

# Steady State Modeling of Doubly Fed Induction Generator

Bhola Jha<sup>1</sup>, Dr. K. R. M Rao<sup>2</sup>

<sup>1</sup>Dept. of Electrical Engg., G. B. Pant Engg. College, Pauri-Garhwal, India

<sup>2</sup>Dept. of Electrical Engg., M. J. College of Engg. & Tech., Hyderabad, India

## Abstract

In Doubly Fed Induction Generator (DFIG), stator is connected to the grid and rotor is appropriately controlled to maintain constant voltage and frequency at the grid. DFIG finds wide applications in wind energy and the wind speed keeps on changing; therefore the operation of DFIG becomes very much interesting and challenging. The rotor voltage should change in accordance with wind speed to pump the power to the grid at constant voltage and frequency. This rotor voltage is provided by Voltage Source Converter (VSC) control. In this paper, the converters are not modeled in detail, though, instead, the VSC is represented by its output – the rotor voltage – and this is modeled as the output of the VSC control. This paper presents the simulation and experimental results of the rotor voltage to be injected for different rotor speed. The results are cross-checked and found in line with the expectations.

**Keywords:** DFIG, VSC, Phase shifting transformers, steady state modeling.

## I. INTRODUCTION

Harnessing the wind energy is one of the major applications of Doubly Fed Induction Generator (DFIG). The power available in the wind rotates the device known as wind turbine that provides the mechanical energy to the generator (DFIG) for generating electricity. Wind turbines can either operate at fixed speed or variable speed. For a fixed-speed wind turbine the generator is directly connected to the electrical grid. Cage induction generator is usually employed for fixed speed wind turbine. For a variable speed wind turbine the generator is connected to grid through power electronic converter. This means that the power electronic converter decouples the generator from the grid. Synchronous generator and doubly fed induction generator are employed for variable speed wind turbine. There are several advantages for using variable-speed operation of wind turbines [1-2]. Most of the wind turbine manufactures are developing larger DFIG-based variable speed wind turbines in the MW range [3-7] due to many advantages. The wound-rotor doubly-fed electric machine is the only electric machine that is able to operate with rated torque to twice synchronous speed for a given frequency of excitation (i.e., 7200 rpm @ 60 Hz and one pole-pair versus 3600 rpm for singly-fed electric machines). Higher speed for a given frequency of excitation is a quantifying metric that shows lower cost, higher efficiency, and higher power density [7].

Balancing/stabilizing the DFIG for variable input (wind speed) is one of the challenges for wind turbine manufacturers. Voltage and frequency may adversely affect the stability of system if the speed changes which usually happens in case of wind energy. Therefore, to run the system stably, rotor voltage should change whenever there is change in rotor speed due to change in wind speed. Therefore, in DFIG, rotor is to be appropriately controlled such that power can be pumped to grid at constant voltage and frequency under any circumstances for its stable operation. This paper presents the simulation and experimental results of magnitude of rotor voltages which are to be injected for different rotor speed. Actually, this rotor voltage is provided by Voltage Source Converter (VSC) control. But, in this paper, the converters are not modeled in detail, though, instead, the VSC is represented by its output – the rotor voltage – and this is modeled as the output of the VSC control.

## II. MODELING OF DOUBLY FED INDUCTION GENERATOR

The schematic diagram of DFIG is shown in Fig.1. Based on the above diagram of the DFIG,  $dq$ -axis equivalent circuit, mostly known by the  $d$ - $q$  model [8-10] can be drawn as shown in Fig.2 and Fig.3. The circuit is denoted by the standard parameters which are usually used.



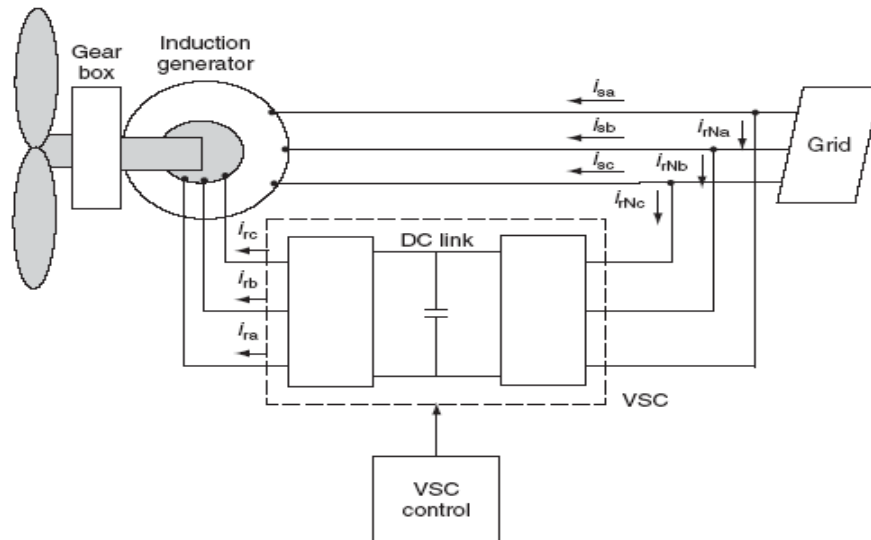


Fig.1: Schematic diagram of DFIG

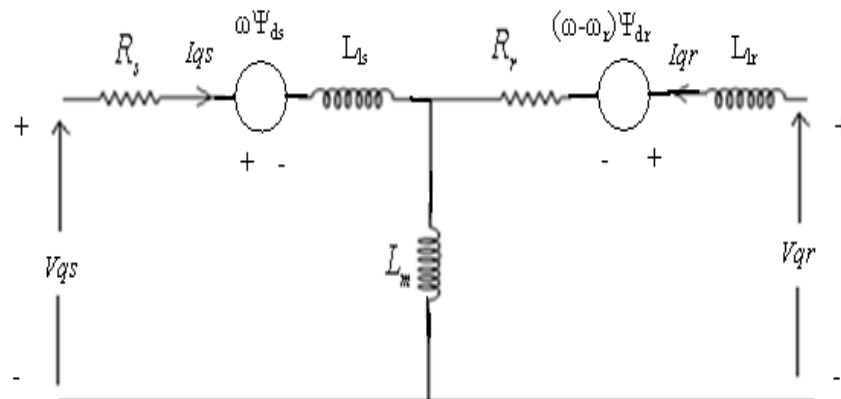


Fig.2: q-axis model of DFIG

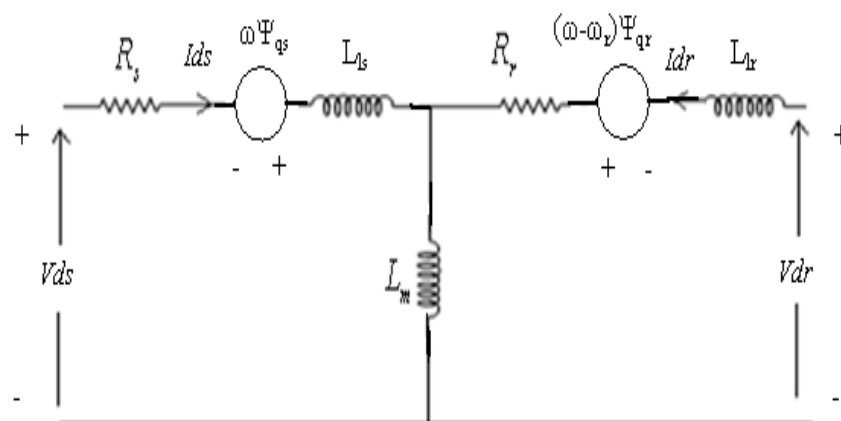


Fig.3: d-axis model of DFIG



Voltage equations are as follows from both circuits.

$$V_{qs}(t) = I_{qs}R_s + \left(\frac{\omega}{\omega_b}\right) \Psi_{ds} + \left\{\frac{d}{dt}(\Psi_{qs})\right\}/\omega_b$$

$$\text{Let } \frac{d}{dt} = p$$

$$V_{qs}(t) = I_{qs}R_s + \left(\frac{\omega}{\omega_b}\right) \Psi_{ds} + \{p(\Psi_{qs})\}/\omega_b$$

$$V_{ds}(t) = I_{ds}R_s - \left(\frac{\omega}{\omega_b}\right) \Psi_{qs} + \{p\Psi_{ds}\}/\omega_b$$

$$V_{qr}(t) = I_{qr}R_r + \left\{\frac{\omega - \omega_r}{\omega_b}\right\} \Psi_{dr} + \{p\Psi_{qr}\}/\omega_b$$

$$V_{dr}(t) = I_{dr}R_r - \left\{\frac{\omega - \omega_r}{\omega_b}\right\} \Psi_{qr} + \{p\Psi_{dr}\}/\omega_b$$

Where flux linkages are related as

$$\Psi_{qs} = X_{ls}I_{qs} + X_m(I_{qs} + I_{qr})$$

$$\Psi_{ds} = X_{ls}I_{ds} + X_m(I_{ds} + I_{dr})$$

$$\Psi_{qr} = X_{lr}I_{qr} + X_m(I_{qs} + I_{qr})$$

$$\Psi_{dr} = X_{lr}I_{dr} + X_m(I_{ds} + I_{dr})$$

Total stator and rotor voltages are

$$V_s = V_{ds} + jV_{qs}$$

$$V_r = V_{dr} + jV_{qr}$$

Using above equations

$$V_{ds} = I_{ds}R_s - \left(\frac{\omega}{\omega_b}\right) (X_{ls}I_{qs} + X_m(I_{qs} + I_{qr})) + \{p(X_{ls}I_{ds} + X_m(I_{ds} + I_{dr}))\}/\omega_b$$

$$V_{qs} = I_{qs}R_s - \left(\frac{\omega}{\omega_b}\right) (X_{ls}I_{ds} + X_m(I_{ds} + I_{dr})) + \{p(X_{ls}I_{qs} + X_m(I_{qs} + I_{qr}))\}/\omega_b$$

Using the following identity

$$(p + j\omega)(I_{ds} + jI_{qs}) = (pI_{ds} - \omega I_{qs}) + j(pI_{qs} + \omega I_{ds})$$

The above equations can be written as

$$V_s = [R_s + \left(\frac{X_1}{\omega_b}\right)(p + j\omega)]I_s + \left[\left(\frac{X_m}{\omega_b}\right)(p + j\omega)\right]I_r$$

$$\text{Where } I_s = (I_{ds} + jI_{qs}), I_r = (I_{dr} + jI_{qr}) \text{ and } X_1 = (X_{ls} + X_m)$$

Similarly,

$$V_{dr} = R_r I_{dr} - \left(\frac{\omega - \omega_r}{\omega_b}\right) [X_{lr} I_{qr} + X_m(I_{qs} + I_{qr})] + \frac{p}{\omega_b} [X_{lr} I_{dr} + X_m(I_{ds} + I_{dr})]$$

$$V_{qr} = R_r I_{qr} + \left(\frac{\omega - \omega_r}{\omega_b}\right) [X_{lr} I_{dr} + X_m(I_{ds} + I_{dr})] + \frac{p}{\omega_b} [X_{lr} I_{qr} + X_m(I_{qs} + I_{qr})]$$

In the same fashion as discussed above, rotor voltage can be written as

$$V_r = [R_r + \left(\frac{X_2}{\omega_b}\right)(p + j(\omega - \omega_r))]I_r + \frac{X_m}{\omega_b} [p + j(\omega - \omega_r)]I_s$$

$$\text{Where } X_2 = (X_{lr} + X_m)$$

Now as we have got  $V_s$  and  $V_r$  in terms of  $p$ , so to get the whole equations at zero state, the ' $p$ ' or ' $\left(\frac{d}{dt}\right)$ ' term is neglected and the new equations obtained are given below:



$$V_{s,o} = [R_s + \left(\frac{\omega}{\omega_b}\right)jX_1]I_s + \left[\left(\frac{\omega}{\omega_b}\right)jX_m\right]I_r$$

$$V_{r,o} = [R_r + \left(\frac{\omega - \omega_r}{\omega_b}\right)jX_2]I_r + \left[\left(\frac{\omega - \omega_r}{\omega_b}\right)jX_m\right]I_s$$

The above obtained equations in the matrix form as

$$\begin{bmatrix} V_{s,o} \\ V_{r,o} \end{bmatrix} = \begin{bmatrix} [R_s + \left(\frac{\omega}{\omega_b}\right)jX_1] & [\left(\frac{\omega}{\omega_b}\right)jX_m] \\ [\left(\frac{\omega - \omega_r}{\omega_b}\right)jX_m] & [R_r + \left(\frac{\omega - \omega_b}{\omega_b}\right)jX_2] \end{bmatrix} \begin{bmatrix} I_s \\ I_r \end{bmatrix}$$

Now, *abc to dq* and vice versa transformations is applied in order to achieve the desired result.

$$V_{qd0,s} = K_s V_{abc,s}$$

$$K_s = \left(\frac{2}{3}\right) \begin{bmatrix} \cos \theta & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\ \sin \theta & \sin(\theta - 2\pi/3) & \sin(\theta + 2\pi/3) \\ 1/2 & 1/2 & 1/2 \end{bmatrix}$$

A very important point to be noted here is, when  $\omega$  is constant there exist a direct relation between  $\omega$  and  $\theta$  as

$$\theta = \omega \int dt$$

But, if  $\omega$  varies with time, then another relation exists, as

$$\theta(t) = \omega t + \theta(0)$$

But in this case  $\omega$  is constant so the direct relation i.e.  $\theta = \omega t$  is used.

$$V_{abc,s} = V_{\max} \{ \cos \omega t - (i - 1) 2\pi/3 \}$$

Using the relation  $V_{\max} = \sqrt{2}V_{rms}$ , here  $\sqrt{2}V_1$  we have the following relation,

$$V_{abc,s} = \sqrt{2}V_1 \{ \cos \omega t - (i - 1) 2\pi/3 \}$$

On substituting the values of  $i = 1, 2, 3$  respectively we have,

$$V_{as} = \sqrt{2}V_1 \cos \omega t$$

$$V_{bs} = \sqrt{2}V_1 \cos(\omega t - 2\pi/3)$$

$$V_{cs} = \sqrt{2}V_1 \cos(\omega t + 2\pi/3)$$

$$\text{Now, } \begin{bmatrix} V_{qs} \\ V_{ds} \\ V_{os} \end{bmatrix} = \left(\frac{2}{3}\right) \begin{bmatrix} \cos \theta & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\ \sin \theta & \sin(\theta - 2\pi/3) & \sin(\theta + 2\pi/3) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} \sqrt{2}V_1 \cos \omega t \\ \sqrt{2}V_1 \cos(\omega t - 2\pi/3) \\ \sqrt{2}V_1 \cos(\omega t + 2\pi/3) \end{bmatrix}$$

Using the identity,

$$[\cos 2\omega t + \cos 2(\omega t - 2\pi/3) + \cos 2(\omega t + 2\pi/3)] = 3/2$$

And by solving the above matrix equations we get,

$$V_{q0,s} = \left(\frac{2}{3}\right) \sqrt{2}V_1 \left[\frac{3}{2}\right] = \sqrt{2}V_1$$

Solving in similar way, we have

$$V_{d0,s} = 0$$



Now since

$$V_{s0} = V_{d0,s} + jV_{q0,s}$$

$$\text{So, } V_{s0} = j\sqrt{2}V_1$$

On the similar way we have to apply the following changes to the rotor side also,

$$V_{qd0,r} = K_r V_{abc,r}$$

$$K_r = \left( \frac{2}{3} \right) \begin{bmatrix} \cos \beta & \cos(\beta - 2\pi/3) & \cos(\beta + 2\pi/3) \\ \sin \beta & \sin(\beta - 2\pi/3) & \sin(\beta + 2\pi/3) \\ 1/2 & 1/2 & 1/2 \end{bmatrix}$$

Here,  $\beta = (\theta - \theta_r)$  where  $\theta_r$  is the rotor angle,  $\omega_r = \frac{d}{dt}(\theta_r)$

Again

$$V_{abc,r} = \sqrt{2}V_2 \{ \cos(\omega t - \delta_0 - (i-1)2\pi/3) \}$$

On substituting the values of  $i=1, 2, 3$  respectively we have,

$$V_{ar} = \sqrt{2}V_2 \cos(\omega t - \delta_0)$$

$$V_{br} = \sqrt{2}V_2 \cos(\omega t - \delta_0 - 2\pi/3)$$

$$V_{cr} = \sqrt{2}V_2 \cos(\omega t - \delta_0 + 2\pi/3)$$

On substituting, we have

$$V_{q0,r} = \sqrt{2}V_2 \cos(\delta_0)$$

And

$$V_{d0,r} = \sqrt{2}V_2 \sin(\delta_0)$$

$$\text{Since } V_{r0} = V_{d0,r} + jV_{q0,r}$$

On substituting, we have

$$V_{r0} = \sqrt{2}V_2 \sin(\delta_0) + j(\sqrt{2}V_2 \cos(\delta_0))$$

$$V_{r0} = \sqrt{2}V_2 [\cos(\frac{\pi}{2} - \delta_0) + j\sin(\frac{\pi}{2} - \delta_0)]$$

Now,

$$\begin{bmatrix} j\sqrt{2}V_1 \\ [\sqrt{2}V_2 [\cos(\frac{\pi}{2} - \delta_0) + j\sin(\frac{\pi}{2} - \delta_0)]] \end{bmatrix} = \begin{bmatrix} [R_s + (\frac{\omega}{\omega_b})jX_1] & [(\frac{\omega}{\omega_b})jX_m] \\ [(\frac{\omega - \omega_r}{\omega_b})jX_m] & [R_r + (\frac{\omega - \omega_b}{\omega_b})jX_2] \end{bmatrix} \begin{bmatrix} I_s \\ I_r \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_s \\ I_r \end{bmatrix} \quad (A)$$

We know the stator current and its phase angle from the given power, stator voltage and power factor. After knowing stator current, rotor current can be calculated from the first equation of above matrix. Once the stator current and rotor current are known, rotor voltage can be determined for given rotor speed from the second equation of above matrix.

### III. RESULTS ANALYSIS

The main objective of this paper is to find the rotor voltage to be injected for constant load of different power factor under varying wind speeds. In other words, the injection of rotor voltage should change as the wind speed changes in order to pump the power to grid at constant frequency. The simulation results of injection of rotor r. m. s voltage under varying rotor speed are shown in Fig.4 and Fig.5 for the output of 3kW and 5kW respectively. The experimental waveforms of rotor voltage at different speeds are shown in Fig.6, Fig.7, Fig.8 and Fig.9. The plot of experimental r. m. s value of rotor voltage versus speed is shown in Fig.10. The rotor voltage is found minimum (ideally zero) at the synchronous speed (750 r. p. m) and increases in both direction i. e. sub synchronous and super synchronous speed. This is also found that rotor voltage is same for  $(750 \pm \Delta)$  r. p. m, here,  $\Delta$  are taken as 50 r. p. m, 100 r. p. m, 150 r. p. m and 200 r. p. m.



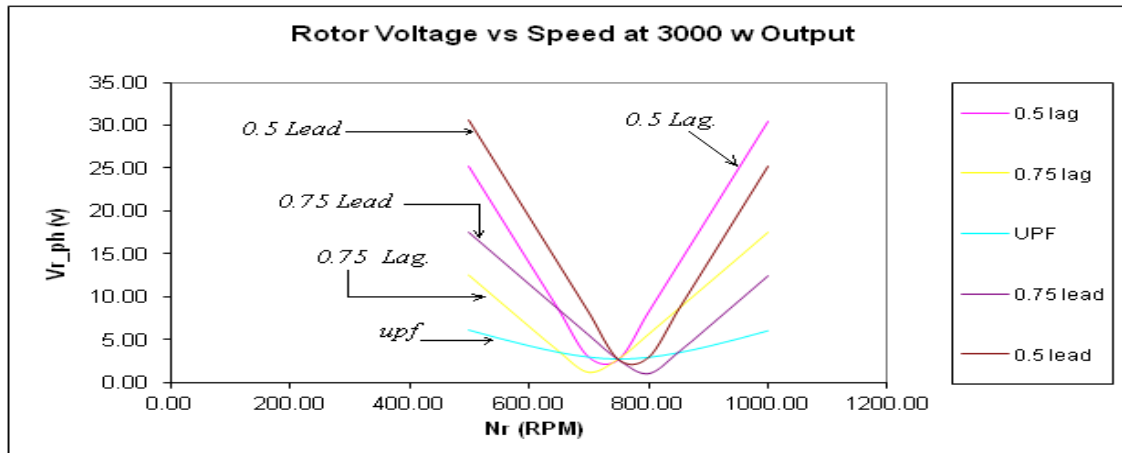


Fig.4: Simulation results of rotor voltage versus speed for 3kW

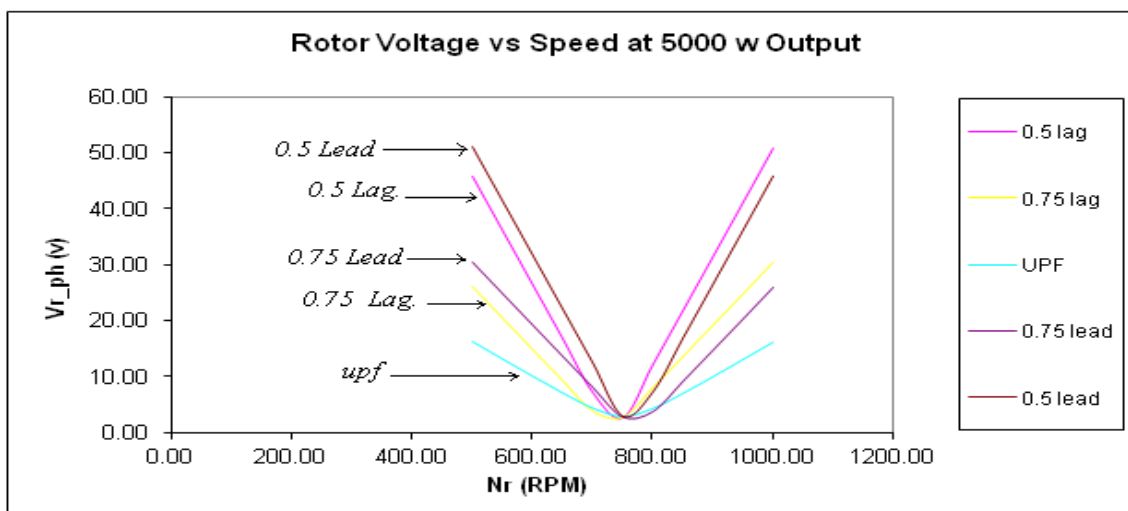


Fig.4: Simulation results of rotor voltage versus speed for 5kW

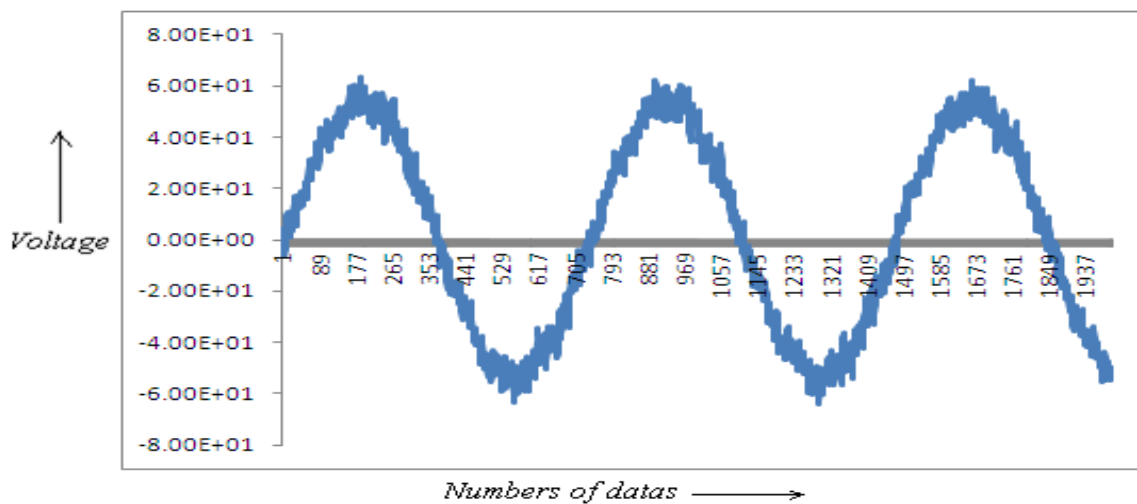


Fig.6: Experimental waveform of rotor voltage (RMS = 38.1135V) for 950 rpm



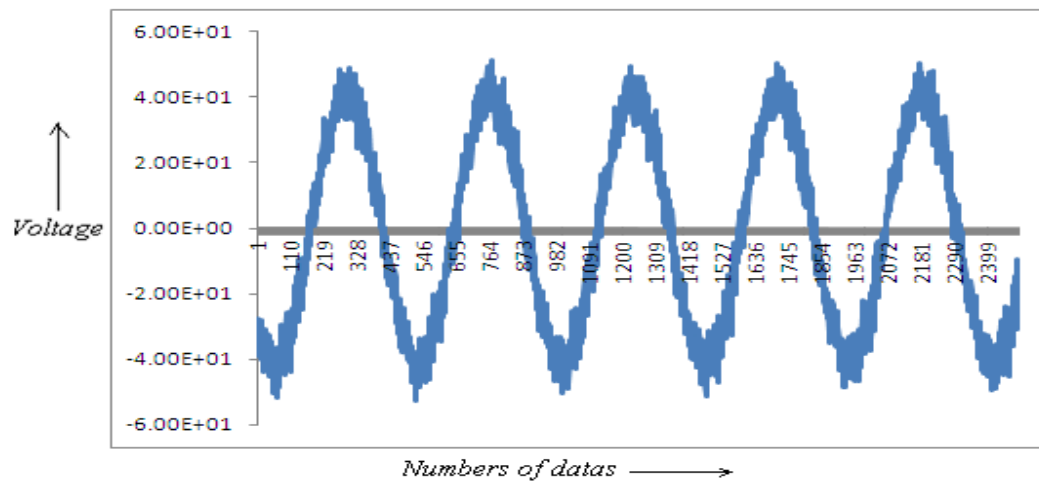


Fig.7: Experimental waveform of rotor voltage (RMS = 29.048V) for 900 rpm

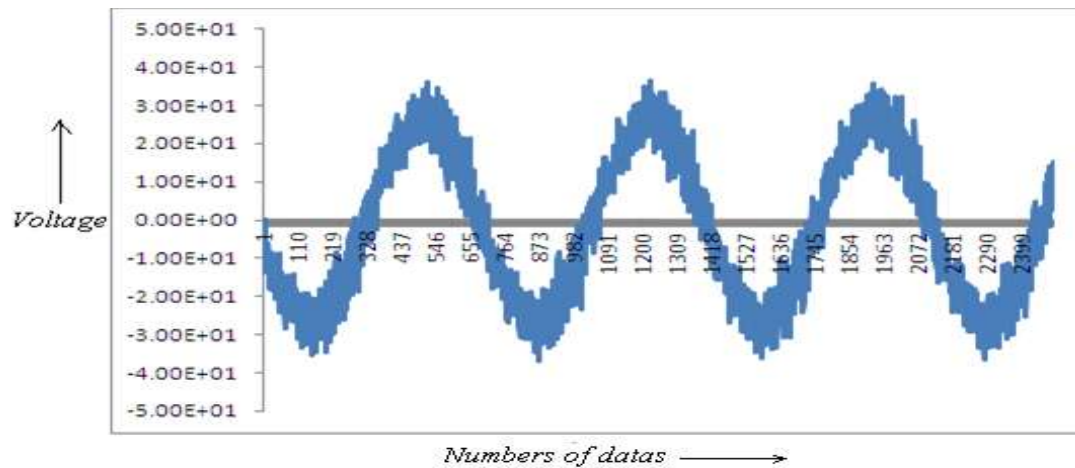


Fig.8: Experimental waveform of rotor voltage (RMS = 19.049V) for 850 rpm.

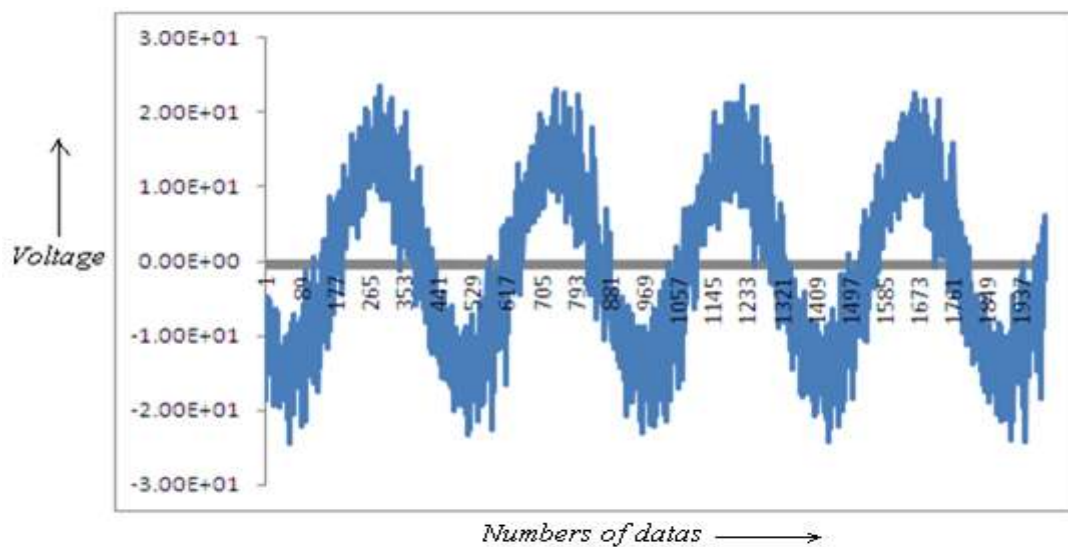


Fig.9: Experimental waveform of rotor voltage (RMS = 11.13V) for 805 rpm.



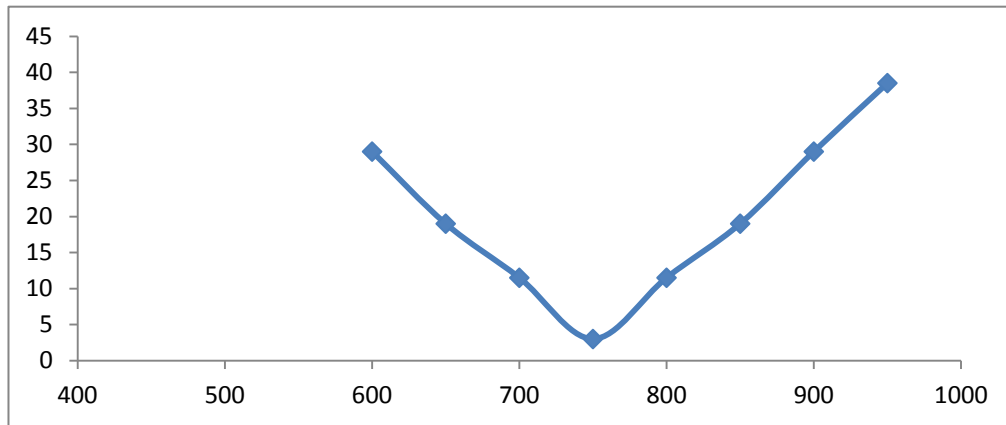


Fig.10: Experimental results of rotor voltage versus speed

The r. m. s value of rotor voltages can be seen on the waveforms which are 38.37V, 29.04V, 19.04V and 11.13V for 950 r. p. m, 900 r. p. m, 850 r. p. m and 800 r. p. m respectively. This is found that magnitude of injected rotor voltage is minimum for unity power factor load in both, sub synchronous speed and super synchronous operation. In sub synchronous mode, magnitude of injected rotor voltage is more for leading power factor load as compared to lagging power factor load. But, in super synchronous mode, magnitude of injected rotor voltage is more for lagging power factor load as compared to leading power factor load. For given speed, e. m. f injection increases if power factor decreases. This is observed that the injected rotor voltage is more for more power output.

#### IV. EXPERIMENTAL DESCRIPTION

The block diagram of experimental set up is shown in Fig.11. This is clear from this Fig.11 that there are three powers ports into the rotor of DFIG. One is prime mover power (DC motor), second is power transferred to the rotor from stator via induction and third is the power fed from excitation induction motor. Variable speed and rotor voltage injection to the DFIG are provided through DC motor and excitation induction motor respectively. Machine parameters are mentioned in Appendix 1 and Appendix 2. This is to be noted that the specification of DFIG and excitation induction motor are taken same.

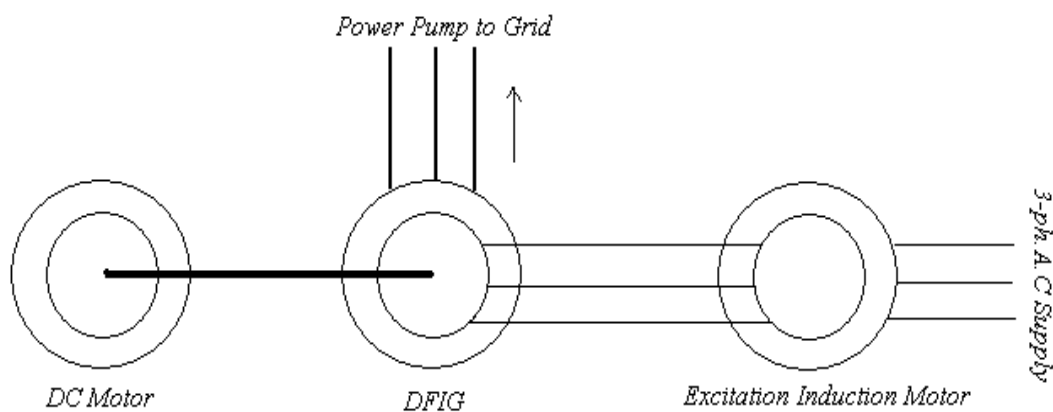


Fig.11: Block diagram of experimental set up

#### V. CONCLUSION

Mathematical modeling for the steady state analysis of the doubly fed induction generator is developed. The converters are not modeled in detail, though. Instead, the voltage source converter (VSC) is represented by its output – the rotor voltage – and this is modeled as the output of the VSC control. Rotor voltage is found minimum at the synchronous speed and it increases if the speed is away from synchronous speed. In sub synchronous mode, magnitude of injected rotor voltage is more for leading power factor load as compared to lagging power factor load and vice-versa for super synchronous speed. This is also concluded that the e. m. f injection increases as the power factor decreases for a given speed. Also, e. m. f injection is more for more output power. Simulation results are cross-checked with the experimental results.

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## APPENDICES

Appendix 1: Specification of DFIG

S. N.	Parameters	Values
1	Power	7.5 kW
2	Stator voltage	415 V
3	Stator current	19.5 A
4	Rotor voltage	234 V
5	Rotor current	20.5 A
6	Stator resistance	1.5 $\Omega$
7	Stator leakage reactance	4.05 $\Omega$
8	Magnetizing reactance	39.06 $\Omega$
9	Rotor leakage reactance referred to stator	4.05 $\Omega$
10	Rotor resistance referred to stator	0.35 $\Omega$
11	Speed (in rpm) & frequency (in Hz)	725 & 50
12	Number of poles & phase	8 & 3

Appendix 2: Specification of DC Motor

S. N	Parameters	Values
1	Power	75 HP
2	Armature voltage	440 V
3	Armature current	140 A
4	Field voltage	220 V
5	Field current	3.5 A

