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Hybrid observer design for a class of hybrid dynamic systems

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ABSTRACT

Hybrid observer design is addressed for a class of discrete-time linear switched systems whose switching mechanism is unknown. The hybrid observation problem consists in determining the estimation of the current mode and the continuous observer for continuous behavior from a finite set of input-output data. First, the current mode is estimated using a classification algorithm that associates the current data to its appropriate submodel. Second, the continuous observer is determined by the resolution of linear matrix inequalities. A comparative study of the proposed approach with the k-means method was achieved in simulation. A numerical example was reported to evaluated the proposed method.

Keywords: Clustering algorithm, Discrete state, Hybrid dynamic systems, Hybrid observer estimation, Switched linear system.

1. INTRODUCTION

Hybrid systems are dynamic systems that explicitly and simultaneously involve behaviors or models of continuous and event dynamic type. Continuous behavior is the result of the natural evolution of the physical process, while event or discrete behavior may be due to the presence of switching, operating phases, transitions, computer program codes, etc.

The method of interaction between these two behaviors define three main classes according to (19), namely, the discrete approach, the mixed approach and the continuous approach. In fact, the first approach consists in approximating the continuous dynamics to be lead as a discrete event system. While the second method considers both continuous and discrete behaviors in the same structure as the hybrid Petri network (22) and the hybrid automate (23). However, the third approach is to approximate the discrete dynamics of the hybrid system by differential equations to model the occurrence of discrete events in order to use the continuous systems theory. This approach can be modeled by the piecewise affine (PWA) model (25), the switched linear model, the Markov jump linear (MJL) model (24).... etc. The hybrid dynamic systems (SDH), with their three configurations, have received great attention in the last years since they are very useful for automatic control applications such as chemical process, embedded systems, telecommunication networks and so on. In fact, several approaches have been proposed in the literature for hybrid system presentations, analysis, modeling, observations and control. This paper deals with the problem of hybrid systems observation. The hybrid observer design for hybrid dynamic systems (SDH) is composed of two variables, the first is the discrete state $q(k)$ uses to model the event part, while the second is the continuous state $x(k)$ relating to the continuous part. Though, the hybrid state estimation involves both the estimation of the discrete state and the continuous state $\hat{q}(k)$, $\hat{x}(k)$. For the observer design problem of hybrid systems, many methods were proposed. The previous approaches, for the reasons of simplicity, assume that the discrete state is available a priori and then used a Luenberger observer or a high order sliding-mode observer to estimate the continuous state (5, 6). Another method, presented in (3), consists in designing an observer for the switched systems. Its stability is guaranteed by polytopic Lyapunov functions. Furthermore, the approach presented in (1) consists in developing a method for an active mode observability that may randomly switched between the different modes taken from a finite set. The purpose of this method is to determine the control sequences (discerning control sequences) that prove the reconstruction the switching sequence on the basis of observations. The observer design presented in (2) consists of two stages. First, the discrete state estimation is assumed as a maximum value of a likelihood criterion. Second, the estimated discrete state is embedded in a Kalman filtering framework to estimate the continuous state. A recent result for a class of Markovian Jump Linear Systems (MJLSs) in the presence of bounded polyhedral disturbances was developed in (4). this method is called a set-membership estimation algorithm. In fact, it allows to find the smallest consistent set of all possible states, which is shown to be expressed by a union of multiple polytopes. Previous works on state estimation for hybrid systems were dedicated to the class of piecewise affine systems (PWA) (7,11). In (7), the proposed method is shown in a hybrid observation schema which corresponds to the combination of the commutation law construction and the linear piecewise observer. In (20), a recursive state estimation is developed for hybrid systems described by the state-space model in the form of the conditional probability (density) functions. This method is based on the factorized form of Bayesian filtering. For the linear switching systems, in (21), the authors deal with the joint problem of state estimation and event under irregular sampling. The solution is established extending Shannon's sampling theorem. However, for hybrid systems modeled by differentials Petri Nets, a method was proposed in (9). The synthesis of an observer is based on an observation schema combining the observer of Petri Net and a Luenberger continuous observer. The discrete observer returns the discrete mode by estimating the discrete marking and provide the active mode to the continuous observer.

In this paper, a new method is presented for observer design of switched linear systems defined in discrete-time. Our aim is to develop a hybrid state estimation from a finite set of the system input-output. In the first time, the proposed method estimates the current mode under evolution based on a classification algorithm. It consists in selecting the representative points of classes (submodels). These points are considered as class centers. Once the first component of the class centers called decision variables are selected, one proceeds to estimate the discrete state. Indeed, the discrete state $q(k)$ is determined by the minimum value of S criterion which involves the difference between the system output and the decision variable of the appropriate class. Finally, we present the continuous observer which is based on the switched observer approach.

The paper has the following structure. Section II deals with the problem statement. Section III studies the observability of switching system. The proposed method is developed in section IV. In section V, we present the simulations in order to illustrate the effectiveness of the proposed method.

2. PROBLEM STATEMENT

We consider a class of hybrid dynamics systems described by the following state space model :

$$\begin{cases} x(k+1) = A_i x(k) + B_i u(k) \\ y(k) = C_i x(k) \end{cases} \quad (1)$$

where $i \in S=\{1, ..., s\}$ is the discrete state of the system, s is the number of submodels, $x(k) \in \mathbf{R}^n$, $y(k) \in \mathbf{R}$ and $u(k) \in \mathbf{R}$ are respectively the continuous state, the output and the input of the system, A_i, B_i and C_i are the parameter matrices associated with the submodel indexed by i. In the method to be presented, no constraint is imposed on the switching mechanism. In fact, the switches can be exogenous, deterministic, state-driven, event-driven, time-driven or totally random. However, we assume that the order n and the number of submodels s are known a priori.

Given data $\{y(k), u(k)\}_{k=1}^{N}$ generated by a switched linear model of the form as in (1), we are interested in estimating a hybrid state. This observer gives the evolution of the current mode $\hat{q}(k)$ and the estimation of the state vector $\hat{x}(k)$ According to figure 1, the discrete observer uses a classification procedure and a decision mechanism. The classification procedure provides the class centers $(y_1^*, y_2^*, \dots, y_s^*)$ which will be exploited by the decision mechanism in order to identify the discrete state $\hat{q}(k)$. Then, the state vector estimation $\hat{x}(k)$ is expressed by a switched observer. The stability analysis of the switched linear observer involves a Poly-Quadratic stability principle (16). This concept allows to check the asymptotic stability of the state vector estimation $\hat{x}(k)$ by means of polytopic quadratic Lyapunov functions.

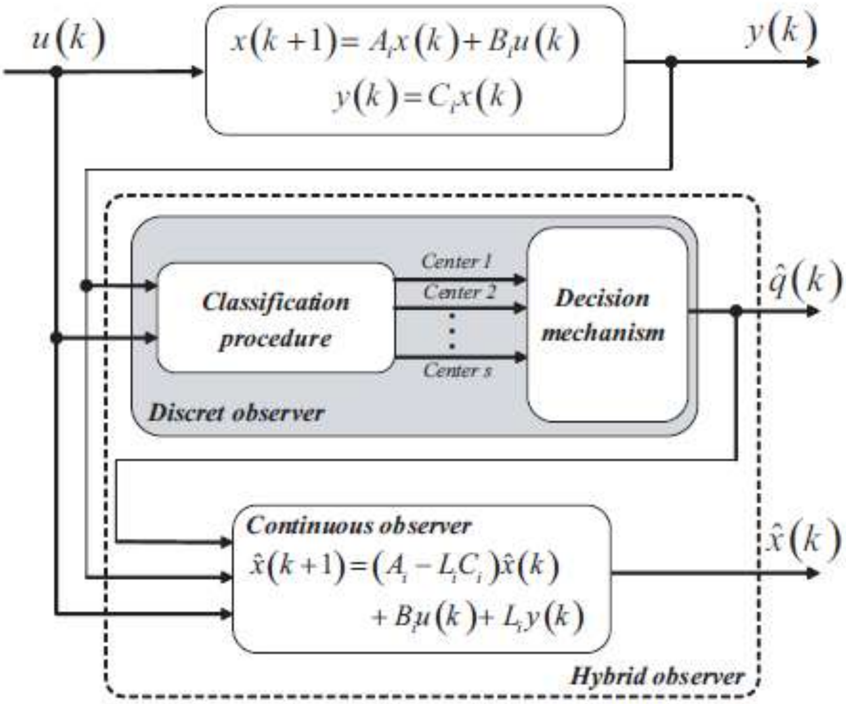

Fig. 1: The proposed observer structure

3. OBSERVABILITY OF THE SWITCHED SYSTEMS

The observability notion of switched linear systems needs to take into account several different problems related to observed variables of the system, namely, the state and the mode. In fact, the mode sequence can be observable or unobservable, and in the second case, another problem arises in the ability to recover the mode with the state. Thus, many observability concepts are defined in (17). The necessary and sufficient conditions for observability expressed by the authors are only a transposition of the conditions already established to characterize the decidability. To explain this result it is necessary to introduce some definitions.

Let a path θ defined as a finite modes sequence $\theta=\theta_1\theta_2\ldots\theta_{N_1}$ with N_1 is the path length, and Θ_N the set of all path of length N. We pose by $\theta_{[i,j]}$ the infix of θ between i and j (i.e. $\theta_{[i,j]}=\theta_i\theta_{i+1}\ldots\theta_j$), we use $\theta\theta'$ to denote the concatenation of θ with θ', and we pose $\Phi(\theta)\triangleq A_{\theta_{N_1}}\ldots A_{\theta_1}$ called the transition matrix of a path θ. By convention, we let $\theta_{[i,i-1]}=\varepsilon$ (ε is empty word) and $\Phi(\varepsilon)=I$. Let $O(\theta)$ an observability matrix of a path θ and a matrix $\Gamma(\theta)$

$$O(\theta)\triangleq\begin{pmatrix} C_{\theta_1} \\ \vdots \\ C_{\theta_{N_1}}A_{\theta_{N_1-1}}\ldots A_{\theta_{N_1}} \end{pmatrix}$$

$$\Gamma(\theta)\triangleq\begin{pmatrix} 0 & \ldots & 0 & 0 \\ C_{\theta_2}B_{\theta_1} & \ldots & 0 & 0 \\ C_{\theta_3}A_{\theta_2}B_{\theta_1} & \ldots & 0 & 0 \\ \vdots & \ldots & 0 & \vdots \\ C_{\theta_{N_1}}\Phi\left(\theta_{[2,N_1]}\right)B_{\theta_1} & \ldots & C_{\theta_{N_1}}B_{\theta_{N_1-1}} & 0 \end{pmatrix}$$

Which ultimately allows us to define :

$$Y(\theta,x,U)=O(\theta)x+\Gamma(\theta)U$$

Where $U=\left[u(1)\ldots u(N_1)\right]^T$ is a control vector and $Y(\theta,x,U)=\left[y(1)^T \quad \ldots \quad y(N_1)^T\right]$ is an output vector in $\mathbb{R}^{(N_1\times 1)}$ and $x=x(1)$ is the initial state. In order to characterize the switched system observability, we must introduce a few definitions. Namely $\Re(M)$ is the subspace spanned by the rows of the matrix *M*.

Definition (17) (Controlled-Discernibility (CD))

Two different paths θ and θ' of length N_1 are controlled-discernible (CD) if

$$(I-P)\left(\Gamma(\theta)-\Gamma(\theta')\right)\neq 0$$

where P is the matrix of any projection on $\Re\left(\left[O(\theta) \quad O(\theta')\right]\right)$.

The result of (17) on the observability of SLS systems is as follows:

Theorem (17) *So that the SLS (1) to be strongly mode observable at N_1 requires that the following items be reciprocal:*

1) *There exists an integer N_1' and a vector U such that for all $x\in\mathbb{R}^n$ and all $\theta\in\Theta_{N_1+N_1'}$*

$$\theta_{[1,N_1]}\neq\theta'_{\left[1,N_1'\right]}\Rightarrow Y(\theta,x,U)\neq Y'(\theta',x',U')\,\forall x'\in\mathbb{R}^n$$

2) *Two different paths θ and θ' of length N_1 are forward controlled-discernible (FCD) if there exists an integer N_1' such that $\theta\lambda$ and $\theta'\lambda'$ are controlled discernible for any pair of paths λ and λ' of length N_1'. The smallest such integer is the index of FCD.*

4. THE PROPOSED METHOD

Several existing methods estimate the active discrete state using a set of data defined on an observation horizon. This set is assumed to be pure i.e his elements are generated by the same submodel. Though, this assumption is not always true in practice because these elements may be generated by more submodels. To resolve this problem, they introduce the dwell time to ensure that the data belongs to the same submodel.

In this paper, we propose a novel approach which consists in associating the current output $y(k)$ to its appropriate submodel. This assignment is insured thanks to a clustering algorithm.

A. The discrete state estimation

The discrete state is estimated as the index of the submodel that generated the pair of data $(u(k),y(k))$ in the sense of a certain decision criterion $J_i(k)$ designed by :

$$\hat{q}(k)=\underset{i=1,\ldots,s}{\arg\min}\,J_i(k) \tag{2}$$

The criterion $J_i(k)$ can be chosen in many different ways. Since no model has been specified for the discrete state dynamics in switching linear system (1), $J_i(k)$ is taken here as a function of a square error between the system output $y(k)$ and the decision variable associated to the i^{th} class y_i^* :

$$J_i(k)=\left|y(k)-y_i^*\right|^2 \tag{3}$$

Criterion (3) can be interpreted as the distances from the data $y(k)$ to the decision variable y_i^* of i^{th} class. Before summarizing the state estimation algorithm, we must take the following assumption.

Assumption 1. At each instant index k and for any couple (i, j) of discrete states indexes,

$$J_i(k) = J_j(k) \Rightarrow i = j \tag{4}$$

The objective of assumption 1 is obviously to remove any ambiguity in the inference of the discrete state if the true submodel, is exactly known. We suggest the use of the Chiu's clustering algorithm (10,12), also called the method of mountains (13,14). In this work, we extended this method to classify a set of regression's vectors (15, 26) $\varphi(k)\left(\varphi(k) = \left[y(k)\; y(k-1)\; u(k-1)\right]^T, k = 1,\ldots,N\right)$ which consists in associating to each $\varphi(k)$ a potential P_k defined by :

$$P_k = \sum_{\substack{l=1 \\ l \neq k}}^{N} \exp\left(\frac{-4\left\|\varphi(k) - \varphi(l)\right\|^2}{r_a^2}\right) \tag{5}$$

Where r_a is a positive coefficient that controls the decay of the potential. Indeed, the potential decreases exponentially when $\varphi(l)$ moves away to $\varphi(k)$. Otherwise, the coefficient r_a defines the radius of a class. The first class center, denoted φ_1^* is the data whose potential P_1^*, expressed by equation (6), is the maximum. The potential of the neighboring points of the center while gradually decreasing away from the latter. To avoid selecting the data in the neighborhood of the center φ_1^* as other centers of classes, the classification procedure changes the value of each potential given by the following formula:

$$P_k \leftarrow P_k - P_1^* \exp\left(\frac{-4\left\|\varphi(k) - \varphi_1^*\right\|^2}{r_b^2}\right) \tag{6}$$

The parameter r_b ($r_b > 0$) must be strictly greater than r_a to promote the operation on the selection of other distinctly different classes of first and its close. The center of second class is selected as the data having the maximum modified potential given by relation (7). Let φ_2^* the second center and P_2^* the associate modified potential. Similarly, we selected the c^{th} center φ_c^* having P_c^* as a potential and the potentials are modified by the following formula:

$$P_k \leftarrow P_k - P_{c-1}^* \exp\left(\frac{-4\left\|\varphi(k) - \varphi_{c-1}^*\right\|^2}{r_b^2}\right) \tag{7}$$

Chiu introduced two positive parameters ε_1 and ε_2 $(\varepsilon_1 > \varepsilon_2)$ for conditioning the choice of centers. Indeed, at each instant, the selection procedure of each class center obeyed the following inequalities:

- If $P_c^* > \varepsilon_1 P_1^* \Rightarrow$ selection authorized.
- If $P_c^* < \varepsilon_2 P_1^* \Rightarrow$ selection achieved.
- If $\varepsilon_2 P_1^* < P_c^* < \varepsilon_1 P_1^*$ and if : $\dfrac{Min\left(\left|\varphi_c^* - \varphi_1^*\right|, \left|\varphi_c^* - \varphi_2^*\right|, \ldots, \left|\varphi_c^* - \varphi_{c-1}^*\right|\right)}{r_a} \leq 1 - \dfrac{P_c^*}{P_1^*}$

where φ_c^* is the current center and $\varphi_1^*, \varphi_2^*, \ldots, \varphi_{c-1}^*$ are the already selected centers. The retained center has, in these conditions, the highest potential after the rejection of the old potential P_{c-1}^* (the value of P_{c-1}^* was set to zero).

B. Continuous Observer

The purpose is to design a switched observer for the switched linear system. In fact, we consider the following observer defined by :

$$\begin{cases} \hat{x}(k+1) = A_i\hat{x}(k) + B_i u(k) + L_i\left(y(k) - \hat{y}(k)\right) \\ \hat{y}(k) = C_i\hat{x}(k) \end{cases} \tag{8}$$

The gain matrices $L_i, i = 1,\dots,s$ have to be computed such that the estimation state $\hat{x}(k)$ is asymptotically converged to the system state $x(k)$ whatever the initial conditions, i.e :

$$\forall \varepsilon(0) \in \mathbb{R}^n \quad \lim_{k\to\infty} \varepsilon(k) = 0 \tag{9}$$

Where $\varepsilon(k) = x(k) - \hat{x}(k)$ is the estimation error and its dynamic behaviors defined by :

$$\varepsilon(k+1) = (A_i - L_iC_i)\,\varepsilon(k) \tag{10}$$

The switched observer design is reduced to the computation of the gain matrices $L_i, i \in \{1,\dots,s\}$ which ensure the asymptotic stability for the switched system (10). To solve this problem, we use the concept of Poly-Quadratic stability (16). In fact, the stability is checked by means of particular quadratic Lyapunov functions taking into account the switching nature of system (1). To recall this concept, the estimation error dynamic becomes :

$$\varepsilon(k+1) = \tilde{A}(\xi_k)\,\varepsilon(k) \tag{11}$$

The structure of dynamical matrix $\tilde{A}$ is assumed to depend in polytopic way on the parameter ξ_k :

$$\tilde{A}(\xi_k) = \sum_{i=1}^{s} \xi_k^i \tilde{A}_i \tag{12}$$

Where $\tilde{A}_i = A_i - L_iC_i$. The components of the parameter vector ξ_k^i appear as indicator functions given by (3) :

$$\xi_k^i = \begin{cases} 1 & \text{when the switched system} \\ & \text{is described by matrix } A_i \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

with $i \in \{1,\dots,s\}$ and $\xi_k = \left[\xi_k^1,\dots,\xi_k^s\right]$. To check asymptotic stability of system (11), Poly-quadratic stability uses Lyapunov function with a polytopic structure similar to that of the system description :

$$V(\varepsilon(k),\xi_k) = \varepsilon^T(k)P(\xi_k)\varepsilon(k), \quad \text{with } P(\xi_k) = \sum_{i=1}^{s}\xi_k^i P_i \tag{14}$$

where $P_i,\ i = 1,\dots,s$, are symmetric positive definite constant matrices of appropriate dimensions. Using this fact, the following theorem gives sufficient condition to build such a switched observer.

Theorem(3) *If there exist symmetric matrices S_i, matrices F_i and G_i solutions of:*

$$\begin{pmatrix} G_i + G_i' - S_j & G_i'A_i - F_i'C_i \\ A_i'G_i - C_i'F_i & S_i \end{pmatrix} > 0 \ \forall (i,j) \in \{1,\dots,s\}^2 \tag{15}$$

then a switched observer (8) for system (1) exists and the resulting gains L_i are given by $L_i = G_i'^{-1}F_i'$ with $P_i = S_i^{-1}$.

In summary, the proposed approach consists of two steps : one is offline procedure which allows to determine the class centers using the Chui's algorithm; and two is an online operations which consists in estimating the discrete state and the continuous state based on the results of the first step.

Algorithm 1

Offline part

Calculate the potential P_k as in (5) for $k = 1,\dots,N$

Select the first decision variable y_1^* (the vector center's first component)

Repeat

Calculate the modified potential as in (6)

Select the c^{th} center

Until $\left(\varepsilon_2 P_1^* < P_1^* < \varepsilon_1 P_1^*\right)$ and $\left(\dfrac{Min\left(\left|\varphi_c^* - \varphi_1^*\right|, \left|\varphi_c^* - \varphi_2^*\right|, \dots, \left|\varphi_c^* - \varphi_{c-1}^*\right|\right)}{r_a} \le 1 - \dfrac{P_c^*}{P_1^*}\right)$

Online part

For $k = 1,\dots,N$

Compute criteria $J_i(k)$ as in (3) for $i = 1,\dots,s$

Estimate the discrete state $\hat{q}(k)$ such as $J_{\hat{q}(k)} = \min\limits_{i=1,\dots,s} J_i(k)$

End for

According to $\hat{q}(k)$ update $\hat{x}(k)$ as in (8)

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5. SIMULATION : A DOUBLE CART WITH ELASTIC COUPLING

In this section, we give a switched linear example in order to illustrate our method. Let us consider the mechanical system (18) described in figure (fig2).

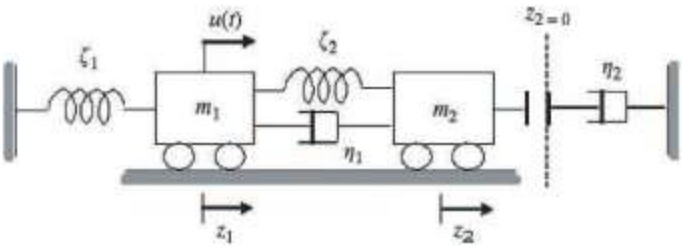

Fig. 1 The A double cart with elastic coupling.

The linear switching model described in continuous time of the plant under investigation is :

- for $z_2(t) \le 0$:

$$m_1\ddot{z}_1 = \zeta_1 z_1 + \zeta_2\left(z_2 - z_1\right) + \eta_1\left(\dot{z}_2 - \dot{z}_1\right) + u \text{ and}$$
$$m_2\ddot{z}_2 = -\zeta_2\left(z_2 - z_1\right) - \eta_1\left(\dot{z}_2 - \dot{z}_1\right) \tag{16}$$

- for $z_2(t) > 0$:

$$m_1\ddot{z}_1 = -\zeta_1 z_1 + \zeta_2\left(z_2 - z_1\right) + \eta_1\left(\dot{z}_2 - \dot{z}_1\right) + u \text{ and}$$
$$m_2\ddot{z}_2 = -\zeta_2\left(z_2 - z_1\right) - \eta_1\left(\dot{z}_2 - \dot{z}_1\right) - \eta_2\dot{z}_2 \tag{17}$$

Let $x^T = (x_1, x_2, x_3, x_4) = (z_1, \dot{z}_1, z_2, \dot{z}_2)$. Then, the continuous time state representation of (17) and (18) become :

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{\zeta_1+\zeta_2}{m_1} & -\frac{\eta_1}{m_1} & \frac{\zeta_2}{m_1} & \frac{\eta_1}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{\zeta_2}{m_2} & \frac{\eta_1}{m_2} & -\frac{\zeta_2}{m_2} & -\frac{\eta_1}{m_2} \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} u(t)$$
$$y(t) = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} x(t) \qquad \text{if } x_3(t) \le 0 \tag{18}$$

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{\zeta_1+\zeta_2}{m_1} & -\frac{\eta_1}{m_1} & \frac{\zeta_2}{m_1} & \frac{\eta_1}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{\zeta_2}{m_2} & \frac{\eta_1}{m_2} & -\frac{\zeta_2}{m_2} & -\frac{\eta_1+\eta_2}{m_2} \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} u(t)$$
$$y(t) = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} x(t) \qquad \text{if } x_3(t) > 0 \tag{19}$$

We will consider these two configurations :

[1]. The system without damper 1, i.e $(\eta_1 = 0)$.

[2]. The system with damper 1, i.e $(\eta_1 \ne 0)$.

Let $m_1 = m_2 = 1kg$, $\zeta_1 = \zeta_2 = 1\, Nm^{-1}, \eta_2 = 1\, Nsm^{-1}$

Case 1:

The parameters matrices are given in the sampling phase, for a sampling period $T_e = 0.5s$, by :

$$A_1 = \begin{pmatrix} 0.7627 & 0.4596 & 0.1174 & 0.02006 \\ -0.8992 & 0.7627 & 0.4396 & 0.1174 \\ 0.1174 & 0.02006 & 0.8801 & 0.4797 \\ 0.4396 & 0.1174 & -0.4596 & 0.8801 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} 0.1199 \\ 0.4596 \\ 0.00254 \\ 0.01778 \end{pmatrix}, \quad C_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 0.7625 & 0.4596 & 0.1176 & 0.01778 \\ -0.9014 & 0.7625 & 0.4418 & 0.09981 \\ 0.09981 & 0.01778 & 0.8979 & 0.3776 \\ 0.342 & 0.09981 & -0.3598 & 0.5203 \end{pmatrix}$$

$$B_2 = \begin{pmatrix} 0.1199 \\ 0.4596 \\ 0.002305 \\ 0.01778 \end{pmatrix}, \quad C_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}$$

For the simulation, we applied a sinusoidal control. Figure (3) shows the evolution of the real mode, the Chiu's and K-means estimated of discrete state. We can notice that the chiu's estimated of discrete state gives a better estimate than the k-means estimated. The vectors of regression class's center obtained with two algorithms are given in table 1.

Table 1: The centers regression vectors

	Chiu	K-means
φ_1^*	$\begin{bmatrix} -1.1448 \\ -1.1448 \\ -0.7705 \end{bmatrix}$	$\begin{bmatrix} -0.8446 \\ -0.8449 \\ -0.7528 \end{bmatrix}$
φ_2^*	$\begin{bmatrix} 0.7283 \\ 0.7283 \\ 0.4818 \end{bmatrix}$	$\begin{bmatrix} 0.5497 \\ 0.5447 \\ 0.4998 \end{bmatrix}$

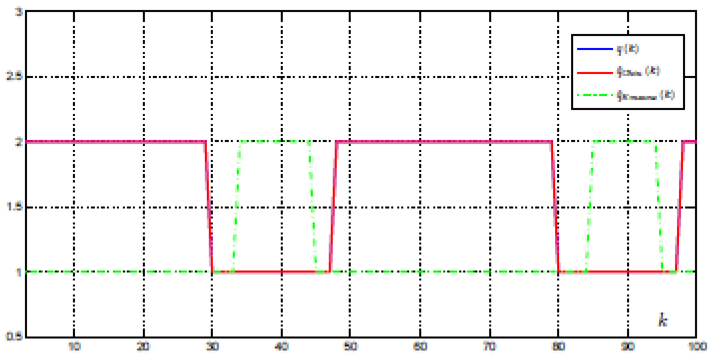

Fig. 3: The evolution of the real Mode, the Chiu estimated mode and the K-means estimated mode

The LMI (12) is feasible. The continuous observer's gains are given as follow :

$$L_1 = \begin{pmatrix} 1.0788 \\ -0.2894 \\ 0.0024 \\ -0.0772 \end{pmatrix}, \quad L_2 = \begin{pmatrix} 1.1797 \\ -0.3073 \\ 0.04999 \\ 0.0442 \end{pmatrix}$$

The estimation errors of the components of the state vector are illustrated in figure (5). We found the asymptotic convergence of the estimation errors for the components of the state vector. The trace of the state vector's variables and their estimated, shown in figure (4), prove their convergence.

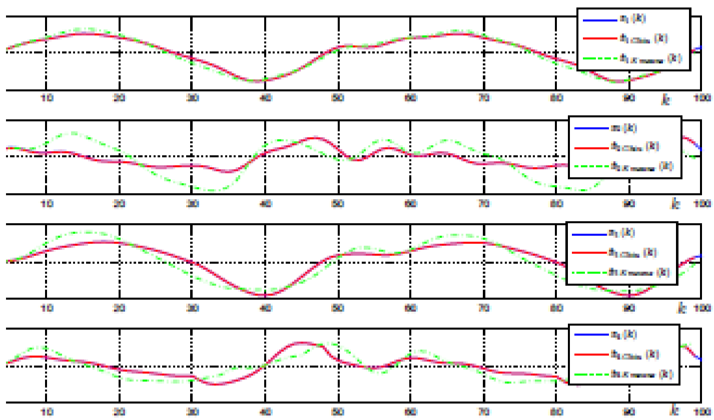

Fig. 4 The evolution of state vector's components and their Chiu and K-means estimated

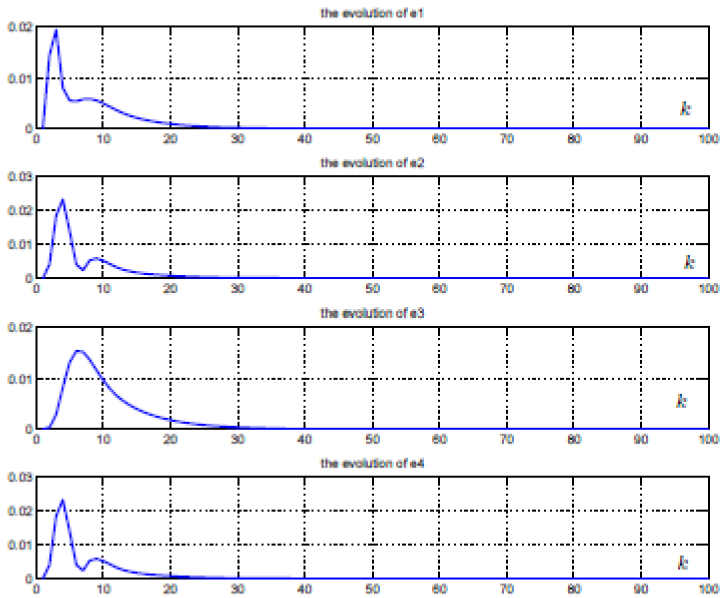

Fig. 5 The Chiu's algorithm estimation error of state vector

The Mean Square Error *MSE* and the Variance-Accounted-For *VAF* are calculated by the following formulas for evaluating the accuracy of the hybrid state estimation.

$$MSE = \frac{1}{N}\sum_{k=1}^{N}\left(y(k) - \hat{y}(k)\right)^2$$

$$VAF = \max\left\{1 - \frac{\operatorname{var}\left(y(k) - \hat{y}(k)\right)}{\operatorname{var}\left(\hat{y}(k)\right)}, 0\right\} \times 100\%$$

Table 2 gives the performance indexes computed by the Chiu's and k-means's methods. We can note that the chui classification method ensures best estimate of the system total state.

Table 2: Performance Index

	Chiu	K-means
MSE	9.760810^{-6}	0.0166
VAF	99.9987%	97.4658%

Case 2:

The parameters matrices are given in the sampling phase, for a sampling period $T_e = 1s$, by :

$$A_1 = \begin{pmatrix} 0.4311 & 0.5546 & 0.2238 & 0.3183 \\ -0.791 & 0.1947 & 0.2363 & 0.4602 \\ 0.1294 & 0.3183 & 0.7493 & 0.649 \\ 0.0125 & 0.4602 & -0.3308 & 0.4186 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} 0.3451 \\ 0.5546 \\ 0.1213 \\ 0.3183 \end{pmatrix}, C_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 0.418 & 0.5335 & 0.2417 & 0.2417 \\ -0.8253 & 0.1262 & 0.2918 & 0.2918 \\ 0.08957 & 0.2417 & 0.8118 & 0.4439 \\ -0.03943 & 0.2918 & -0.2023 & 0.1656 \end{pmatrix}$$

$$B_2 = \begin{pmatrix} 0.3403 \\ 0.5335 \\ 0.09861 \\ 0.2417 \end{pmatrix}, C_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}$$

For the same control input used, figure (6) shows the evolutions of the real and estimated switching signal. we note that the estimated discrete state by Chiu algorithm provides a better estimate than that given by the K-means algorithm. In table 3, we give the vectors of regression class's center given by chui and K-means algorithms.

Table 3: The centers regression vectors

	Chiu	K-means
φ_1^*	$\begin{bmatrix}-1.2640\\-1.0093\\-0.9823\end{bmatrix}$	$\begin{bmatrix}-0.8273\\-0.7963\\-0.7469\end{bmatrix}$
φ_2^*	$\begin{bmatrix}0.7110\\0.8724\\0.5878\end{bmatrix}$	$\begin{bmatrix}0.5610\\0.5392\\0.4938\end{bmatrix}$

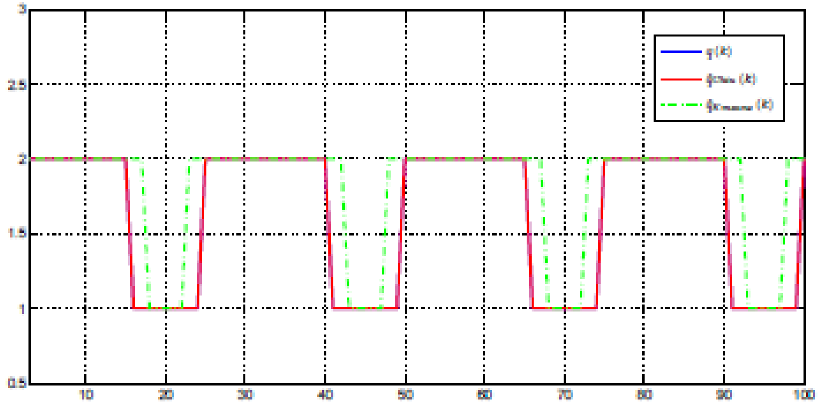

Fig. 6 The evolution of the real mode, the Chiu estimated mode and the Kmeans estimated mode

The LMI (12) is feasible. The continuous observer's gains are given as follow :

$$L_1 = \begin{pmatrix}-0.1821\\-0.0481\\0.7242\\0.3326\end{pmatrix}, \quad L_2 = \begin{pmatrix}-0.4173\\-0.4889\\0.2515\\-0.1463\end{pmatrix}$$

The estimation errors of the components of the state vector, are illustrated in figure (8). We found the asymptotic convergence of the estimation errors for the components of the state vector. The trace of the state vector's variables and their estimated, shown in figure (7), prove their convergence.

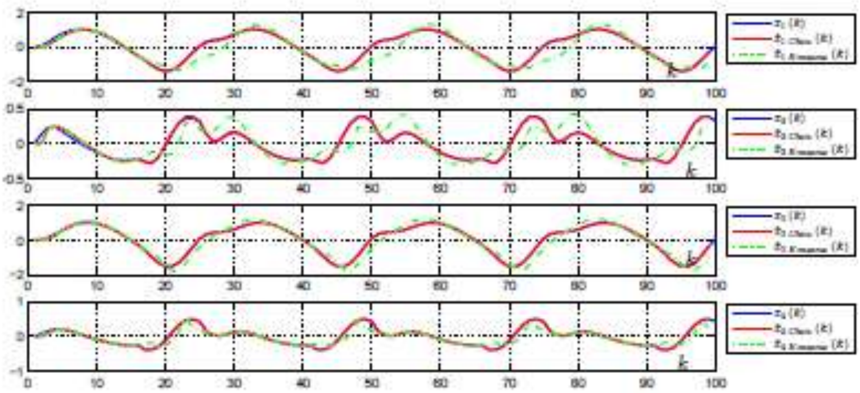

Fig. 7 The evolution of state vector's components and their Chiu and K-means estimated

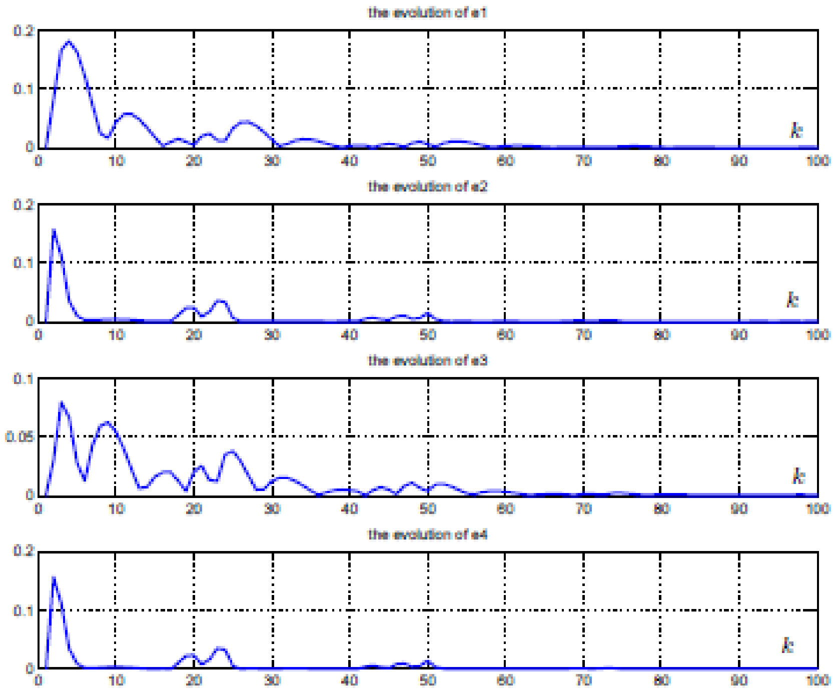

Fig. 8 The Chiu's algorithm estimation error of state vector

In table 4, the performance indexes are computed for Chui and k-means algorithms. Where it is clear to notice that, in this case too, the chui algorithm provides better estimation for hybrid state.

Table 4: Performance Index

	Chiu	K-means
MSE	0.0015	0.1419
VAF	99.87%	81.7529%

6. CONCLUSION

In this paper, the joint estimation problem of the discrete state and the continuous state for a class of switching discrete-time linear systems was developed. Using an approach based on the clustering algorithm to associate the current output to its appropriate submodel. Then, for estimating the continuous observer, we used the LMI approach. Eventually , an experimental validation with a mechanical system "double cart with elastic coupling" are presented. Simulation results showed that our proposed method has proved a better estimate of the hybrid state than that used with the K-means algorithm.

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