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Extraction of Near Equal Power Moving Radar Targets Using a Conjugate Gradient-Multistage Wiener Filter in the Fractional Fourier Transform Domain

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ABSTRACT

The Fractional Fourier Transform (FrFT) is a useful tool that separates signals-of-interest (SOIs) from interference and noise in non-stationary environments. This requires estimation of the rotational parameter 'a' for the signal in the time-frequency plane, which is typically chosen as the value that minimizes the mean-square error (MSE) between the desired SOI and its estimate. The FrFT is most suited for applications in which the time and frequency content of an SOI and the undesired interference vary differently. In this paper, we describe an algorithm that applies the FrFT to separating multiple near equal power moving targets in a radar system from each other, as well as from the clutter-filled environment. The algorithm utilizes the fact that echoes from moving targets, which are chirps in the FFT domain are tones in the FrFT domain, when rotated to the proper 'a'. We find the optimum value by searching over all 'a' for the maximum peak in the energy in the FrFT. We then notch out the signal everywhere except for the peak. This gives an initial estimate of the desired signal to use as a training sequence. We then apply the joint Conjugate Gradient-Multistage Wiener Filter (CG-MWF) to cancel out the interference from the received signal, also rotated to the same optimum 'a'. This algorithm is robust in clutter because clutter does not correlate with the chirp signal in the FrFT domain, and thus does not impair our ability to estimate the chirp signal peaks. We show that the performance of the proposed algorithm is robust using up to three closely spaced moving targets in clutter for signal-to-clutter ratios (SCRs) down to 0 dB and when interfering targets are within 0.5 dB of the stronger ones.

Keywords: Conjugate Gradient-Multistage Wiener Filter (CG-MWF), Fractional Fourier Transform (FrFT), Radar.

1. INTRODUCTION

The Fractional Fourier Transform (FrFT) has been applied to problems in fields such as quantum mechanics, optics [1], image processing, data compression, and signal processing for communications ([7] and [11]). The FrFT is a very useful tool for separating a signal-of-interest (SOI) from interference in a non-stationary environment [11]. The simpler problem of extracting a single target in clutter using time delay correlation methods is presented in [6] and using clutter map cancellation in [4]. Extracting a single weak target in clutter by performing clutter subtraction is given in [5]. Multiple strong targets are separated with the FrFT using multiple channels in a biostatic SAR in [9] and in [15]. Finally, [13] estimates a signal by computing a peak and comparing it to the side lobe level and removes it by applying a narrow, band stop filter to find weaker signals. This algorithm, however, is limited in performance and cannot cancel strong interferers.

The problem of separating multiple moving targets of near equal power levels received by a monostatic radar system in clutter lends itself nicely to implementation by the FrFT because moving target echoes are chirp signals, which can be easily separated in the FrFT domain. When signals are near equal power, an adaptive filter in the FrFT domain can improve interference cancellation. In this paper, we propose to use the FrFT to extract a target when interfering targets are near equal power and there are high levels of clutter, by first rotating to the proper axis 't_a' using rotational parameter 'a' for the desired target, assumed to be slightly stronger in power than the interfering targets, in which the chirp becomes a tone. We notch out the signal at all values other than this optimum 'a', denoted a_{opt}, to remove the interference and obtain an initial estimate of the target to be used as a training signal. We then use this training signal and the received signal translated to the same FrFT domain a_{opt} as the inputs to a Conjugate Gradient-Multistage Wiener Filter (CG-MWF) to suppress the interference further. Note that when the interference is weak, the training signal itself will be a good estimate of the desired target. However, when interference is strong, some of the interference may be present in the

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domain a_{opt}, requiring further filtering. Following filtering, we rotate back to the time domain to obtain the estimated chirp target. The paper outline is as follows: Section 2 briefly reviews the FrFT and its relation to the Wigner Distribution (WD), which is a useful visual tool for understanding the FrFT. Section 3 describes the signal model and Section 4 discusses the proposed algorithm for detection using the FrFT. Section 5 has simulation results showing the robust performance of the proposed FrFT method. Conclusions and remarks on future work are given in Section 6.

2. BACKGROUND: FRACTIONAL FOURIER TRANSFORM (FrFT)

The FrFT of a continuous time domain function f(x) of order 'a' is defined as in [11]. In discrete time, we model the N ×1 FrFT of an N × 1 vector **x** as

$$\mathbf{X}_a = \mathbf{F}^a \mathbf{x}, \tag{1}$$

where $0 < |a| < 2$, $\mathbf{F}^a$ is an $N \times N$ matrix whose elements are given by ([3] and [11])

$$\mathbf{F}^a[m,n] = \sum_{k=0, k \neq (N-1+(N)_2)}^{N} u_k[m] e^{-j\frac{\pi}{2}ka} u_k[n], \tag{2}$$

and where $u_k[m]$ and $u_k[n]$ are the eigenvectors of the matrix **S** defined by [3]

$$\mathbf{S} = \begin{bmatrix} C_0 & 1 & 0 & \dots & 1 \\ 1 & C_1 & 1 & \dots & 0 \\ 0 & 1 & C_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \dots & C_{N-1} \end{bmatrix}, \tag{3}$$

and

$$C_n = 2cos(\frac{2\pi}{N}n) - 4. \tag{4}$$

Numerous methods are presented in the literature for implementing the FrFT efficiently (see for example [2] and [3]).

The Wigner Distribution (WD) is a time-frequency representation of a signal, and may be viewed as a generalization of the Fourier Transform, which is solely the frequency representation. The WD of a signal x(t) can be written as

$$W_x(t,f) = \int_{-\infty}^{\infty} x(t+\tau/2) x^*(t-\tau/2) e^{-2\pi j \tau f} d\tau. \tag{5}$$

It is well-known that the projection of the WD of a signal x(t) onto an axis t_a gives the energy of the signal in the FrFT domain 'a', $|\mathbf{X}_a(t)|^2$ (see e.g. [7] or [8]). Letting $\alpha = a\pi/2$, this is written as

$$|\mathbf{X}_\alpha(t)|^2 = \int_{-\infty}^{\infty} W_x(tcos(\alpha) - fsin(\alpha), tsin(\alpha) + fcos(\alpha)) df. \tag{6}$$

In discrete time, the WD of a signal x[n] is written as [10]

$$W_x[\frac{n}{2f_s}, \frac{kf_s}{2N}] = e^{j\frac{\pi}{N}kn} \sum_{m=l_1}^{l_2} x[m] x^*[n-m] e^{j\frac{2\pi}{N}km}, \tag{7}$$

where $l_1 = \max(0, n-(N-1))$ and $l_2 = \min(n, N-1)$, for non-periodic signals.

Radar echoes from moving targets are chirp signals, and we show the WD of a chirp signal x(t) in Fig. 1. The figure illustrates how the FrFT may be used to detect chirp signals. If we rotate to the right axis 't_a', the chirp projects onto the axis as a strong tone. The right axis can be found by computing the energy in the FrFT using Eq. (6) at all values of 'a' and searching for the peak.

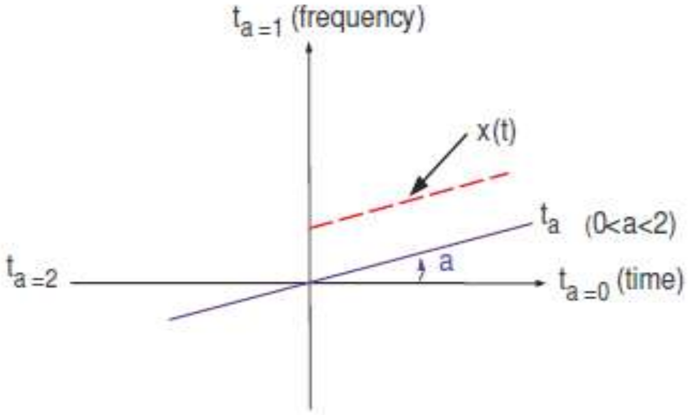

Fig. 1: Wigner Distribution of Chirp Signal x(t)

3. SIGNAL MODEL

We follow the signal model presented in [13]. Assume that there is an airborne radar platform such as a plane moving along the y-axis at constant speed V and a moving target k at an initial distance $R_{0,k}$ away, at time t = 0, moving with speed v_k and acceleration a_k, potentially in a different direction as the plane (see Fig. 2). After a time t the platform and target have moved distances of Vt and $v_k t + ½a_k t^2$, respectively. The components of the speed of the target along the x- and y-axes are $v_{r,k}$ and $v_{c,k}$, and similarly $a_{r,k}$ and $a_{c,k}$ for the acceleration. At time t, target k is now at a distance from the platform whose horizontal component has decreased by $v_{r,k}t + ½a_{r,k}t^2$ and likewise the vertical component has decreased by $v_{c,k}t + ½a_{c,k}t^2$. Accordingly, the distance between the target and radar can be approximated by [13]

$$R_k(t) \approx R_{0,k} - v_{r,k}t + [(V - v_{c,k})^2 - R_{0,k}a_{r,k}]\frac{t^2}{2R_{0,k}}. \tag{8}$$

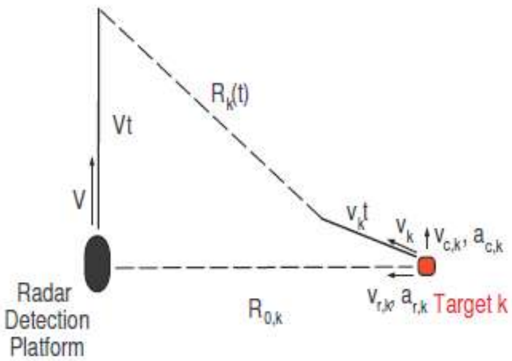

Fig. 2: Radar Detection Platform and Moving Target Scenario

As discussed in [13], the echo of the moving target as received by the radar system can be approximated by a chirp signal that takes the form

$$x_k(t) = e^{j2\pi f_{d,k}t}e^{j\pi\Gamma_k t^2}, \tag{9}$$

where

$$f_{d,k} = \frac{2v_{r,k}}{\lambda}, \tag{10}$$

$$\Gamma_k = \frac{2}{\lambda R_{0,k}}[-(V - v_{c,k})^2 + R_{0,k}a_{r,k}], \tag{11}$$

λ is the wavelength of the radar, related to its frequency f by $\lambda = c/f$, and $c = 3\times10^8$ m/s is the speed of light. We further assume there are no strong point scatterers, so that the ground (i.e. surface) clutter may be modeled as additive white Gaussian noise (AWGN), using a desired signal-to-clutter ratio (SCR). If we now assume a scenario with K moving targets, we can write the composite received echo signal as

$$x(t) = \sum_{k=1}^{K} A_k x_k(t) + n(t), \tag{12}$$

where n(t) is a combination of clutter and noise using a given SCR, and A_k is the amplitude of the target. If we assume, without loss of generality, that signal k = 1 is the main target, we can model the amplitudes A_k for k = 2,3,...,K using an assumed carrier-to-interference ratio (CIR) by writing $A_k = 10^{-CIR_k/20}$. In this paper, we assume near equal power signals, so $0 \leqslant CIR_k \leqslant 2$ dB, $\forall k$.

4. PROPOSED ECHO SEPARATION METHOD USING FrFT AND CG-MWF

The proposed algorithm applies the CG-MWF [14] in the FrFT domain. In discrete time, the samples of the received signal are written as $\mathbf{x}(i)$, where sample index i ranges from i = 1,2,...,N, and N is the total number of collected samples. A block diagram of the adaptive filtering problem is shown in Fig. 3. We wish to estimate a desired signal $\mathbf{d}_0(i)$ from a received signal $\mathbf{x}_0(i)$. The filter weights will be determined by applying the CG-MWF, and $\mathbf{d}_0(i)$ is the estimate of the desired signal. Normally a training sequence would be used, i.e. $\mathbf{d}_0(i) = \mathbf{x}_1(i)$ here, but since we do not have this signal, it has to be estimated. We propose to do this by translating the received signal to the FrFT domain over the range of $0 \leqslant a \leqslant 2$ and searching for the peak. We then rotate to the value of 'a', a_{opt}, which produced the peak and notch out the signal everywhere except at the peak value. Hence we get an estimate of the true target signal. The second input is $\mathbf{x}_0(i)$, which is the received signal, also translated to the same optimum FrFT domain. The portion of the algorithm used to obtain $\mathbf{d}_0(i)$ and $\mathbf{x}_0(i)$ is described in Table 1. The unconstrained adaptive filter will then compute weights $w_{CG,1}(i)$ to produce a better estimate of $\mathbf{x}_1(i)$ from $\mathbf{d}_0(i)$ and $\mathbf{x}_0(i)$. The output is the estimate $\hat{\mathbf{d}}_0(i)$, and $\epsilon_0(i) = \mathbf{d}_0(i) - \hat{\mathbf{d}}_0(i)$ is the estimation error. The CG-MWF algorithm is summarized in Table 2 ([14] and [12]), where we set $\sigma_l = 0$, $\epsilon = 0.001$, and $N_{CG} = 1$ is the filter rank.

The estimate of the received chirp target, still in the FrFT domain 'a_{opt}', is given by

$$\hat{\mathbf{D}}_0(i) = \mathbf{w}_{a,CG,1}(i)^H X_{peak}. \tag{13}$$

Finally, we rotate back to the time domain to obtain

$$\hat{\mathbf{d}}_0(i) = \mathbf{F}^{-a_{opt}}\hat{\mathbf{D}}_0(i). \tag{14}$$

Mean-square error (MSE) may be computed as

$$MSE = \sqrt{\frac{\sum_{i=1}^{M} ||\epsilon_0(i)||^2}{M}}. \tag{15}$$

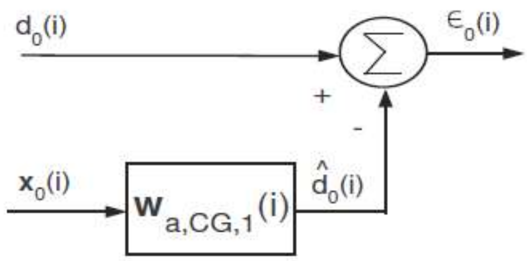

Fig. 3: Unconstrained Adaptive Filter

Table 1: Generation of Inputs for CG-MWF Algorithm

1. For $a = 0 : \Delta a : 2$ % Loop over all a
   $\mathbf{X}(a) = \mathbf{F}^a x(t)$; % Compute FrFT of x(t), given in Eq. (12)
   $X_{max}(a) = \underset{a}{arg\ max}(|\mathbf{X}(a)|)$; % Compute max value
   End
2. $a_{opt} = \underset{a}{arg\ max}\ X_{max}(a)$; % Find peak over all a
3. $\mathbf{X}_{peak} = \mathbf{F}^{a_{opt}} x(t)$; % Rotate to signal-of-interest
4. % Compute $\mathbf{x}_0(i)$ as $\mathbf{x}(i)$ rotated to FrFT domain a_{opt}
   $\mathbf{x}_0(i) = \mathbf{X}_{peak}$;
5. % Compute $\mathbf{d}_0(i)$ by zeroing interference
   $\mathbf{d}_0(i) = \mathbf{X}_{peak}|_{a \neq a_{opt}} = 0$;

Table 2: CG-MWF Recursion Equations

Input: $\mathbf{d}_0(i)$ and $\mathbf{x}_0(i)$
Initialization: $\mathbf{f}(1) = \mathbf{r}(1) = \mathbf{d}_0(0) - \mathbf{x}_0^H(0)\mathbf{w}_{a,CG,1}(0)$
Recursion: For $j = 1, 2, ..., N_{CG} - 1$, $\alpha(j) = \frac{\mathbf{r}^H(j)\mathbf{r}(j)}{\mathbf{f}^H(j)\mathbf{x}_0^H(i)\mathbf{f}(j) + \sigma_I^2}$ $\mathbf{w}_{a,CG,1}(j) = \mathbf{w}_{a,CG,1}(j-1) + \alpha(j)\mathbf{f}(j)$ $\mathbf{r}(j+1) = \mathbf{r}(j) - \alpha(j)\mathbf{x}_0(i)\mathbf{f}(j)$ Note: If $\|\mathbf{r}(j+1)\| < \epsilon$ stop, else continue. $\beta(j) = \frac{\mathbf{r}^H(j+1)\mathbf{r}(j+1)}{\mathbf{r}^H(j)\mathbf{r}(j)}$ $\mathbf{f}(j+1) = \mathbf{r}(j+1) + \beta(j)\mathbf{f}(j)$
Output: $\mathbf{w}_{a,CG,1}(i) = \mathbf{w}_{a,CG,1}(N_{CG} - 1)$

5. SIMULATIONS

We model the three targets given in [13], whose parameters are repeated for convenience in Table III. In the first example, we let SCR = 10 dB, CIR_2 = 1 dB, and CIR_3 = 2 dB. These signals overlap in frequency, so standard FFT-based filtering will not work. Due to the strong interference, we need to make the step size Δa small, and here we determine that Δa = 0.001 is sufficient. If Δa is too large, the resolution problem causes error in estimation due to the interference. We let N = 1,000 and plot the FFT of the received signal as well as the true target echo $x_1(i)$ and its estimate $\hat{\mathbf{d}}_0(i)$ from this algorithm both in time and frequency in Fig. 4. We also compute the mean-square error (MSE) between the true signal and its estimate, which is 0.1473. Note that in this case the algorithm computed a_{opt} = 1.086.

In the second example, we set the SCR = 0 dB, with all other parameters the same. There is no noticeable difference in performance, showing the robustness of the proposed algorithm to clutter. This is because clutter does not correlate with chirps or tones in the FrFT domain, since clutter is uniformly distributed over time and frequency. In the next example, we let SCR = 0 dB, CIR_2 = 0.25 dB, and CIR_3 = 0.5 dB. Once again, there is no noticeable performance loss, and the result is shown in Fig. 5. The MSE is 0.1805, showing the algorithm robustness to both clutter and high levels of interference. The key feature of this algorithm is that signals are separated by using FrFT domain filtering, hence it performs well when signals are close to the same power levels.

Note also that targets can be notched out in the FrFT domain easily by applying the opposite constraint as we did above. That is, we null the signal at a = a_{opt} instead of at a ≠ aopt. This would allow us to search for other signals. When signals are the same power, such a technique might be useful to find a particular target of interest. In this case, targets can successively be notched until the correct target is detected and the filtering technique applied to extract it from the noisy environment.

Table 3: Parameters of Simulated Targets

Target, k	$v_{r,k}[m/s]$	$v_{c,k}[m/s]$	$a_{r,k}[m/s^2]$
1	0.67	5	2.54
2	1.67	−5	3.97
3	2.33	3	1.03

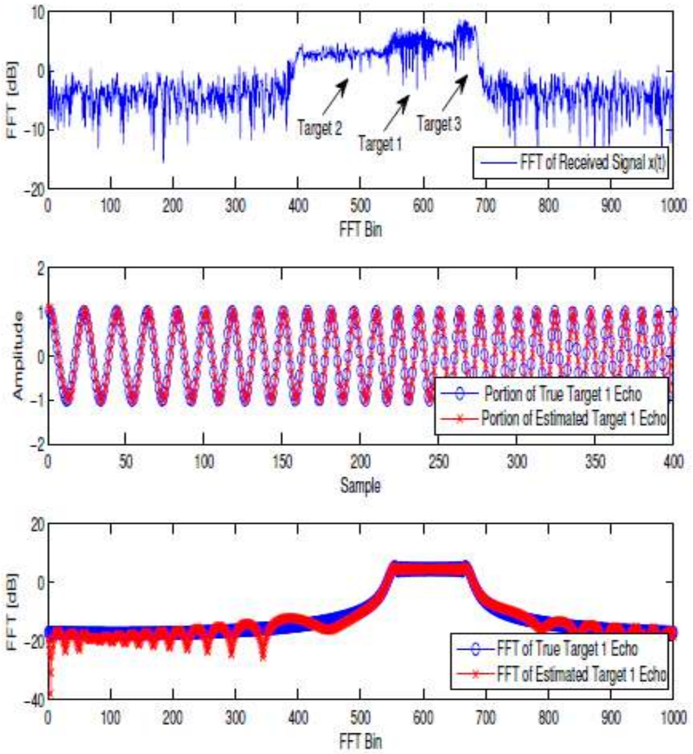

Fig. 4: K = 3 Targets; SCR = 10 dB, CIR_2 = 1 dB; CIR_3 = 2 dB

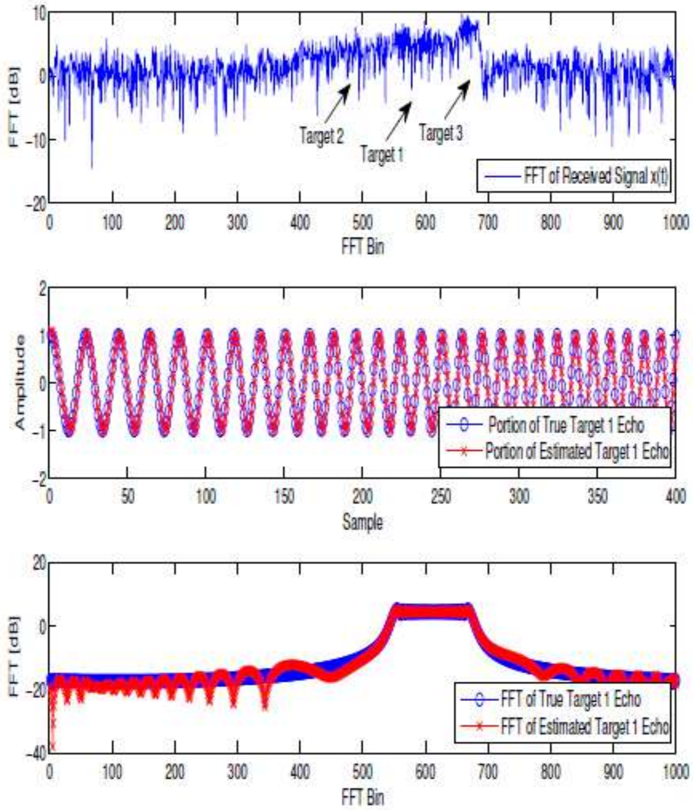

Fig. 5: K = 3 Targets; SCR = 0 dB, CIR_2 = 0.25 dB; CIR_3 = 0.5 dB

CONCLUSION

In this paper, we present a simple, robust algorithm for extracting targets in a moving target environment with clutter and strong interference, which is near to the same power as the target of interest. The algorithm applies the Fractional Fourier Transform (FrFT). Since target echoes are chirp signals, when rotated to the fractional domain 'a' that maximizes the energy in the FrFT (i.e. their projection onto the axis 't_a' in the Wigner Distribution), they become tones. Tones from stronger signals are extracted by zeroing the rest of the signal to use as an input to a Conjugate Gradient-Multistage Wiener Filter (CG-MWF) algorithm. We show the algorithm works for three moving targets when the signal-to-clutter ratio (SCR) is just 0 dB, and the two interfering targets are only 0.25 and 0.5 dB below the desired target. Future work involves applying the CG-MWF algorithm in the FrFT domain to other types of signals for interference mitigation.

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