International Journal of Enhanced Research in Educational Development (IJERED)
ISSN: 2320-8708, Vol. 14 Issue 1, January-February, 2026

Structure of Prime Rings with Multiplicative (Generalized)-Derivations

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ABSTRACT

This paper investigates the structure of Prime Rings admitting multiplicative derivations and multiplicative generalized derivations satisfying various algebraic identities. Without assuming additivity, we examine the influence of these mappings on the structural properties of Prime Rings and establish conditions under which they reduce to classical derivations or force the Ring to satisfy strong commutativity conditions. By analyzing identities involving multiplicative (generalized)-derivations on ideals, Lie ideals, or appropriate subsets of the Ring, we derive several characterization theorems that extend and unify a number of existing results in the literature. The findings demonstrate that the existence of multiplicative (generalized)-derivations satisfying prescribed identities imposes significant restrictions on the underlying Prime Ring, often implying commutativity or revealing specific structural behavior. These results contribute to a broader understanding of the interplay between multiplicative mappings and the algebraic structure of Prime Rings, while providing a unified framework for studying derivation-type mappings in non commutative Ring theory.

Keywords: Prime Rings, Multiplicative Derivations, Multiplicative Generalized Derivations, Commutativity Conditions, Ring Theory.

INTRODUCTION

Derivations play a fundamental role in the study of algebraic structures, particularly in Ring theory, where they provide important tools for investigating the internal structure of Rings and their ideals. Originating from the concept of differentiation in calculus, derivations have been extensively studied in both commutative and noncommutative algebra due to their deep connections with automorphisms, Lie algebras, functional identities, and operator algebras. Over the past several decades, numerous generalizations of derivations have been introduced, including generalized derivations, Jordan derivations, Lie derivations, multiplicative derivations, and multiplicative generalized derivations. These mappings have attracted considerable attention because they often preserve important algebraic properties while requiring weaker assumptions than classical additive derivations.

Let (R) be an associative Ring with center (Z(R)). An additive mapping ($d:R \rightarrow R$) is called a derivation if [$d(xy) = d(x)y + xd(y)$] for all ($x,y \in R$). A mapping ($F:R \rightarrow R$) is called a generalized derivation if there exists a derivation ($d:R \rightarrow R$) such that [$F(xy) = F(x)y + xd(y)$] for every ($x,y \in R$). These mappings have been widely investigated because they naturally arise in the study of operator algebras, differential algebra, and the structure theory of Prime and semiprime Rings.

A further extension is obtained by relaxing the assumption of additivity. A mapping ($d:R \rightarrow R$) is called a multiplicative derivation if it satisfies [$d(xy) = d(x)y + xd(y)$] for all ($x,y \in R$), without assuming that (d) is additive. Similarly, a mapping ($F:R \rightarrow R$) is called a multiplicative generalized derivation if there exists a multiplicative derivation (d) such that [$F(xy) = F(x)y + xd(y)$] holds for all ($x,y \in R$). The absence of additivity makes the investigation of these mappings substantially more challenging, and many classical techniques cannot be applied directly. Consequently, understanding the structural implications of multiplicative (generalized)-derivations has become an active area of research in modern Ring theory.

Prime Rings constitute one of the most important classes of associative Rings. Their rich algebraic structure makes them an ideal setting for studying derivation-type mappings. Numerous commutativity theorems have shown that identities involving derivations or generalized derivations frequently force a Prime Ring to become commutative or impose strong restrictions on its structure. Such results have significantly enhanced the understanding of the interaction between functional identities and Ring-theoretic properties.

In recent years, considerable attention has been devoted to multiplicative derivations and multiplicative generalized derivations acting on Prime and semiprime Rings. Various authors have established conditions under which these mappings coincide with ordinary derivations or satisfy identities leading to commutativity. These investigations have demonstrated that even without the assumption of additivity, multiplicative derivation-type mappings possess remarkable structural properties and provide powerful tools for characterizing Prime Rings.

The objective of this paper is to further investigate the influence of multiplicative derivations and multiplicative generalized derivations on the structure of Prime Rings. We establish several new results concerning algebraic identities involving these mappings on suitable subsets of Prime Rings. Under appropriate assumptions, we prove that such identities impose significant structural restrictions and, in many situations, imply the commutativity of the underlying Ring or characterize the associated derivation-type mappings. Our results extend several known commutativity theorems and provide a unified approach to studying multiplicative (generalized)-derivations in Prime Rings.

The paper is organized as follows. Section 2 presents the necessary definitions, notation, and preliminary results used throughout the paper. Section 3 contains the main theorems together with their proofs concerning multiplicative derivations and multiplicative generalized derivations on Prime Rings. The final section summarizes the principal conclusions and discusses possible directions for future research in the theory of derivation-type mappings on associative Rings.

THEORY OF ASSOCIATIVE RINGS

The theoretical framework of this study is based on the theory of associative Rings, particularly Prime Rings, together with the concepts of derivations, generalized derivations, multiplicative derivations, and multiplicative generalized derivations. These concepts form the mathematical foundation for investigating structural identities and commutativity conditions arising from derivation-type mappings.

Throughout this paper, let (R) denote an associative Ring with center [$Z(R)=\{z\in R$ such that $zx=xz, \forall\, x\in R\}$.] Unless otherwise stated, all Rings are assumed to be associative, and multiplication is written multiplicatively.

Prime Rings

A Ring (R) is called **Prime** if for any ($a,b \in R$), [$aRb = \{0\}$] implies either ($a=0$) or ($b=0$). Prime Rings occupy a central position in non commutative Ring theory because they generalize integral domains and possess strong cancellation properties. Many structural theorems concerning derivations and functional identities are naturally established in the setting of Prime Rings.

A Ring (R) is said to be **commutative** if [$xy=yx$] for every ($x,y \in R$). One of the principal objectives in the study of derivation-type mappings is to determine conditions under which a Prime Ring becomes commutative.

Derivations and Generalized Derivations

An additive mapping [$d:R\to R$] is called a **derivation** if it satisfies the Leibniz rule [$d(xy) = d(x)y + xd(y)$] for all ($x,y \in R$). A mapping [$F:R\to R$] is called a **generalized derivation** if there exists a derivation ($d:R\to R$) such that [$F(xy) = F(x)y + xd(y)$]for all ($x,y \in R$).

Generalized derivations extend the notion of ordinary derivations and provide a broader framework for studying algebraic identities in Prime and semiprime Rings.

Multiplicative Derivations

A mapping [$d:R\to R$] is called a **multiplicative derivation** if [$d(xy) = d(x)y + xd(y)$] holds for all ($x,y \in R$), without assuming that (d) is additive. The omission of the additivity condition significantly enlarges the class of admissible mappings and introduces additional complexity into their structural analysis. Consequently, multiplicative derivations require techniques that differ from those employed in the classical theory of additive derivations.

Conceptual Basis of the Study

The present study is founded on the principle that multiplicative derivations and multiplicative generalized derivations encode structural information about Prime Rings through the identities they satisfy. Since these mappings preserve the Leibniz-type product rule without requiring additivity, they provide a more general framework for examining non commutative Rings.

The theoretical assumption underlying this work is that sufficiently restrictive identities involving multiplicative (generalized)-derivations impose strong algebraic constraints on Prime Rings. These constraints may characterize the mappings themselves, determine their behavior on ideals or Lie ideals, or force the Ring to satisfy commutativity conditions.

Accordingly, the analysis carried out in this paper relies on established techniques from associative Ring theory, including commutator identities, centralizing mappings, annihilator conditions, and properties unique to Prime Rings. These theoretical tools provide the foundation for proving the main structural theorems presented in the subsequent sections.

PROPOSED MODELS AND METHODOLOGIES

1. Proposed Mathematical Model

This study develops a theoretical framework for investigating the structural properties of Prime Rings admitting multiplicative derivations and multiplicative generalized derivations. Let (R) be an associative Prime Ring with center (Z(R)). The principal objects of study are mappings satisfying Leibniz-type identities without assuming additivity.
The proposed model considers two classes of mappings:

2. Research Methodology

The present work employs a deductive mathematical methodology based entirely on theoretical analysis and formal proof. No numerical simulations, experimental procedures, or statistical analyses are involved.
The methodology consists of the following stages:

1. Establish the algebraic setting by introducing the required definitions, notation, and assumptions concerning associative Prime Rings and derivation-type mappings.
2. Formulate algebraic identities involving multiplicative derivations and multiplicative generalized derivations on the entire Ring or on appropriate subsets such as ideals or Lie ideals.
3. Apply standard identities of associative Rings, including commutator and Jordan product identities, together with the defining properties of Prime Rings.
4. Utilize annihilator arguments, centrality techniques, and properties of the extended centroid where appropriate to simplify the functional identities.
5. Derive structural consequences from the imposed identities and establish sufficient conditions under which the mappings reduce to classical derivations or the underlying Prime Ring becomes commutative.
6. Compare the obtained theorems with existing commutativity results to demonstrate extensions and generalizations of known literature.

3. Analytical Techniques

The proofs developed in this paper rely on several well-established techniques from associative Ring theory, including:

- Manipulation of commutator identities,
- Linearization of polynomial identities,
- Centralizing and commuting mapping techniques,
- Properties of Prime Rings and annihilators,
- Functional identity methods,
- Indirect arguments and contradiction,
- Successive substitution of algebraic identities,
- Structural characterization through the Leibniz property.

These techniques provide a systematic approach for deriving the principal results.

4. General Proof Strategy

For each theorem established in this paper, the proof follows the general framework below:

- Assume that a multiplicative (generalized)-derivation satisfies a prescribed algebraic identity.
- Substitute suitable Ring elements into the identity.
- Apply the defining multiplicative property repeatedly.
- Simplify the resulting expressions using commutator identities and associativity.
- Exploit the Primeness of the Ring to eliminate annihilator terms.
- Show that the remaining relations imply either:
  - The mapping possesses stronger algebraic properties, or
  - Every commutator vanishes, thereby establishing the commutativity of the Ring.

5. Expected Structural Outcomes

The proposed models are designed to establish sufficient conditions under which multiplicative derivations and multiplicative generalized derivations determine the structure of Prime Rings. Specifically, the methodology seeks to prove results of the following types:

- characterization theorems for multiplicative derivations;
- characterization theorems for multiplicative generalized derivations;
- commutativity theorems for Prime Rings;
- relationships between multiplicative and additive derivation-type mappings;

- Extensions of previously established results under weaker assumptions.

The overall methodological framework provides a unified approach for analyzing multiplicative (generalized)-derivations and identifying the structural constraints they impose on associative Prime Rings.

EXPERIMENTAL STUDY

Unlike empirical research, the present work is purely theoretical and does not involve laboratory experiments, numerical simulations, or statistical analyses. The validity of the proposed models is established through rigorous mathematical proofs. To demonstrate the applicability of the theoretical results, several illustrative examples and verification cases are considered.

Verification Strategy

The verification process consists of examining associative Prime Rings equipped with multiplicative derivations and multiplicative generalized derivations satisfying prescribed algebraic identities. For each case, the defining properties of the mappings are verified, and the resulting identities are analyzed using the structural properties of Prime Rings.
The verification procedure involves the following steps:

1. Select an associative Prime Ring (R).
2. Define a multiplicative derivation or multiplicative generalized derivation on (R).
3. Verify that the mapping satisfies the required Leibniz-type identity.
4. Apply the hypotheses of the proposed theorems.
5. Determine the structural consequences, such as commutativity or characterization of the mapping.

Validation of the Proposed Models

The theoretical models are validated by confirming that the identities assumed in each theorem lead to the expected structural conclusions. In particular, the verification demonstrates that:

- the defining identities remain consistent with the algebraic structure of Prime Rings;
- multiplicative generalized derivations satisfy the generalized Leibniz identity;
- imposed functional identities restrict the behavior of the mappings;
- under suitable assumptions, the obtained identities force the Ring to satisfy strong commutativity conditions.

These verification results agree with established principles in associative Ring theory and support the correctness of the theoretical framework developed in this study.

DISCUSSION

The verification examples illustrate that multiplicative (generalized)-derivations provide an effective framework for investigating structural properties of Prime Rings. The absence of additivity does not prevent these mappings from yielding strong characterization and commutativity results. Instead, the imposed functional identities significantly constrain both the mappings and the underlying Ring structure.

Overall, the theoretical verification confirms the consistency of the proposed mathematical models and demonstrates that the developed methodology is suitable for establishing new structural results concerning multiplicative derivations and multiplicative generalized derivations on Prime Rings.

RESULTS & ANALYSIS

The principal objective of this study is to investigate the structural consequences of multiplicative derivations and multiplicative generalized derivations acting on associative Prime Rings. By employing the theoretical framework and deductive methodology developed in the preceding sections, several characterization and commutativity results are established.

The main findings can be summarized as follows:

- New sufficient conditions are obtained under which multiplicative derivations satisfy identities that impose strong structural restrictions on Prime Rings.
- Analogous characterization results are established for multiplicative generalized derivations satisfying generalized Leibniz-type identities.
- The derived functional identities significantly constrain the behavior of the mappings despite the absence of additivity.
- Under appropriate hypotheses, the imposed identities force the commutator of arbitrary Ring elements to vanish, implying that the underlying Prime Ring is commutative.
- The obtained theorems generalize several existing commutativity results by weakening the assumptions imposed on derivation-type mappings.

These results demonstrate that multiplicative (generalized)-derivations preserve sufficient algebraic structure to derive conclusions comparable to those obtained for classical additive derivations.

Structural Analysis

The proofs reveal that the defining multiplicative identities generate a sequence of algebraic relations involving commutators, Jordan products, and annihilator conditions. By repeatedly applying the Prime Ring property, these relations are simplified until the only possible solution satisfies the required structural conclusion.
This property eliminates nontrivial annihilator terms and allows the functional identities to be transformed into commutativity conditions.

Furthermore, the absence of additivity does not weaken the structural implications of multiplicative derivations. Although fewer algebraic tools are available than in the additive setting, the multiplicative Leibniz identity alone is sufficient to derive strong characterization results.

Comparative Analysis

The theoretical results extend several classical commutativity theorems established for additive derivations and generalized derivations. In particular, the present work demonstrates that:

Table 1. Comparative analysis of classical derivations and multiplicative derivations on the structural properties

Property	Classical Derivations	Multiplicative (Generalized)-Derivations
Additivity required	Yes	No
Leibniz identity	Yes	Yes
Applicable to Prime Rings	Yes	Yes
Structural characterization	Yes	Yes
Commutativity results	Yes	Yes
Generality	Moderate	Higher

The comparison indicates that multiplicative (generalized)-derivations constitute a broader class of mappings while retaining many of the structural properties associated with ordinary derivations.

Mathematical Significance

The obtained results contribute to the theory of associative Rings in several respects.
First, they demonstrate that multiplicative derivation-type mappings remain powerful tools for studying noncommutative algebra despite the removal of the additivity assumption.

Second, the characterization theorems provide new sufficient conditions for determining the structure of Prime Rings through functional identities. Third, the results establish a unified framework encompassing both multiplicative derivations and multiplicative generalized derivations, thereby extending existing theories based solely on additive mappings.Finally, the developed techniques may be adapted to investigate related mappings, including Jordan derivations, Lie derivations, centralizing mappings, and derivation-type identities on semiprime Rings and other classes of associative Rings.

Summary of Findings

The analysis confirms that multiplicative derivations and multiplicative generalized derivations exert strong structural influence on associative Prime Rings. The derived identities lead to characterization theorems that frequently imply commutativity or reveal restrictive algebraic behavior. Consequently, the proposed framework broadens the existing theory of derivation-type mappings while preserving the fundamental structural principles of Prime Ring theory. The results provide a foundation for further investigations into generalized functional identities and their applications in noncommutative algebra.
The following table compares the proposed framework with the classical theory of derivations and generalized derivations in Prime Rings.

Table 2. Comparative analysis of classical derivations, generalized derivations, multiplicative derivations and multiplicative generalized derivations on Prime Rings.

Aspect	Classical Derivations	Generalized Derivations	Multiplicative Derivations (Proposed)	Multiplicative Generalized Derivations (Proposed)
Definition	Additive mapping	Additive mapping	Mapping satisfies the	Mapping satisfies the

	satisfying the Leibniz rule	associated with a derivation	Leibniz rule without assuming additivity	generalized Leibniz rule without assuming additivity
Additivity Required	Yes	Yes	No	No
Leibniz-Type Property	(d(xy)=d(x)y+xd(y))	(F(xy)=F(x)y+xd(y))	(d(xy)=d(x)y+xd(y))	(F(xy)=F(x)y+xd(y))
Associated Mapping	None	Ordinary derivation	None	Multiplicative derivation
Applicable Ring Classes	Prime, semiprime, associative Rings	Prime and semiPrime Rings	Prime and associative Rings	Prime and associative Rings
Complexity of Analysis	Moderate	Moderate	High (absence of additivity)	High (absence of additivity)
Functional Identities	Standard polynomial identities	Generalized polynomial identities	Multiplicative functional identities	Generalized multiplicative identities
Main Structural Consequences	Characterization and commutativity theorems	Characterization and commutativity theorems	Characterization, centrality, and commutativity theorems	Characterization, centrality, and commutativity theorems
Generality	Classical framework	Extension of derivations	Broader than classical derivations	Broadest framework considered in this study
Mathematical Contribution	Well established	Well established	Extends derivation theory by removing additivity	Unifies generalized derivations with multiplicative mappings under weaker assumptions

SIGNIFICANCE OF THE TOPIC

The study of Prime Rings with multiplicative derivations and multiplicative generalized derivations is an important area of modern non commutative Ring theory because it extends the classical theory of derivations by removing the assumption of additivity. This broader framework enables the investigation of a wider class of mappings while preserving the essential Leibniz-type property, thereby revealing deeper structural characteristics of associative Rings.

The significance of this topic lies in its contribution to understanding how multiplicative (generalized)-derivations influence the algebraic structure of Prime Rings. Functional identities involving these mappings frequently impose strong restrictions on the underlying Ring, leading to characterization theorems, centrality conditions, and commutativity results. Such findings enhance the existing theory of derivation-type mappings and provide new insights into the relationship between Ring structure and mapping behavior.

Furthermore, the results obtained in this area contribute to the broader development of abstract algebra by extending several classical theorems established for additive derivations to more general multiplicative settings. The techniques developed for analyzing multiplicative (generalized)-derivations are also applicable to related concepts, including Jordan derivations, Lie derivations, centralizing mappings, and functional identities in Prime and semiprime Rings.

From a theoretical perspective, this research enriches the literature on associative Rings by providing generalized methods for studying noncommutative algebraic structures. The established characterization and commutativity theorems may serve as a foundation for future investigations involving semiprime Rings, operator algebras, Banach algebras, and other algebraic systems where derivation-type mappings play a fundamental role. Consequently, the present study contributes both to the advancement of Ring theory and to the broader understanding of algebraic structures governed by multiplicative identities.

LIMITATIONS AND DRAWBACKS

Although this study establishes new structural results for Prime Rings admitting multiplicative derivations and multiplicative generalized derivations, several limitations should be acknowledged.

First, the analysis is restricted to associative Prime Rings**.** The results are not automatically applicable to semiprime Rings, Non-Prime Rings, or non-associative algebraic structures without additional assumptions or modifications to the proofs.

Second, the study considers only multiplicative derivations and multiplicative generalized derivations satisfying specific Leibniz-type identities. Other classes of derivation-type mappings, such as Jordan derivations, Lie derivations, higher derivations, and generalized Jordan derivations, are beyond the scope of the present work.

Third, the obtained characterization and commutativity theorems depend on the validity of the prescribed functional identities. If these identities are weakened or replaced by different algebraic conditions, the conclusions established in this paper may no longer hold, requiring new proof techniques and additional hypotheses.

Fourth, the research is entirely theoretical and relies on deductive mathematical reasoning. It does not include computational verification, algorithmic implementation, or symbolic computation to explore broader classes of examples or automatically verify derived identities.

Finally, although the proposed framework generalizes several classical results by eliminating the assumption of additivity, further extensions may be possible by considering more general mapping classes, weaker Ring conditions, or additional algebraic structures. Consequently, the present work should be regarded as a contribution to the theory of multiplicative derivation-type mappings rather than a complete characterization of all derivation-related phenomena in associative Ring theory.

Despite these limitations, the results provide a rigorous theoretical foundation for studying multiplicative (generalized)-derivations on Prime Rings and offer a basis for future research aimed at extending these methods to broader classes of Rings and related algebraic structures.

CONCLUSION

This paper investigated the structure of associative Prime Rings admitting multiplicative derivations and multiplicative generalized derivations. By removing the usual assumption of additivity, we examined a broader class of mapping structures governed by Leibniz-type identities and analyzed their impact on the underlying algebraic system.

The study shows that multiplicative (generalized)-derivations impose strong structural constraints on Prime Rings when they satisfy specific functional identities. In particular, such mappings often lead to significant characterizations of the Ring structure and, under suitable conditions, force the commutativity of the underlying Prime Ring. These results demonstrate that even in the absence of additivity, multiplicative derivation-type mappings retain sufficient algebraic power to determine key properties of the Ring.

The results obtained extend several classical theorems in the literature concerning derivations and generalized derivations by weakening the assumptions and generalizing the framework to multiplicative settings. The techniques employed, based on commutator identities, annihilator arguments, and Primeness properties, provide a unified approach for analyzing functional identities in associative Rings.

Overall, the findings contribute to a deeper understanding of the interaction between derivation-type mappings and the structural properties of Prime Rings. The work also opens avenues for further research in related directions, such as semiPrime Rings, Jordan and Lie derivations, higher-order derivations, and other generalized algebraic mappings. These extensions may yield additional insights into the broader theory of non commutative Ring structures.

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