International Journal of Enhanced Research in Educational Development (IJERED)
ISSN: 2320-8708, Vol. 11 Issue 6, Nov.-Dec, 2023, Impact Factor: 7.326

Population Dynamics of Aphids in Limited Ecological Environment

Dr. Amit Kumar Srivastava[1], Dr. Vindyachal Lal[2]

[1]Former Assistant Professor of Mathematics, D.N.M. Engineering College, Lucknow, U.P. (India)
[2]Associate Professor and Head (Retd.), Department of Mathematics, D.A.V. Post Graduate College, Azamgarh, U.P. (India)

ABSTRACT

Aphids are among the most conspicuous and important insect pests in the green houses and the crops. They excrete sugary fluid called saliva on the leaf which prevents further sucking and movement of the individual, and so causing death due to starvation. In this paper, the population dynamics of aphids is discussed in limited ecological environment considering that the rapid increase in mortality rate of aphids is proportional to the area

$$\int_{o}^{t} n(s)ds$$

covered by saliva excreted by aphids where n(t) is the number of indviduals in the population at some timet.

Keywords: ***Population, aphid, starvation, mortality.***

INTRODUCTION

Aphids (hemiptera : aphidiade) are small, soft-bodied and sap sucking insects that are most destructive insect pests affecting various field crops, fruits and vegitables[1]. Aphids suck the cell saps from leaves and may stunt the plant growth[2] and they play a significant role in transmitting many fungal and viral diseases in plants[3, 4, 5]. Aphids colonies are characterised by rapid increases and declines in abundance[6]. Aphids are easily driven to death when weather is not according to their survival.

Several investigations have been made to study the population dynamics of aphids by using mathematical models[7, 8, 9]. In an experiment, it was found that in the species aphisfabae of the family aphidiade, the total number of individuals in the population rapidly increased in the begining, reaching its maximum on the 5^{th} day and rapidly decreased afterwards as a consequence of high mortality of aphids[10]. The simplest model for population growth of a plant or animal species was given by[11] as follows

$$n' = an , t > 0$$

Where dash denotes differentiation with respect to t and where n(t) is the number of individuals in the population at some time t, and a > 0 is the specific growth rate of the population, that is, increase of birth rate over death rate. On solving it we get a famous population growth equation (called Malthusian exponential growth equation)

$$n (t) = n(0) \exp (at)$$

Where n(0) is the number of individuals at initial stage. This equation implies that population n approaches to infinity when $t \rightarrow \infty$. The space being finite, the population can not be infinite. There is an upper limit of population due to various factors such as limitation of food resources, concentration of rate-limiting substrates in case of bacterial colonies, crowding (population density increase) etc. Considering the limiting factors, the growth rate model was modified by[12] as follows

$$n' = a n - b n^2 , t > 0$$

Where b > 0 reflects the degree to which the growth rate a is reduced due to rate limiting factors.

This equation gives an upper limit of the population. The population n asymptotically approaches to a limiting value a/b (known as carrying capacity) for a > 0 as $t \rightarrow \infty$. Srivastava, Shrivastava and Lal[13] studied a mathematical model for growth of bacterial population under toxic effects assuming that toxic substance produced by bacteria become limiting factor to their further growth by increasing rapidly mortality rate of organism. In the present paper, we study a mathematical model of population growth of aphids which excrete sugary fluid called saliva. The such saliva smeared over the surface of the leaf preventing further sucking and movement of the individual and so causing deaths due to starvation. We have attributed rapid increase of mortality rate to the deterioration of the environment by excretion of saliva by aphids.

Formulation of the Model and its Solution

Let n(t) denote the total number of individuals at time t.

Considering that coverage area by saliva excreted by aphids on the leaf at time t is proportional to the integral $\int_o^t n(t)\,dt$ following[9] and assuming the linear dependence between this integral density and mortality rate, we have

$$n' = a\,n - b\,n^2 - k\,n \int_o^t n(t)\,dt \qquad \text{\_\_\_\_\_ (1)}$$

where k is positive constant and where dash denotes differentiation with respect to t.

This integro-differential equation can be solved for n(t) by specialized technique as follows

Let us put $y(t) = \int_o^t n(t)\,dt$

Then from (1) we get a second order differential as

$$y'' = a\,y' - b\,(y')^2 - k\,y\,y'$$

Multiplying both sides by exp (by) and integrating it between o and t, we get

$$\exp(by)\,y' - n(0) = \exp(by)\,[\,a/b + k/b^2 - (k/b)\,y\,] - [\,a/b + k/b^2\,]$$

since y(o) = o and y'(o) = n(0)

It gives

$$n(t) = y' = n(0)\exp(-by) + [\,a/b + k/b^2 - (k/b)\,y\,] - [\,a/b + k/b^2\,]\exp(-by)$$

$$= [\,a/b + k/b^2 - (k/b)\,y\,] - [\,a/b + k/b^2 - n(0)\,]\exp(-by) \qquad \text{\_\_\_\_(2)}$$

$$= f(y) \qquad \text{say}$$

Hence

$$t = \int_o^y dy/f(y)$$

Equation (2) gives population at time t.

RESULTS AND DISCUSSION

We now analyze the behaviour of curve n (t) obtaining n^1 and finding maximum value of n (t).

From equation (2), We have

$$n(t) = f(y)$$

On

$$n' = f'(y)\,n$$

$$= \{\,[n(0) - a/b\,]\,[\,-b\exp(-by)\,] - k/b\,[\,1 - \exp(-by)\,]\,\} \qquad \text{\_\_\_ (3)}$$

Hence it follows that

(1) If $n(0) \geq a/b$ then n' is always negative and population steadly decreases to zero.

(2) If n (0) < a/b then

$$\left(\frac{dn}{dt}\right)_{t=o} = \left(\frac{dn}{dt}\right)_{y=o} = -b\,[\,n(0) - a/b\,] > o$$

So that population initially increases and reaches a maximum when dn/dt = o

which gives from (3)

$$y_{max} = 1/b \ \log\left\{\frac{k + ab - n(0)\ b^2}{k}\right\}$$

Putting this value in (2) we get maximum population

$$n_{max} = a/b - k/b^2 \ \log\left\{\frac{k + ab - n(0)\ b^2}{k}\right\} < a/b$$

After this the population decreases and tends to zero. Thus we conclude that in both the cases either $n(0) \geq a/b$ or $n(0) < a/b$, the population tends to extinction.

CONCLUSION

In this paper, we have studied a mathematical model of aphids population considering the linear dependence between the area coverage by excretion of saliva by aphids and the mortality rate. It was concluded that in each case either $n(0) \geq a/b$ or $n(0) < a/b$, the population of aphids tends to extinction where n(0) is the initial population and a/b is the carrying capacity.

REFERENCES

[1]. Aheer, G.M.; Ali A. and Munir, M. : Abiotic factors effect on population fluctuation of a late aphid in wheat, Jour. Agric. Res., 46(4), 367-371, 2008.

[2]. Kindler, S.D.; Springer, T.L. and Jensen, K.B.L. : Detection and characterization of the mechanisms of resistance to Russian wheat aphid (Homoptora; aphidiade) in tall wheat grass. J., econ. Ent., 88, 1503-1509, 1995.

[3]. Bukvayova, N.; Henselova, M.; Vajcikova, V. and Kormanova, T. : Occurence of dwarf virus of winter wheat and barley in several regions of Slovakia during the growing seasons 2001-2004. Pl. Soil. Environ., 52; 392-401, 2006.

[4]. Smith, K.M. Plant virus, Springer, Netherland, 1977.

[5]. Slama, K. and Jedlicka, P. : Respiratory metabolim of the pea aphid, Acyrthosiphon pisum (Hemiptera : Aphidiadae), Eur. J., Entomol., 109, 491-502, 2012.

[6]. Dixon AFG; Kindlmann, P. Population dynamics of aphids.In : Dempster PJ, IFG Mc Lean (eds) Insect populations, Kluwer, Dordrecht, 207-230, 1998.

[7]. Barlow, N.D., Modelling aphid population, New Zeland J. Ecol. 4, 52-55, 1981.

[8]. Barlow, N.D. and Dixon A.F.G., Simulation of time aphid population dynamics, centre for Agricultural Publishing and Documentatition, Wege ningen, 1980.

[9]. Sharma, Rajan Kumar and Lal, Vmdhyachal : A mathematical model for population growth of hemiptera under starvation, Ultra Science (International Journal of Physical Science) Vol. 19(3)M, 489-492, 2007. ISSN 0970-9150.

[10]. Okrouhla, M., Portravine ekologieka studie slunecka cheilomenes sulphurea, Ph.D. Thesis, Charles University, Prague, 1983.

[11]. Malthus, T.R., An essay on the principle of population, Johnston, London, 1798.

[12]. Pearl, R., The Biology of Population of Growth, Knopt, New York, 1925.

[13]. Srivastava, Amit Kumar; Shrivastava Rajesh and Lal, V.C. : A mathematical model for Growth of Bacterial Population in which mortality is density and toxicity dependent, Jour. PAS, Vol. 13 (Ser. A), 442-446, 2007, ISSN 0972-3498.