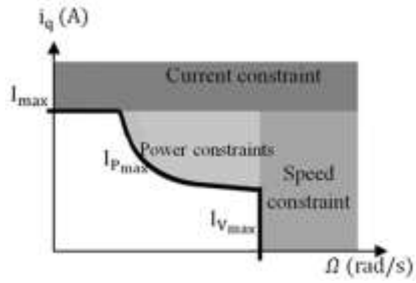

Figure 4. Manufacturer constraints diagram

Where i_q is the stator current reference along the axis q, $I_{\max}$ is the maximum current, $I_{P\max}$ is the current for maximum power and $I_{V\max}$ is the current for maximum speed.

A. Current constraint

The high thermal of the PMSM's conductor's wires insulations is the cause of the stator current's constraint. In order to not exceed the maximum current $I_{\max}$ allowed by the manufacturer, the reference stator current i_{qref} must be always less or equal to $I_{\max}$:

$$\begin{cases} i_{qref} \le I_{\max} \\ \left| x_{1ref} = \begin{bmatrix} 0 \\ i_{qref} \end{bmatrix} \right| \le \left| x_{1\max} = \begin{bmatrix} 0 \\ I_{\max} \end{bmatrix} \right| \end{cases} \tag{16}$$

Referring to the model reference equation (6), the current will be always less or equal to $I_{\max}^{-}$ when the control law (7) is applied $\left(|x_1| \le \left|x_{1\max}^{-}\right|\right)$.

Since i_{qref} is computed from the desired position reference θ_{ref} (15), the inequality (16) may be not satisfied. In such situation, i_{qref} is superior to $I_{\max}$, we therefore set i_{qref} equal to $I_{\max}$ and (17) becomes:

$$I_{\max} = I_{qref} = \frac{-J}{k_1 p\left(L_d - L_q\right) i_d + k_1 p f_f}\left[\left(-3\lambda + \frac{f}{J}\right)\ddot{\theta} + 3\lambda^2\dot{\theta} - \lambda^3\left(\theta - \theta_{ref}\right)\right] - \frac{\left(L_d - L_q\right) i_q}{\left(L_d - L_q\right) i_d + f_f}\left(i_{dref} - i_d\right) + i_q \tag{17}$$

From equation (17), we can write the third order polynomial :

$$P(\lambda) = -\lambda^3\left(\theta - \theta_{ref}\right) + 3\lambda^2\dot{\theta} - 3\lambda\ddot{\theta} + \left[-\frac{f}{J}\ddot{\theta} + \frac{k_1 p\phi_f}{J}\left(I_{\max} - i_q\right)\right] = 0 \tag{18}$$

In section 5, we prove that this polynomial always accept a negative root λ_c. Therefore the triple pole is set equal to λ_c ensuring the global stability for equation (9).

Asymptotic behavior

In the case of current constraint this behavior consider that $i_d \approx 0$, $C_r \approx constant$ and $I_q = I_{\max}$. We obtain the position Laplace transformation of θ:

$$\theta(s) = \frac{K}{s^2\left(s + \tau\right)} \tag{19}$$

with $K = \dfrac{p\phi_f I_{\max} - C_r}{J}$, $\tau = \dfrac{f}{J}$. The position asymptotic behavior is :

$$\theta(t) = -\frac{K}{\tau^2} + \frac{K}{\tau}t + \frac{K}{\tau^2}e^{-\tau t} \tag{20}$$

Equation (7) gives the asymptotic control behavior which is linear to the speed of the system.

$$U = \alpha x_2 + c_2 \tag{21}$$

$$\alpha = \begin{bmatrix} -pL_q I_{\max} & 0 \\ p\phi_f & 0 \end{bmatrix} \text{ and } c_2 = \begin{bmatrix} 0 \\ R_s I_{\max} \end{bmatrix}$$

This linear asymptotic behavior will appear in our simulation results (figure 6).

B. Power constraint

The origin of the power constraint is due to the thyristors and MOSFETs: the main inverters components. This is necessary for the sinusoidal voltages' generation, called pulse width modulation. Knowing that the power is proportional to the voltage:

$$P_{absorbé} = 3UI = P_{active} + P_{erte\ joule} \tag{22}$$

The power's constraint appears in the following form, neglecting Joule losses:

$$P_{active} = P = \dot{\theta} C_e \tag{23}$$

When the power requested by the current control exceeds the maximum power tolerated by the manufacturer, the PMSM may reach power saturation. In order to avoid the power saturation, we impose the following dynamic on the power of the PMSM:

$$\dot{P} + k\left(P - P_{ref}\right) = 0 \tag{24}$$

k is any positive real value, and from (23)and (24) we obtain :

$$P_{ref} = P + \frac{1}{k}\left(\ddot{\theta} C_e + \dot{\theta} \dot{C}_e\right) \tag{25}$$

Using equation (5) and (6), P_{ref} becomes :

$$P_{ref} = \vartheta\left(i_{qref}\right) = P + \frac{1}{k}\left(\ddot{\theta} C_e - \dot{\theta} p k_1 \left[\left(i_q - i_{qref}\right)\left(\phi_f + \left(L_d - L_q\right) i_d\right) + \left(L_d - L_q\right)\left(i_d - i_{dref}\right) i_q\right]\right) \tag{26}$$

When P_{ref} is inferior to $P_{\max}$, the current law i_{qref} (15) ensure that maximum power will not be reached. When P_{ref} is equal or superior to $P_{\max}$, P_{ref} is set equal to $P_{\max}$. Deduced from (26), the reference current is:

$$i_{qref} = \vartheta^{-1}\left(P_{ref} = P_{\max}\right) = I_{P_{\max}} = \frac{k\left(P_{\max} - P\right) - \ddot{\theta} C_e}{\dot{\theta} p k_1 \left(\phi_f + \left(L_d - L_q\right) i_d\right)} + \frac{\left(L_d - L_q\right)\left(i_d - i_{dref}\right) i_q}{\phi_f + \left(L_d - L_q\right) i_d} + iq \tag{27}$$

According to equation (15) and (27), the following equation third degree with the triple pole variable λ is given by:

$$P(\lambda) = -\lambda^3\left(\theta - \theta_{ref}\right) + 3\lambda^2 \dot{\theta} - 3\lambda \ddot{\theta} + \left[\frac{f}{J}\ddot{\theta} + \frac{k\left(P_{\max} - P\right) - \ddot{\theta} C_e}{J\dot{\theta}}\right] = 0 \tag{28}$$

In section 5, we prove that this polynomial always accept a negative root λ_P. Therefore the triple pole is set equal to λ_P ensuring the global stability for equation (9).

Asymptotic behavior

In order to understand the asymptotic behaviors of the whole system in the case of power constraint, we supposed that $i_d \approx 0$, $C_r = \text{constant}$, and $C_e = \frac{P_{\max}}{\dot{\theta}} = p\frac{\left(L_d - L_q\right)}{J} i_d i_q + p\frac{\phi_f}{J} i_q$

The model of PMSM becomes:

$$\begin{cases} U_d = -pL_q i_q \dfrac{P_{\max}}{p\phi_f i_q} = -L_q \dfrac{P_{\max}}{\phi_f} = cte \\ \dfrac{di_q}{dt} = -\dfrac{R_s}{L_q} i_q - \dfrac{P_{\max}}{L_q i_q} + \dfrac{1}{L_q} U_q \\ \ddot{\theta} = -\dfrac{f}{J}\dot{\theta} + \dfrac{P_{\max}}{J\dot{\theta}} - \dfrac{C_r}{J} \\ \dfrac{d\theta}{dt} = \Omega \end{cases} \tag{29}$$

Where $i_q = \frac{P_{\max}}{p\dot{\theta}\phi_f}$ and such $f\dot{\theta} << \frac{P_{\max}}{\dot{\theta}}$ if $\dot{\theta}$ is great, we obtain the position:

$$\theta(t) = \frac{2J}{P_{\max}}\left(\frac{P_{\max}}{J} t + c_3\right)^{\frac{3}{2}} \tag{30}$$

c_3 is a constant depending on the load and in the initial position of the penetration power phase.

C. Speed constraint

The motor bearings are the cause of the speed limit. the speed constraint defined by the manufacturer imposes that the MSPM speed must be always less that a maximum value $\Omega_{\max}$:

$$\dot{\theta} \le \Omega_{\max} \tag{31}$$

As the system is moving towards θ_{ref} , we are interested on the instantaneous references speed Ω_s satisfying the following equation :

$$\dddot{\theta} + 2l_1\ddot{\theta} + l_1^2\left(\dot{\theta} - \Omega_s\right) = 0 \tag{32}$$

l_1 is any positive real value and from (1), (6), and (32) where Ω_s is

$$\Omega_s = g\left(i_{qref}\right)$$

$$\Omega_s = \left(\frac{2}{l_1} + \frac{f}{l_1^2 J}\right)\ddot{\theta} + \dot{\theta} - \frac{pk_1}{Jl_1^2}\left[\left(i_q - i_{qref}\right)\left(\phi_f + \left(L_d - L_q\right)i_d\right) + \left(L_d - L_q\right)\left(i_{dref} - i_d\right)i_q\right] \tag{33}$$

Ω_s is the speed where the system converges.

When Ω_s is inferior to $\Omega_{\max}$, the current law i_{qref} (15) ensure that maximum speed will not be reached. In such situation Ω_s is equal or superior to $\Omega_{\max}$, therefore Ω_s is set equal to $\Omega_{\max}$. Deduced from (33), the reference current is:

$$i_{qref} = g^{-1}\left(\Omega_s = \Omega_{\max}\right) = I_{V_{\max}} = i_q - \frac{1}{\phi_f + \left(L_d - L_q\right)i_d}\left(\frac{Jl_1^2}{pk_1}\left[-\Omega_{\max} + \left(\frac{2}{l_1} + \frac{f}{l_1^2 J}\right)\ddot{\theta} + \dot{\theta}\right] - \left(L_d - L_q\right)\left(i_{dref} - i_d\right)i_q\right) \tag{34}$$

According to equation (17) and (34), the following third degrees equation with the triple pole variable λ :

$$P(\lambda) = -\lambda^3\left(\theta - \theta_{ref}\right) + 3\lambda^2\dot{\theta} - 3\lambda\ddot{\theta} + \left[-\left(2l_1 + \frac{f}{l_1^2} + \frac{f}{J}\right)\ddot{\theta} + l_1^2\Omega_{\max} - l_1^2\dot{\theta}\right] = 0 \tag{35}$$

In section 5, we prove that this polynomial always accept a negative root λ_Ω . Therefore the triple pole is set equal to λ_Ω ensuring the global stability for equation (9).

Asymptotic behavior

In order to understand the asymptotic behaviors of the whole system in the case of power constraint, we supposed that $i_d \approx 0$ and $C_r = \text{constant}$, what wants to say that according to the model :

$$\begin{cases} i_q = \dfrac{f\Omega_{\max} + C_r}{p\phi_f} = \text{constant} \\ R_s i_q + p\phi_f\Omega_{\max} = U_q \\ -pL_q i_q \Omega_{\max} = U_d \\ \dfrac{d\theta}{dt} = \Omega = \Omega_{\max} \end{cases} \tag{36}$$

with : $\theta(t) = \Omega_{\max}t + \theta_0$ and $U = \text{constant}$. This linear asymptotic behavior will appear in our simulation results (figure 6). When current saturation is reached, the state variable increases proportionally with time and in a linear way. Moreover, speed tends towards a constant value, acceleration and Jerk tend towards a value zero.

5. STABILITY STUDY

The global stability of the PMSM equipped with the nonlinear control in (7), is ensured if the following polynomial has at least one stable root λ

$$P(\lambda) = \left(\theta_{ref} - \theta\right)\lambda^3 + 3\dot{\theta}\lambda^2 - 3\ddot{\theta}\lambda + \dddot{\theta} = 0 \tag{37}$$

Our case of study is concerning a constant reference position task: the PMSM always initials its angular position at zero. According to a study of third degree equation (9) with variable λ and depending on the sign of the coefficient equation (position θ, speed $\dot{\theta}$, acceleration $\ddot{\theta}$ and jerk $\dddot{\theta}$), there are three poles cases control:

- Control without limitation then the pole $\lambda = \lambda_0$ ($\lambda_0 = -10$).
- Control with saturation (for the three saturation cases) and $\theta < \theta_{ref}$ then the pole is real and negative with

$\lambda < \dfrac{\dot{\theta}}{\theta - \theta_{ref}}$.

- Control with saturation (for the three saturation cases) and $\theta > \theta_{ref}$ then the pole is real and negative with $\lambda < \dfrac{-\sqrt{\dot{\theta}^2 - \ddot{\theta}\left(\theta - \theta_{ref}\right)} + \dot{\theta}}{\theta - \theta_{ref}}$.

These results are computed according to the Cardan method [17]. We clearly see that the pole is always negative even forward and backward movements; this study is verified by the exact values of poles illustrated in figure 8.

6. CONTROL STRATEGY : ARTIFICIAL INTELLIGENCE ALGORITHME

We propose the following strategies consisting of an acceleration phase, a constant speed phase and a deceleration phase:

- Phase 1 : maximum current followed by maximum power.
- Phase 2 : constant speed.
- Phase 3 : minimum power followed by minimum current .

During the PMSM simulation movement, a current calculation of $i_{qref\,0}$, $I_{P\max}$ and $I_{V\max}$ is carried out at each iteration. If i_{qref} exceeds $I_{\max}$ or $I_{P\max}$ or $I_{V\max}$, i_{qref} is selected in order to verify the three constraints.

The following algorithm provides how the selection of the reference current i_{qref} is performed and the corresponding non linear control.

//

Step1 : initialization value

$I_{P_{\max}} = 0;\ I_{V_{\max}} = 0$;

$C_e = p\left(L_d - L_q\right) i_d i_q + p\phi_f i_q$: Eq (5)

$P = \dot{\theta} C_e$: Eq (23)

Step2 : computed current reference value

$i_{qref\,0} = f(\lambda = -10)$: Eq (15) *//current reference without limitation*

<u>If</u> $P > 0$ then $I_{P_{\max}} = \vartheta^{-1}\left(P_{\max}\right)$: Eq (27) *// current limit for the maximum positive power*

Else $I_{P_{\max}} = \vartheta^{-1}\left(-P_{\max}\right)$: Eq (27) *// current limit for the maximum negative power*

<u>End if</u>

<u>If</u> $\dot{\theta} > 0$ then $I_{V_{\max}} = g^{-1}\left(\Omega_{\max}\right)$: Eq (34) *// current limit for the maximum positive speed*

Else $I_{V_{\max}} = g^{-1}\left(\Omega_{\max}\right)$: Eq (34) *// current limit for the maximum negative speed*

<u>End if</u>

Step3: chose current reference

$i_{qref} = i_{qref\,0}$ *// current without limitation*

// power limitation

<u>If</u> $P_{ref} > P_{\max}$ & $\dot{\theta} > 0$ & $i_{qref} < I_{P\max}$ then $i_{qref} = I_{P\max}$ *//chose a current limit for maximum power*

Elseif $P_{ref} > P_{\max}$ & $\dot{\theta} < 0$ & $i_{qref} > I_{P\max}$ then $i_{qref} = I_{P\max}$ *//chose a current limit for maximum power rotation in the opposite direction*

Elseif $P_{ref} < -P_{\max}$ then $i_{qref} = I_{P\max}$ *// chose a current limit for maximum power deceleration phase*

<u>End if</u>

//speed limitation

<u>If</u> $\Omega_s > \Omega_{\max}$ then $i_{qref} = I_{V\max}$ *//chose a current limit for maximum speed*

Elseif $\Omega_s < -\Omega_{\max}$ then $i_{qref} = I_{V\max}$ *//chose a current limit for maximum speed*

<u>End if</u>

//current limitation

<u>If</u> $i_{qref} > \mathrm{I}_{\max}$ then $i_{qref} = I_{\max}$ *//chose a current limit for maximum current*

Elseif $i_{qref} < -\mathrm{I}_{\max}$ then $i_{qref} = I_{\max}$ *//chose a current limit for maximum current*

<u>End if</u>

Step4: Control system

$$U=\begin{bmatrix}U_d & U_q\end{bmatrix}^T=l\left(x,x_{1ref}\right)=g_1^{-1}\left[-f_1\left(x\right)-k_1\left(x_1-x_{1ref}\right)\right] \quad : \text{Eq (7)}$$

Return step1

7. SIMULATION AND DISCUTION

The simulation were realized with the same pole placement (triple pole $\lambda_0=-10$) and $\left(x_{2ref}\right)_{simulation1}=\begin{bmatrix}0\\1000Rad\end{bmatrix}$

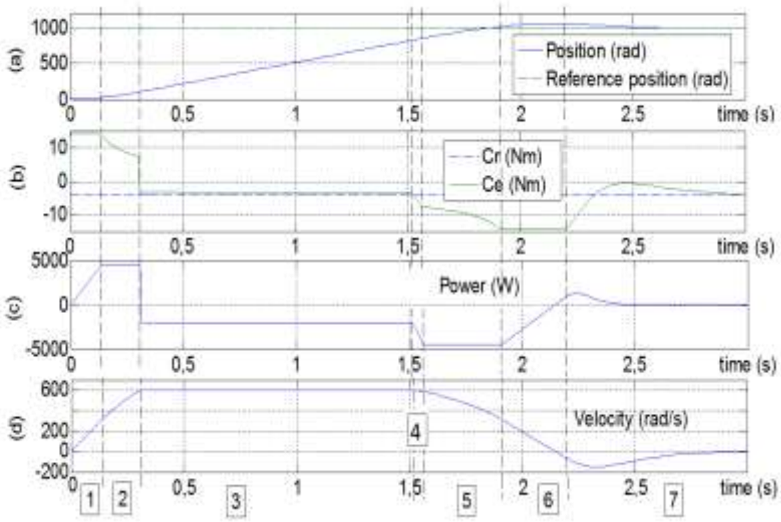

Figure 5. PMSM's behavior under the NCL: (a) Angular position (b) Angular speed (c) Resistive and electromotive torque's (d) Active power

Figure 5 presents the position, speed, power, and torque of the PMSM during the simulation movement. This movement is divided into 7 parts. In parts 1 and 6, there is a current constraint, parts 2 and 5 there is a power constraint, part 3 is a speed constraint and in parts 4 and 7 there is a control without constraint.

The simulation result presented in figure 5, shows that the PMSM reaches its desired final position. The three constraints are continuously not violated by the non linear proposed control proving its effectiveness.

Figure 6 represent the non linear control of the PMSM. The asymptotic behavior previously exposed (equation (20), (21), (29), (30) and (36)) is apparent in figure 5 and 6 during all phases.

Figure 7 represents the reference constraint current i_{qref} during the PMSM movement. The different values of i_{qref} are selected based on the proposed algorithm of section 6.

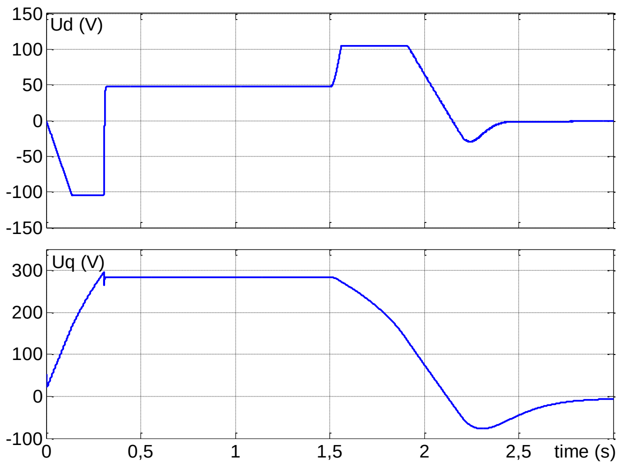

Figure 6. Control law

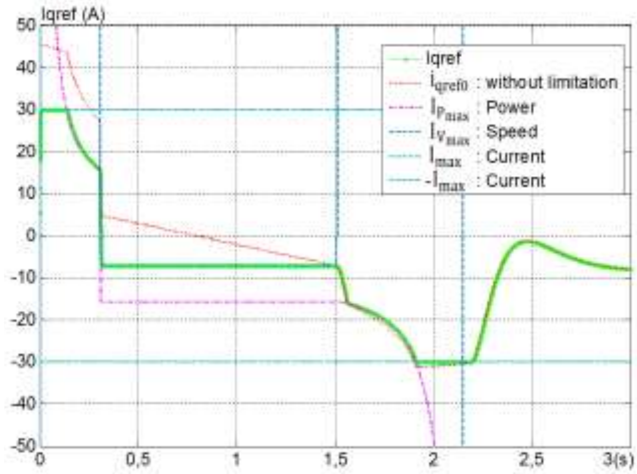

Figure 7. Different Reference current constraint

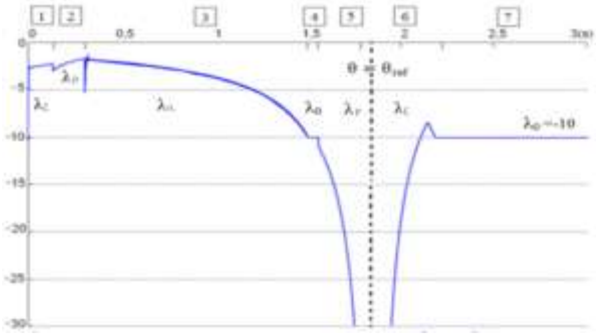

Figure 8. Triple pole's value variation

Figure 8 represents the computed triple pole (9) trought MATLAB during the simulation movement of the PMSM. We note that the pole value is continously mainained negative. this is in accordance to the stability analysis priviously presented in (37).

8. CONCLUSION

In this article, a nonlinear control of a PMSM has been developed as the machine is pushed to it maximum current, power and speed saturation limits. This is done in order reach a final desired position with a minimum time response. Simulation results proves the effectiveness of the proposed non linear control since the PMSM realizes its desired movement without exceeding the saturation levels of current, power and speed.
The simulation results were in accordance with the asymptotic behavior analysis provided for each of the saturation phases. A stability analysis is also presented in this paper. It was proven with the proposed non linear control that the PMSM maintains its stability at all phases of the movement. In fact, the stability analyses of PMSM demonstrate that the triple pole is maintained negative during these phases.
An important element in our algorithm is the setting value of the triple pole. This values set a $\lambda_0 = -10$ in this paper. This values has an important impact on time response especially in the presence of saturation.

APPENDIX

Their values for the studied motor are as follows:

$R_s = 0.6\Omega$ is the stator's resistance, $L_d = 0.0014H$ is the stator inductance along the axis d, $Lq = 0.0028H$ is the stator inductance along the axis q, $f = 0.001$ is the friction's coefficient, $J = 0.008Kgm^2$ is the rotor's inertia, $\varphi_f = 0.12Wb$ is the rotor's magnets flow, $p = 4$ is the number of pole's pairs.

The value of saturation defined by the manufacturers, $I_{\max} = 30A$ is the current saturation, $P_{\max} = 4500W$ is the power saturation, $\Omega_{\max} = 600rad/s$ is the speed saturation,

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