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Exploring the contribution of Sub-infinitesimals to the Theoretical Division by Zero

Emmett D. Lane

Research Scholars Program, Harvard Student Agencies, In Collaboration with Learn with Leaders

ABSTRACT

The concept in which division by zero results in a defined and logically consistent answer as opposed to the currently accepted "undefined" answer sprung forth from a theory that was developed by applying division to infinitesimal numbers in the hyperreal number line, leading to the discovery of sub-infinitesimals, which are an extension of the concept of the number zero, with the latter assuming the role of the data of numbers. Sub-infinitesimals themselves have no magnitude as they are simply zero, however they serve as a system of identification for numbers describing how in certain situations a number will "react" to certain operations. When considering division by three, which results in a never-ending quotient, speculation that division by 3 was an "imperfect" operation that cannot give a definite answer without a remainder. Sub-infinitesimal data provides adequate reasoning for how certain numbers can exist, such as the quotient of ⅓ being different from an arbitrary number of values 0.333333… as well as how √2 squared would give an exact value of 2 instead of a number very close to 2. Sub-infinitesimal data also describe how numbers which are equivalent in magnitude could be different solely on how these numbers were obtained. 1 could be described as simply 1, but also as 3-2, e^0, abs($e^{\pi i}$) and so on and so forth, and although all these numbers are equal in magnitude, the means in which resulted in the same result for each number is very different from each other. Although in the aforementioned cases the sub-infinitesimal data which differentiates 1 from being the sum or a product of an equation serves no "true" purpose, the idea of numbers being equal in value but different in sub-infinitesimal data is essential when considering division by zero, as that idea allows us to express how such an operation can function consistently. Acknowledging the latter, this paper will present the theory by asking as well as answering three questions, each of which serving as the stepping stones in describing division by zero. These questions are as follows: how many sides does a circle have, how can we tell the difference between the quotient of ⅓ and an arbitrary value of 0.33333… and if equal numbers are truly "equal" to each other. From this we gather enough information to logically solve division by zero, which is done by utilising sub-infinitesimal data of π to define division by zero to be a specific infinite value pertaining to the sub-infinitesimal geometry of a circle, being the amount of "zero-triangles" in a circle.

Thesis statement: Division by zero is possible, and the answer to division by zero is consistent algebraically.

INTRODUCTION

Division by zero is a mathematical phenomenon that is both a hated as well as a loved phenomenon by mathematicians and non-mathematicians around the globe. Not being considered a fully unsolved question, but never considered truly answered. Division by zero has been and still is a perplexing topic, since logically, when presenting the problem both in and out of a real-world context, the wording itself of the problem is simple absurdity: How many times would you have to subtract nothingness from something to get nothing? Could you equally divide an object into nothingness, and how many times would you have to add up that nothingness to that object again? Logically, no matter what approach is taken, whether it would be the cutting up of apples or pears, no good explanation was concocted in describing how division by zero could be solved or how many times any object needed to be cut until its parts were nothingness. Although in real-life physics, theoretically if an apple were cut up so much that all we were left with were apple chunks that were smaller than a planck length, then we would have split an object into parts that can be considered "nothingness" (Vsauce, 2015). However, in the context of mathematics, smaller than Planck length is still far from absolute nothingness. When attempting to solve division by zero using conventional algebra, and staying within the realms of the finite (using real numbers), we can attempt to solve division by zero using a simple equation as seen below:

$$1/0=k$$

Consider 1/0 to be equal to some constant *k*. If we multiply both sides of the equation by the denominator zero, and by applying the rule of 0*k=0, we would get the following equation:

1=0 which is a logical fallacy, as 1 is not equal to zero.

Facing the problem in a new perspective, the "complex number perspective", we could equate 1/0=∞, which is similar to how √-1 was equated to i. This logic begins to fall apart when attempting to divide other numbers by zero. Primarily, ∞ is not really a number, it is a concept, and even if we were to treat it as a number, it still leaves the case that 1/0 isn't even equal to ∞, as if we were to graph the function f(x)=1/x, we would notice that as x approaches 0, f(x) tends both to negative infinity as well as positive infinity. Thus, is it ∞ or -∞? Well, certain mathematicians claimed that the solution was both at the same time, which led to the unofficial solution being unsigned infinity (*How to Divide by Zero,* 2020). Which in a way solves part of the problem, however it still leaves the main issue of the matter being that having any kind of infinity as a solution does not work as a multiplicative inverse of 0 due to the absorption property of 0 and ∞. Any number multiplied by 0 is 0, and logically any number multiplied by ∞ is still ∞. Even if we define 1/0 as ∞, that still leaves every other number divided by zero as undefined, due to every number divided by zero equating to ∞, leading to the fallacy that all numbers are equal to each other which is not a useful deduction. This led to the modern-day notion that division by zero is undefined. There are only certain cases in which division by zero was defined to be ∞, but that only functions on manipulated algebra, such as wheel theory or stereographic projections, which allows division by zero to equate to ∞ ("Riemann Sphere," 2022) (Carlström, 2001). Those do not serve as definitive solutions because at that point the meaning of ∞ is completely lost, only serving as a means to end the question rather than solving it. Despite this, there are shreds of truth to these solutions, as the solution we've come up with to solve division by zero are numbers which are infinite in magnitude. Explaining how to divide by zero isn't a straightforward and simple answer however, as new concepts are introduced and explained in order for the theory to bear a semblance of cohesion. The theory on how to divide by zero was developed around three questions, each of which arguing for the existence of sub-infinitesimals (It is important to note that throughout the paper the words "sub-infinitesimal" and "sub-infinitesimal data" are used interchangeably. Although they both are describing the same concept, the word sub-infinitesimal is used when alluding to sub-infinitesimals as numbers whilst the term sub-infinitesimal data is used to describe the function of sub-infinitesimals). These questions are as follows: how many sides does a circle have, how to perfectly divide by numbers with never terminating quotients, and if there is a way to tell the difference between identical numbers? Our theory on division by zero begins with the fundamental issue of the problem being the operation itself that is being performed with that being division, and how even when we remain within the realm of finite numbers we end up with certain inconsistencies, such as never terminating quotients.

DISCUSSION

The theory on division by zero is rooted upon the concept of division itself, which is why we should begin with explaining what division actually is. Division itself is considered to be iterated subtracting, in which division is really how many times does a number need to subtracted in order to reach zero, so in the case of 10/5, we need to subtract 5 twice from 10 in order to reach 0, which leads us to a quotient of 2. Although, it might be more accurate to define division as how many times can you "fit" a number into another. Not all divisions are perfect however, as certain divisions end up in numbers being smaller than 1, which lead to the formalisation of rational numbers into our number system long ago. With rational numbers, one was able to show quotients of numbers without the usage of remainders. However, rational numbers were not the death of remainders, as there were many cases where the quotient would never terminate, and that meant numbers that couldn't be perfectly expressed without remainders, such as **⅓.** (thisparticular number will also be used as a proof that sub-infinitesimals exist).Although it can be argued that never terminating numbers are simply a flaw of the base-10 system we have developed our mathematics in, as in a base-12 system, ⅓ would simply equate to 0.4. In a way, that does solve the problem of not being able to "perfectly" express a quotient using a finite amount of numbers, however that doesn't really solve the problem, as we cannot "fit" 3's into 10, regardless of the base the system we are using. Gathering this information, one might ask themselves that even despite this, can the quotient of ⅓ end? Well, that is not possible, as evidenced by the fact that any number bigger or smaller than an endless sequence of 3's following a decimal point will not give 1 if multiplied thrice. In the case of defining 0.333333… with a 4 somewhere in the sequence of 3's we will get a number that will be larger by 1 if multiplied by 3. For example, 0.33333334*3 would give $1+2*10^{-9}$, which is larger than 1. If we were to go "below" and we were to choose 0.33333333… (0 followed endlessly by 3's) and multiplying that number three times we get 0.99999999 and, in this case, we would get a number smaller than 1. This insignificant difference between what ⅓ should multiply to and what it technically multiplies to is something that we consider is overlooked by mathematicians, because despite it being considerable to 1, it is not 1. The logical reasoning is that it is simply impossible to perfectly fit 3 into 10, and that in a way is true, and we could end our whole ordeal here. However, we believe that this line of reasoning to be false, and that it is possible to divide by three without leaving a remainder. The concept begins with dividing ⅓ using long division, which although it is an elementary method, is enough to convey our idea on removing the remainder. What is done is that we repeat long division a theoretical infinite amount of time and using a bit of intuitive thinking we can imagine how long division would affect numbers that are infinitesimal in magnitude (Infinitesimal defined as (*Infinitesimal | Mathematics,* 2016). Seen below is an illustration of 1 divided by 3 utilising long division.

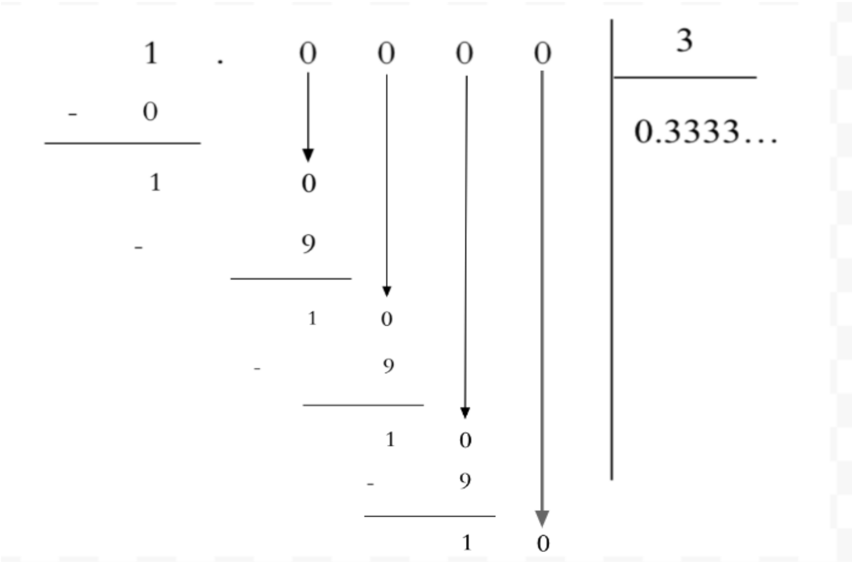

Figure 1: Long division is performed with 1 as dividend and 3 as divisor.

As we can see, if we were to continue subtracting multiples of three from smaller and smaller "quantities" of 1 we would get a never-ending sequence of a 0 followed by an infinite amount of 3's, as seen in the quotient section of the long division. This is caused by the fact that multiples of three cannot be used to express ⅓ independent of a remainder. This is what led to the conclusion that division by three without some value left over isn't really possible, that it isn't a "perfect" division as within the context of a base-10 system, there will always be a remainder whenever dividing by 3. However, we believe this to not be the full picture: it is possible to divide perfectly by 3, however this involves "stretching" our concept of division to include infinitesimals, as well as even going beyond infinitesimals.

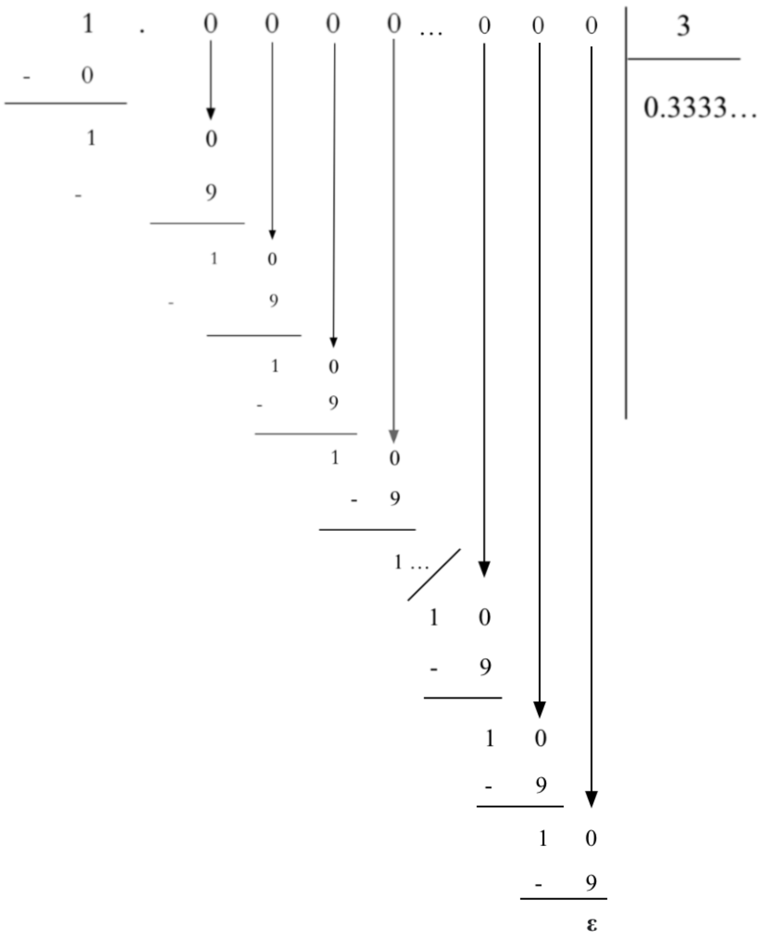

Figure 2: Long division is performed and "extended" to include infinitesimal values.

In this image we see 1 being divided by 3 by making use of long division, and how the process is never terminating (A diagonal line and a few dots were used to represent the process of long division being repeated an “infinite” amount of times), since the pattern of multiplying by 3 and subtracting 9 from 10 repeats for finite numbers, we deduced it as proof that the pattern of 0.33333333 followed by an endless amount of 3’s should remain as such even if we were to theoretically continue subtracting until infinitesimals were reached, we should still get an arbitrarily small “1” as a remainder (a **ε** was used as it is the symbol for the infinitesimal “equivalent” of 1).Here is where the division would stop, giving us an infinite array of 3’s with a remainder value of an infinitesimal **ε**. The quotient cannot get any more precise than that and conventionally, this is where the process terminates. Here is where our new theory “solves” division by three, getting rid of the remainder by introducing a new mathematical concept; **sub-infinitesimal data**. It is seen below:

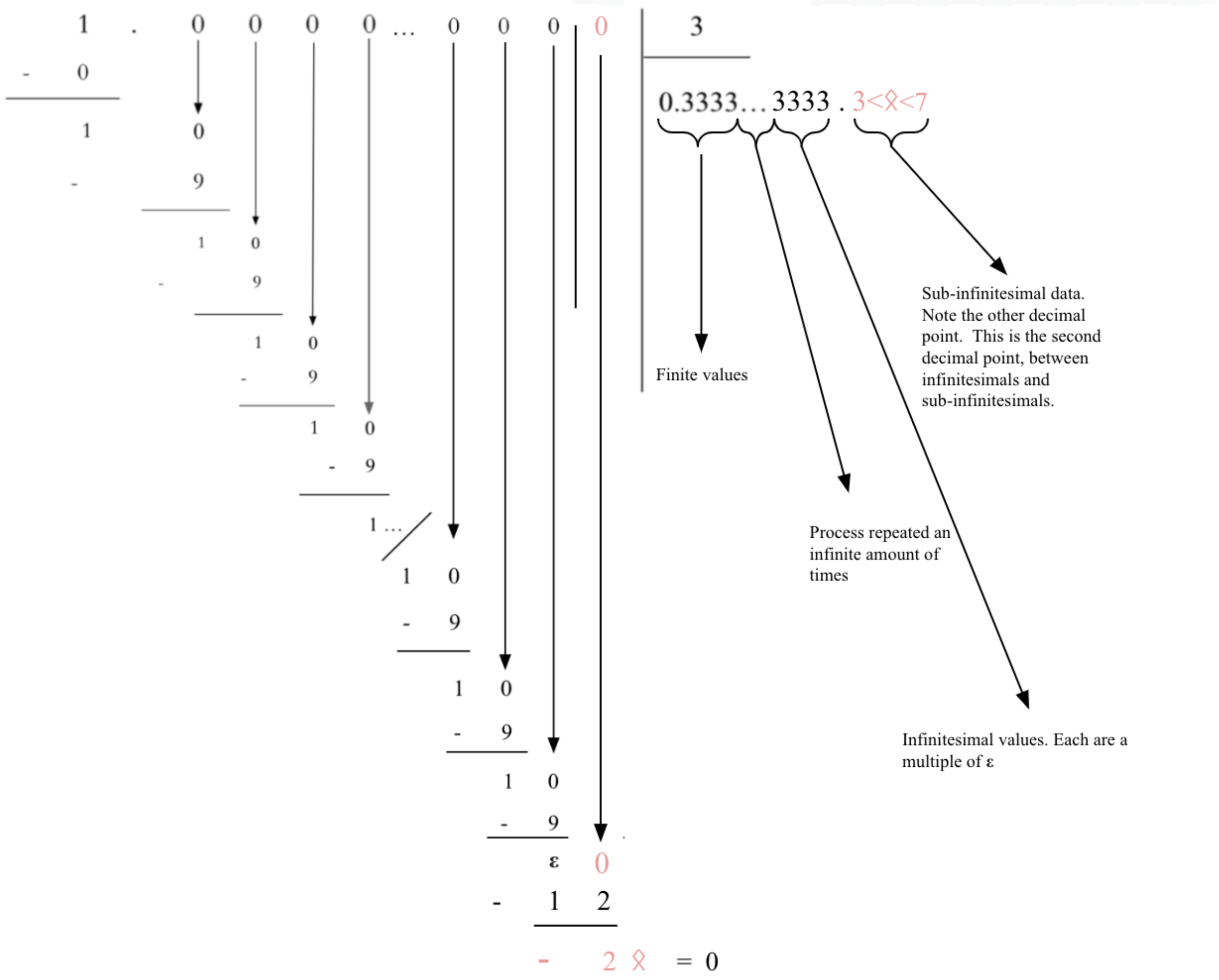

Figure 3: Long division being extended a second time to include a value less than ε

As seen in the figure above, the step taken was to divide 1 by 3 once more beyond infinitesimal values. The pink numbers seen above represent sub-infinitesimal data and the line is representative of when the values of the 1 that are being divided become sub-infinitesimal in magnitude. To summarise, so far, we’ve established that the ⅓’s never terminating quotient is the result of a division that cannot be expressed without a remainder. Figure 2 served as the idea that if we were to continue long division to infinitesimals, the pattern wouldn’t change as that would make it so the quotient would no longer multiply to the dividend. From this, our long division terminates, as we cannot get any smaller than an infinitesimal. Despite this, we believe that it is possible to keep dividing beyond infinitesimals, as showcased in Figure 3. To begin our explanation, we will explain the lowering of the pink zero: this was used for demonstration purposes, as to show that the same process which was used for non-zero real and infinitesimal values could be applied to sub-infinitesimals. The primary goal of this was to showcase the subtraction of 1.2 **ε** from 1 **ε**. That subtraction contradicts one of the main rules of long division, that being subtracting by a multiple bigger than the divisor, as well that resulting in a negative remainder. The truth is, 1.2 **ε** isn’t technically the number being subtracted. It could have been 1.5**ε** or even 1.8 **ε**. It can really be any number (1***ε**≤ ᛟ<2***ε**) as long as it satisfies the condition of subtracting the infinitesimal **ε** and leaving a remainder that is smaller than **ε**. Infinitesimals are described as numbers bigger than zero and smaller than real numbers, so in theory numbers which are smaller than infinitesimals, would be simply zero. So, the remainder is sub-infinitesimal in magnitude, which means that it is equivalent to **0**, meaning that we’ve perfectly divided our dividend of 1 by 3**.** The subtraction of the last infinitesimal value means that there is nothing left to subtract, as we have a remainder that is 0 and thus our process is over. This conventionally shouldn't be possible, as 0 multiplies to a value bigger than 0 (3*(3<ᛟ<7)). This contradicts one of the golden rules of 0, being multiplication by 0 resulting in a number different from 0, in this case, a negative infinitesimal. This is because in actuality, the 0 that is being multiplied by three is **sub-infinitesimal data** which when undergone a certain process (in this case, multiplication by 3), will form a value. This data is what differentiates this as the quotient of ⅓ as opposed to an arbitrary value of 0.333333… multiplied by three. In the quotient section, we see the 0.333333… with the ellipsis showcasing the process being repeated an infinite number of times, then we see the 3's again after the ellipsis, to signify that they are multiples of the infinitesimal value **ε.** Then, after the rightmost "final" infinitesimal 3 (3***ε)** the quotient dips into the sub-infinitesimal values, where 3 <ᛟ< 7 is found in the quotient section of the long division. This is the aforementioned sub-infinitesimal data;however, it is represented inaccurately. 3 <ᛟ< 7 was shown as the natural numbers between 3 and 7 when multiplied by 3 would give value between 10 and 20. This 10 and 20 is in actuality 1.0 and 2.0 **ε**, and needs to meet the condition that when the **ε** is subtracted by the multiple the remainder is smaller than **ε**. In this case, the number 4 was used as when multiplied by 3 its product is 12, and in the context of subtracting 1.2 infinitesimals from 1 infinitesimal, we would get a value of - 0.2 infinitesimals, which as previously stated, equals to zero. Due to this being the first "manifestation" of sub-infinitesimals, sub-infinitesimals will be further explained in more detail in the following paragraph.

Sub-infinitesimals can be loosely thought of the numbers functioning as the inverse to nilpotency in infinitesimals ("Smooth Infinitesimal Analysis," 2022), bearing a slight resemblance to that theory however being completely independent of smooth infinitesimal analysis and matrix algebra. By making use of the idea that $\boldsymbol{\varepsilon}^2=0$ is true but $\boldsymbol{\varepsilon}=0$ isn't necessarily true, we extended that concept to include $\boldsymbol{\varepsilon}/a=0$ (with $a\neq0, a\neq \text{abs}(x)\leq1$ with $x \in \mathbf{C}$) and also as a way to "retrieve" the **ε** from the 0 without causing contradictions. The basic theory of sub-infinitesimals is that they are the values below infinitesimals, serving as the "decimals" of infinitesimal values. They are what is found between infinitesimals as seen below:

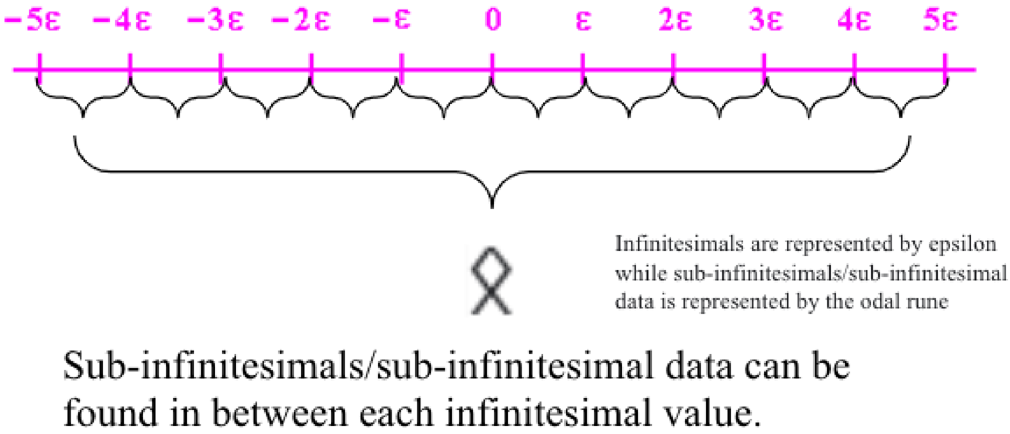

Figure 4: Sub-infinitesimals placement in the hyperreal number line.
Source: (*Números Hiperreales*, 2005) (edited)

They are described as the values lesser than an infinitesimal, and can be thought as the infinitesimal equivalent of decimal numbers. Due to the definition of infinitesimals, being numbers that are bigger than 0 but smaller than any finite number, this means that sub-infinitesimals are equivalent to zero. However, if they are zero, how can they produce a value as seen with the case of ⅓, since 0 multiplied by any number will always be 0, despite figure 3 stating that 0*3=**ε** which would lead to the conclusion that 0=**ε**, which is false. The idea that 0**a*=0 is true without a given context, however in certain contexts 0**a* can equal to a value bigger than 0, which leads to the second aspect of sub-infinitesimals, being their role as the data/identification of numbers, as seen with the case of ⅓. That infinitesimal value doesn't appear out of the blue, as it's the sub-infinitesimal data of ⅓. Think of it like so: when the 1 was divided into 3 equal parts, the final infinitesimal value was then again cut into 3 parts, each of which resulting into 0.33333…**ε** since that results in a number smaller than **ε**, these values become zero-equivalent, as they are sub-infinitesimals. Because of that, they became sub-infinitesimal data, which when undergone an operation (such as multiplication), will result in a **ε** forming, which allows ⅓ to multiply to 1 instead of 0.99999… This is not a fallacy, as the 0 being multiplied here is sub-infinitesimal data. Sub-infinitesimals are simply a new way of looking at zero, and what zero can do. The crux of the theory about sub-infinitesimals is that they are not values. You cannot add them or subtract them or multiply or divide or use any mathematical operation to yield results that are useful, because they are not values with a magnitude. For example, 1+1= 2, but sub-infinitesimal data added to another number will not yield any specific value: it would just be 0+0=0, so 0.5ᛟ+0.5ᛟ does not necessarily mean 1**ε**. This also means that all sub-infinitesimals are all "equal" (in terms of magnitude) to each other, so for example 0.9 **ε =** 0.1 **ε** = 0.0000000001 **ε** and also any other denomination of **ε** (since 0=0**).** However, each sub-infinitesimal above is different in terms of **data.** This explains the colouring of the number from figure 3 as well, because we didn't exactly subtract 1.2 infinitesimals from 1.0 infinitesimal. Since sub-infinitesimals manifest themselves as data instead of values, all that was needed in figure 3 was the data saying that the quotient to the solution is in fact the quotient, not some arbitrary similar value. Since addition and multiplication don't affect sub-infinitesimals in a predictable manner, it's best to generalise as "an infinitesimal will form in this specific context". This also explains why we needed a "negative" remainder when subtracting the final infinitesimal when dividing by three, because although logically 0.999999… **ε** is closer to **ε** than 1.2 **ε**, sub-infinitesimals do not have magnitude, so 1 **ε** - 0.999999… **ε** is still equal to 1**ε**, which is why values such as 1.2 **ε** and above are needed to subtract 1 **ε**, hence the "negative" remainder. The main idea of sub-infinitesimals is that they are **data about the numbers they are sub-infinitesimal to**. For example, 1=1 is undoubtedly true, however is one apple equivalent to one orange? The simple answer would be no, as although they are equal in numbers, both are different in what they are made of. This sort of information is another aspect of sub-infinitesimals/sub-infinitesimal data, named as the "useless data" of sub-infinitesimals, defined useless due to the fact that these sub-infinitesimals do not "react" to mathematical operations. Useless data, despite being labelled useless, is one of the most fascinating aspects of sub-infinitesimal data, because the information this data can hold, in theory, is limitless. The data in these numbers can tell us information far beyond even the scope of mathematics. The prime example of useless data was the 1 orange and 1 apple example, however if one were to write a number 1 on a board and 1 on a sheet of paper, although both numbers would equate to each other, a part of their sub-infinitesimal data would describe (trivial) information such as who wrote the number, where they wrote the number and when the number was written. As previously mentioned, if one were to write the number "1" on a board and "1" on a sheet of paper, those two numbers would have different sub-infinitesimal data. Furthermore, as time passes, numbers would also "age", changing their sub-infinitesimal data. Sub-infinitesimals are data, so presented in a numerical form (Figure 16/17), the data would appear chaotic, and constantly changing (updating in a way). There is also the case for which certain numbers have more sub-infinitesimal data than others, as they are more prevalent in mathematics and used more daily, as seen in figure 5 (it must be noted that this concept of sub-infinitesimals is more philosophical than mathematical). The other type of information that can be found within sub-infinitesimals is what is known as "useful data" which is data which when undergone a mathematical operation will have an effect on the end result. Sub-infinitesimal data falling under the category of "useful" is not exclusive to division, or even ⅓ as a matter of fact, but affects all numbers, even numbers which don't need sub-infinitesimal data to multiply (or any operation) back to that specific number (seen in the section where we divide by zero). ⅓ was used simply because of its property of being a predictable sequence of numbers to work with to prove the existence of sub-infinitesimals. All numbers contain some form of "useful" sub-infinitesimal data, however, it must be noted that transcendental numbers, such as π and $e$are especially important, due to the fact that they contain unique sub-infinitesimal data giving them unique properties.

The below figure shows numbers 1 and ½ with a space in between them representing their sub-infinitesimal data.

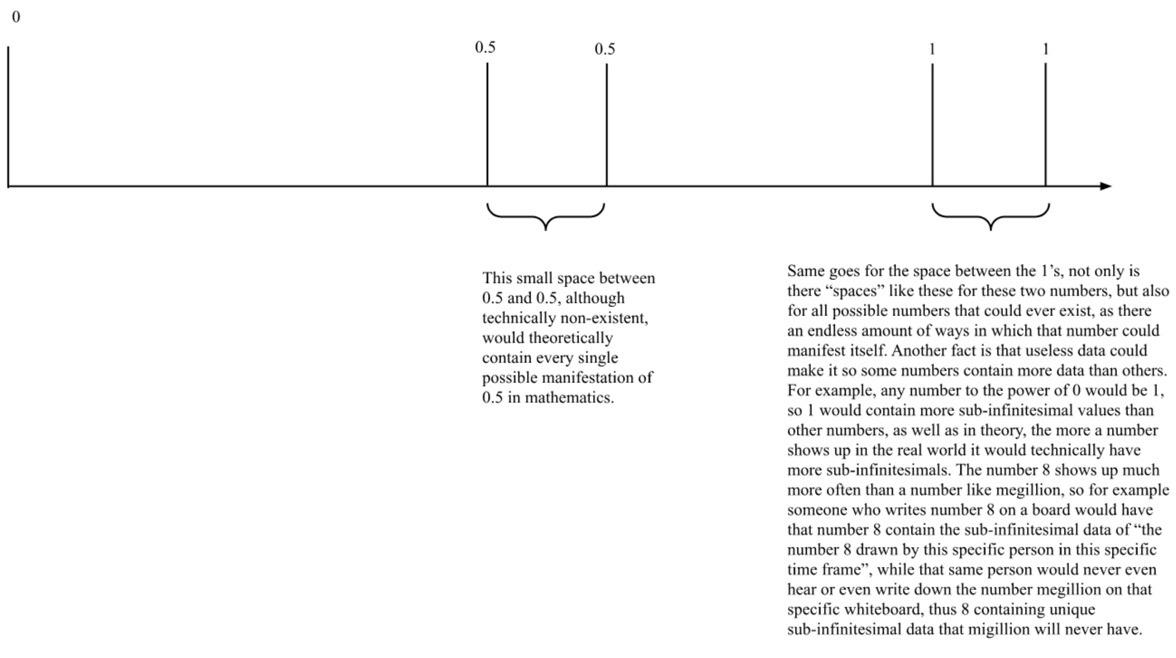

Figure 5: Numbers 1 and ½ on a number line with sub-infinitesimal spaces between.

Lastly, the "final" sub-infinitesimal data category is known as "identification" data; although technically this should still fall under the category of "useful" data, for clarification we separated it from useful data. Identification data serves the purpose of a system of identification for numbers, so for example, what are these numbers the result of, or where do they "come from" in a mathematical context. They describe what series of operations a number has gone through, and what "state" a number is. Examples of this identification are how, for example, the number 4 can be obtained in the following ways: 2+2=4, 8-4=4 and 2*2=4. Although these equations lead to the same results, the steps taken to arrive at the end are all different from each other. This means that although these answers are identical, their sub-infinitesimal data will all differ from each other. Of course, the equations can be rearranged, which will cause the sub-infinitesimal data of each solution to be changed, because sub-infinitesimal data isn't permanent, it changes all the time. Again, this doesn't just apply to these two equations but every single possible equation which results in a four, even the following definite integral: ∫ 2x dx {0,2} would give a value with unique sub-infinitesimal data. The other aspect of this is the "state" a number is in, taking 4 for example, it can manifest itself on a number line, or could be a straight line of magnitude of 4 units, or the area of a square. Each "manifestation" will have unique sub-infinitesimal data, each pertaining to the "state" 4 is in. To simply conclude, sub-infinitesimals aren't values with magnitude, but are purely data about numbers which contains information about numbers, that information possibly going even beyond mathematics.

Having established what sub-infinitesimals are, it is time we pass onto talking about how sub-infinitesimals play a role in addition, subtraction and multiplication. In addition, as well as subtraction, adding and subtracting sub-infinitesimals to any number will just give that number, because remember, sub-infinitesimals are zero-equivalent. Multiplication however, is an interesting case, since multiplying any nonzero number by a sub-infinitesimal will yield a "unique" zero. The reasoning is seen below:

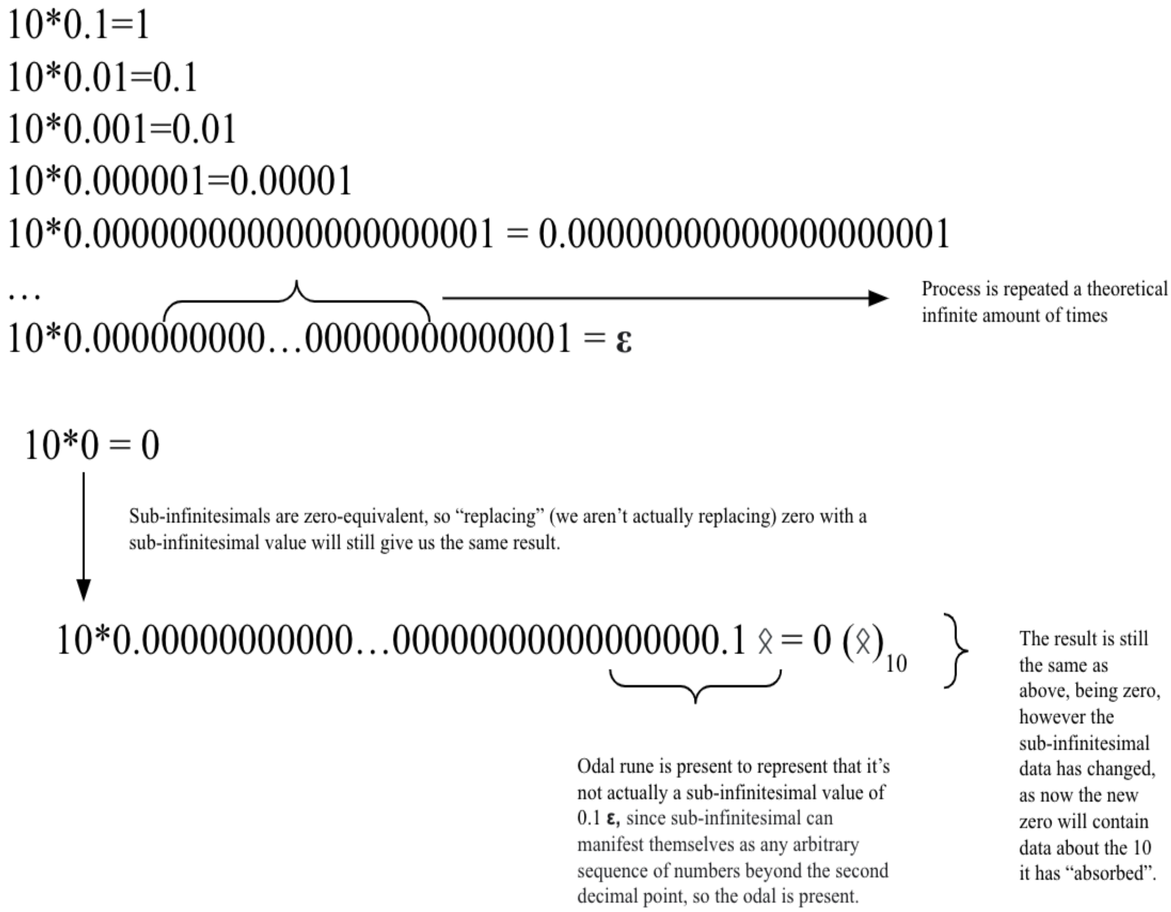

Figure 6: Number 10 being multiplied by smaller and smaller values until multiplied by zero.

In the image, think back to 10 being "swept" into the smaller values as it was being multiplied by smaller values. The idea is that when 10 is multiplied by 0, one can think of it as the 10 isn't being fully eradicated, but that it's being swept into the sub-infinitesimals, making it so the new zero will contain new sub-infinitesimal data which describe it having been multiplied by 10 (as well as whatever sub-infinitesimal data was present within 10, as well as information about the operation it has undergone and more). This also applies to any number, in which case the zero will contain different sub-infinitesimal data which will correspond to the number it was multiplied to. Already knowing this can give us insight on how division by zero can make sense, as it solves the issue of the distributive property, as seen with the screenshot below.

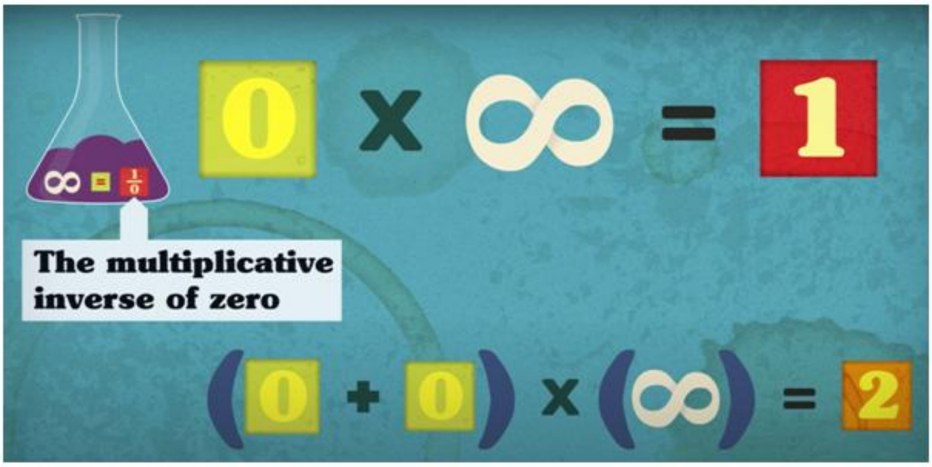

Figure 7: Screenshot of the distributive property and division by zero
Source: (*Why Can't You Divide by Zero? - TED-Ed*, 2018)

Although we are not equating 1/0 or 2/0 to ∞, above we see how if 1/0 = ∞, then (0*∞)+(0*∞) should equate to 2. By the distributive property, we can rearrange the equation to (0+0)*∞. Since 0+0 is equal to 0, we get 0*∞ = 2, despite defining 0*∞ to be 1. This is where sub-infinitesimals change the approach we take to this problem, as the 0+0 (0*2) will contain different sub-infinitesimal data that the zero in 0*∞. It is important to note that ∞ is not the solution to our theory, the solution to 1/0 will be seen beyond this section, as well as working as a "key" containing unique infinite data, which will "morph" the zero into the sub-infinitesimal data needed to perform the operation. It is important to note how by the existence of sub-infinitesimals, this solves an aspect of the problem of division by zero. This change in perspective is valuable, as it is a step closer to solving this problem and defining 1/0.

After having disclosed sub-infinitesimals, what they are and what they do, we will move onto sub-infinitesimals and geometry. Since sub-infinitesimals are a new theory on zero, it becomes natural that from that idea the concept of zero-dimensional space changes. Conventionally, the only geometric figure that is considered 0-dimensional is a dot, however, with the advent of sub-infinitesimals, any geometric figure regardless of dimension can be 0-dimensional, as long as it's magnitude, area or even volume (and even hyper-volume) is lesser than an infinitesimal (if one side is larger or some part of the figure is larger than an infinitesimal it is nolonger considered 0-dimensional), then it'll be equivalent to a 0-dimensional dot, however can be still be "defined" as a geometric figure with its respective dimensions thanks to its sub-infinitesimal data. Remember, sub-infinitesimals are equal in magnitude, so the below shapes are all the same in terms of surface area and perimeter, however the different "data" they carry allow them to appear as distinct (note that they can be "morphed" into another figure, changing sub-infinitesimal data due to the 0=0 property).

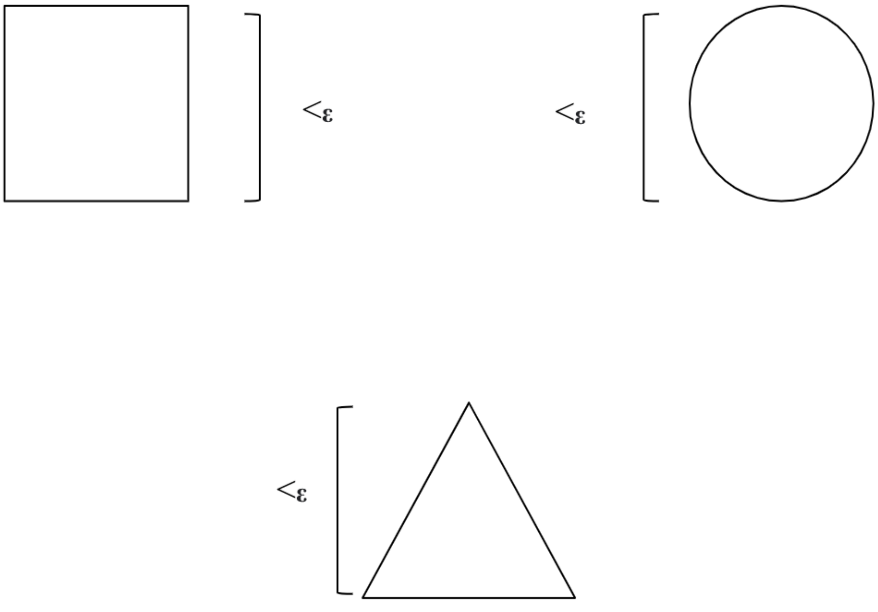

Figure 9: Various "0-dimensional" shapes are drawn with sub-infinitesimal edges

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Because of the sub-infinitesimal data being present, these 0-D shapes can still be visualised as seen above. In terms of sub-infinitesimal logic, any shape of any "dimension" could have been substituted and the outcome would still be the same, as long as the geometry of the figure remains sub-infinitesimal in magnitude. Sub-infinitesimal geometry is what we shall use to explain division by zero, and the importance the sub-infinitesimal geometry of a circle has. Here we ask and answer our final of the three questions asked in the introduction, that being what is the number of sides a circle possesses (referring to polygonal sides). We believe that this question, although seeming to be obvious and simple, is the key to solving division by zero. First let us ask our question: how many sides does a circle have? Many would claim it possesses an infinite number of sides, while many say that it only has 1 side, and some even claim it has no sides. The true answer is actually a mixture of the infinite and 0 sides, as the number of sides is indeed an infinite number, however that infinite number makes it so it has zero sides. Here's the explanation.

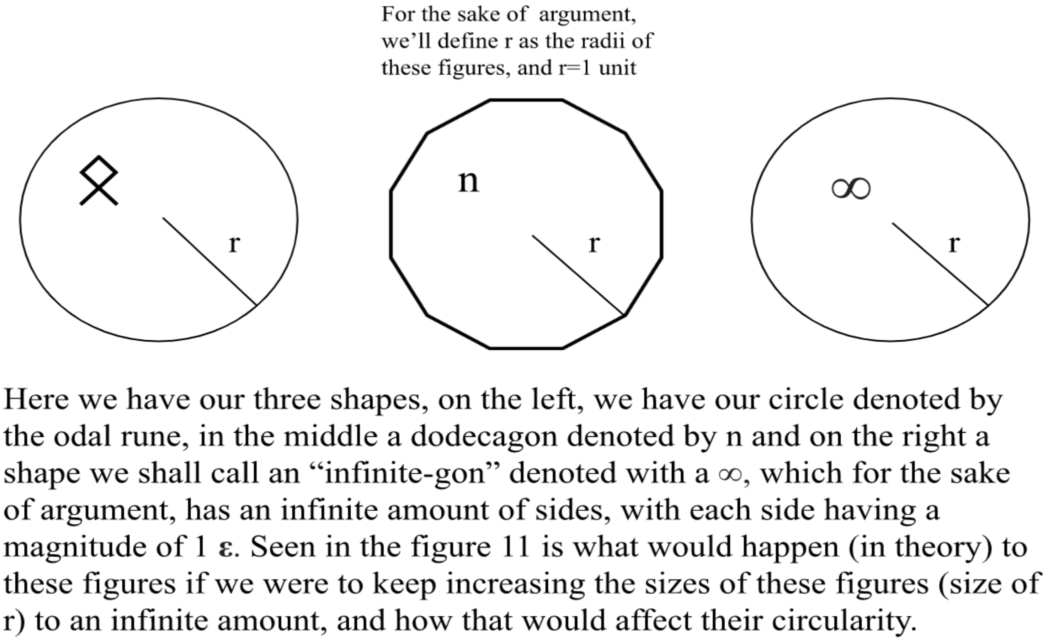
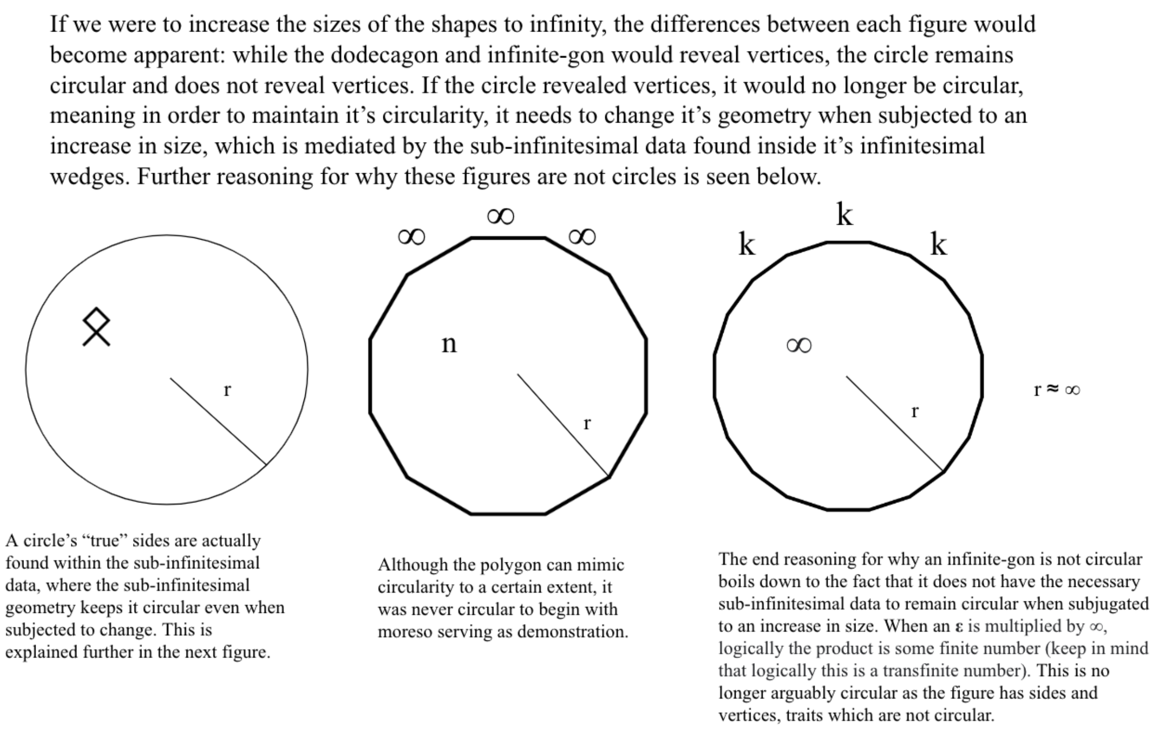


Figure 10: Three figures, one being a circle, the other mimicking circularity and the final differing from a circle.



.

Figure 11: Proof for why a circle has 0 sides

This serves as proof as to why the circle cannot simply be described as having an infinite number of sides, as when the infinite-gon is subjugated to an increase in size, it loses its circularity, meaning it wasn't circular in the first place. A circle retains its circularity when subjugated to an infinite increase in size due to the fact that a circle's "true" geometry is mediated by its sub-infinitesimal data. This is explained in figure 12.

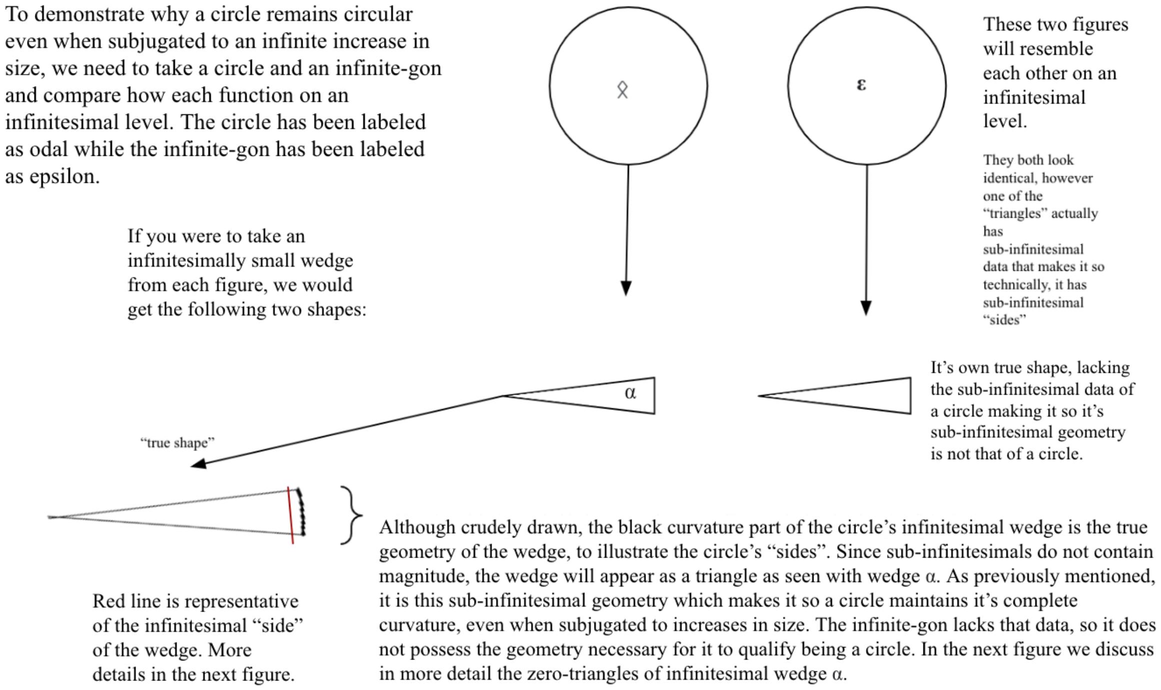


Figure 12: Explanation to a circle's number of sides.

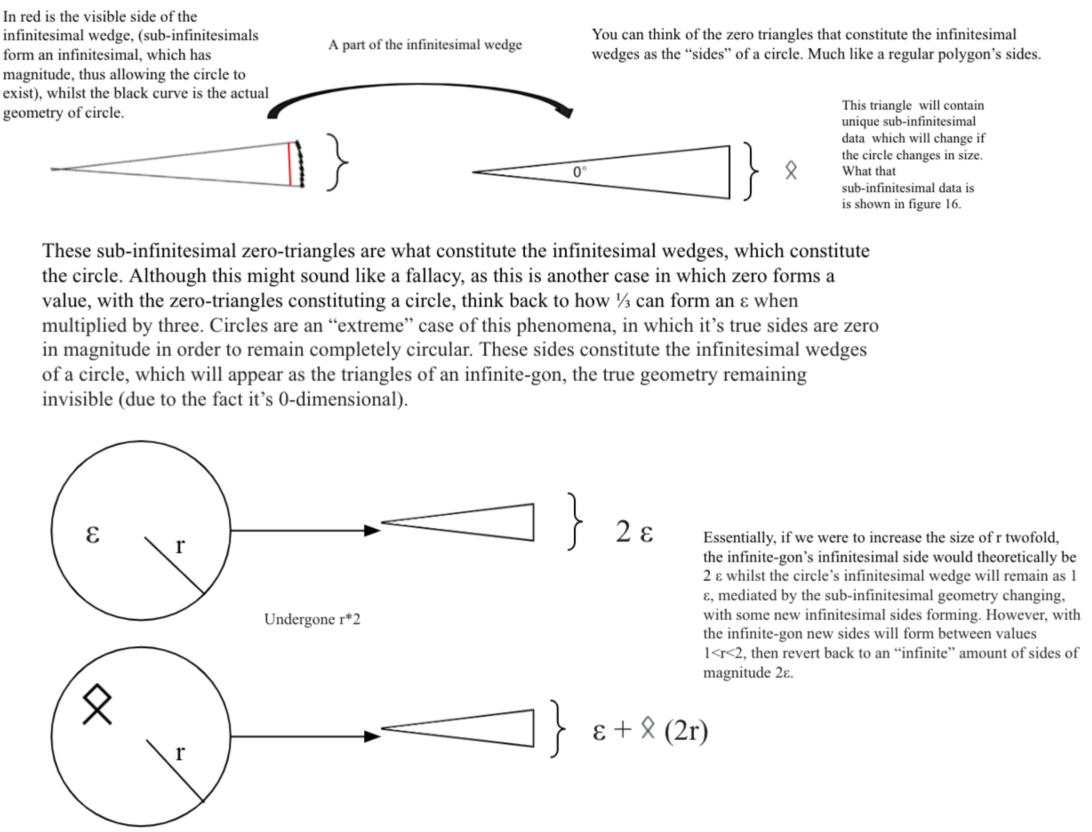


Figure 13: Describing an infinitesimal wedge of a circle and how it is comprised of "zero-triangles"

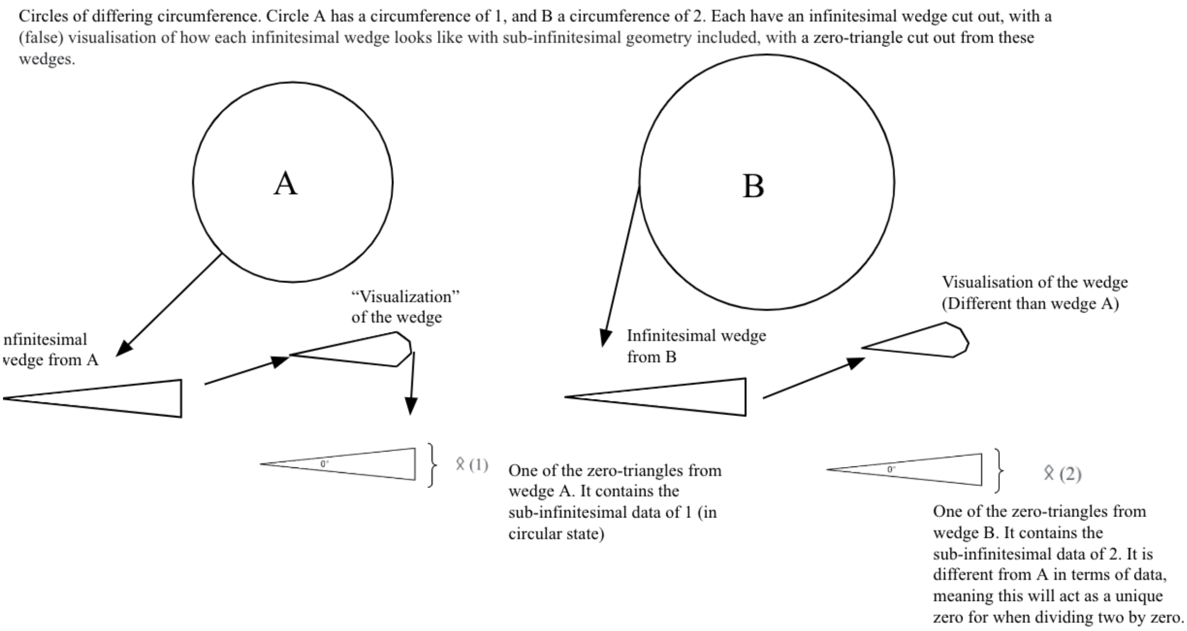


Figure 14: Comparison between the infinitesimal wedges of a circle of circumference 1 and 2.

The main idea behind the sub-infinitesimal geometry is to convey the fact that these two circles with differing circumferences, will have zero-triangles of differing sub-infinitesimal data. The previously established number of sides for a circle was that it is infinite, however that would mean that the larger circle would be "less" circular on account of the fact that the circle of a circumference of 1 would have more sides on a smaller area, even though both are infinite. This is also the beginning of the concept of "specific infinite values" which will be further discussed below. Returning back to the two circles above, although it could be argued that the circle with a larger circumference will contain "more" zero-triangles, we believe it to be more accurate to describe the sub-infinitesimal geometry (data) to be different, however both will share π's sub-infinitesimal data, which will also be explained below as well.

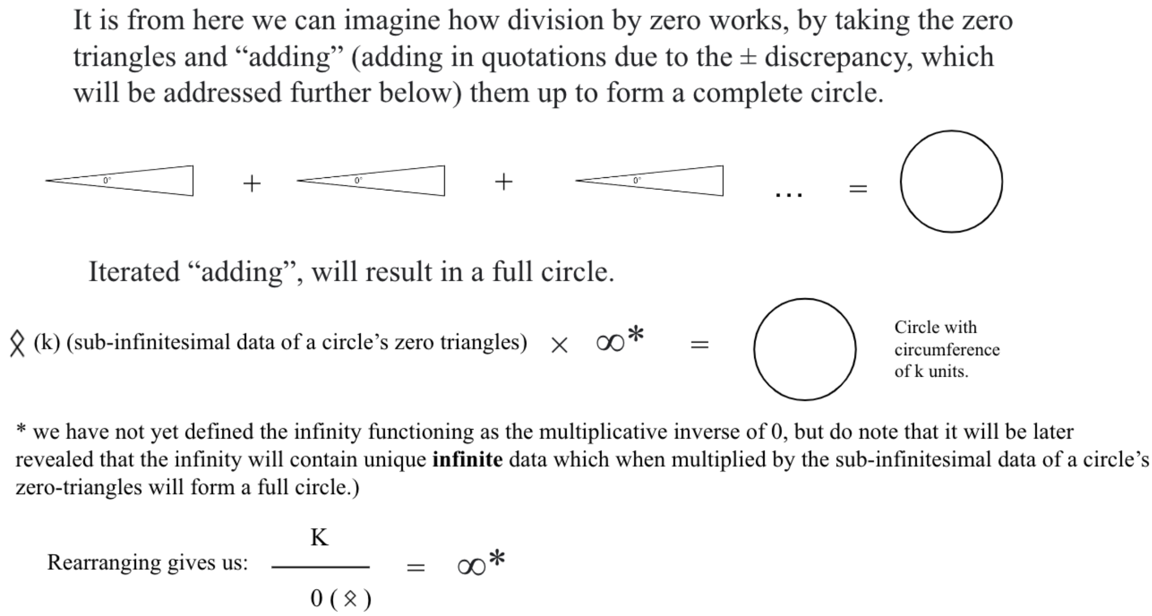


Figure 15: Basic premise of division by zero.

The aforementioned zero-triangles, as seen above, are what we will use in order to determine what some number k divided by 0 would equate to. In theory, all that needs to be done now is to "add" the zero-triangles to a corresponding specific infinite value (shown below), with that specific infinite number of times operating as the multiplicative inverse to that number divided by zero. This is where we must admit that in truth, we do not have the "true" answer to division by zero. Although we are very close, in order to determine any number divided by zero, what we need is the sub-infinitesimal data of π, as we believe it contains the sub-infinitesimal data pertaining to the sub-infinitesimal geometry found in circles, which are the zero-triangles, however currently we have no method to determine what the infinitesimal

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values of π are, let alone the sub-infinitesimal. We don't even know if it is even possible to determine the sub-infinitesimal data of π. There is also the case for how that data manifests itself in other numbers, with the figure below highlighting what we are missing to determine what any number divided by zero is.

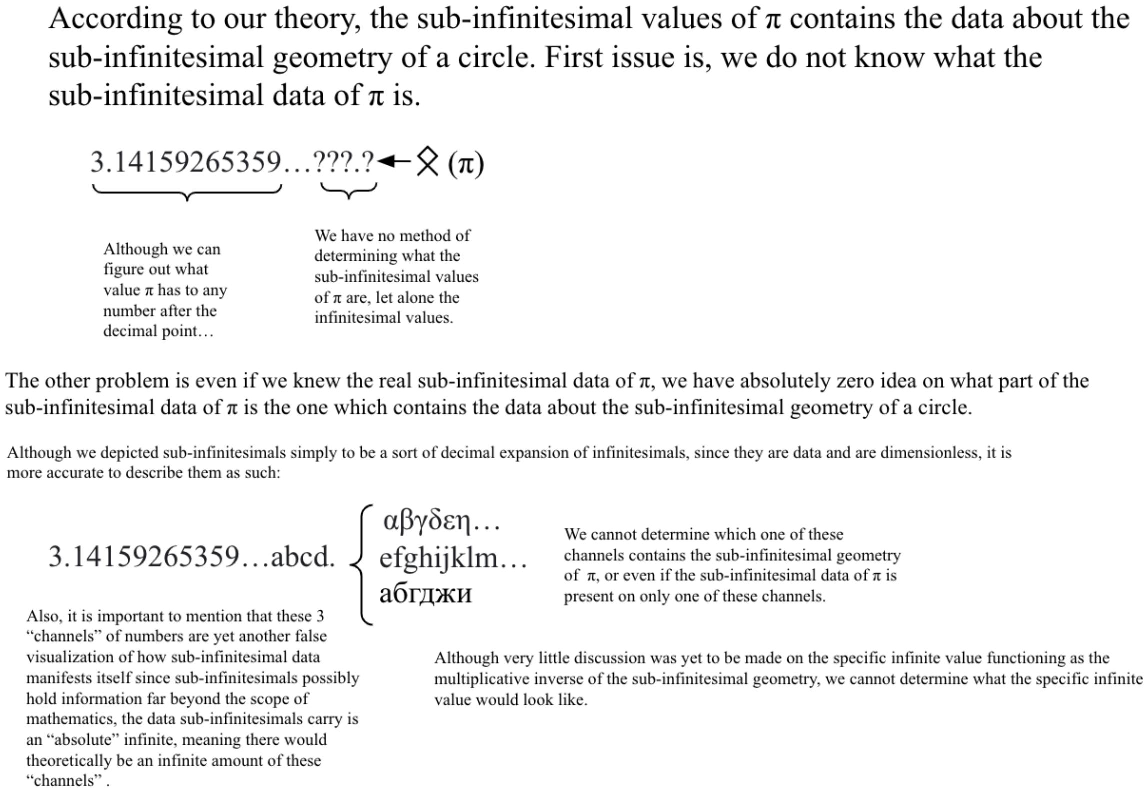


Figure 16: The necessary data we are missing to determine division by zero.

Without that data, we cannot truly determine what division by zero is, which in actuality isn't much of an issue as we can still define division by zero without determining what each specific infinite value is equal to. However, for demonstration purposes, we've come up with the "false system" of division by zero, as a way of defining division by zero without actually determining what it is. We will still define division by zero by the "true system" of division by zero, we will just show what division by zero would look like if we had the data necessary using the false system. The first assumption of dividing by zero using the false system, is that all of π's sub-infinitesimal data pertaining to the geometry of a circle is found in its "real" channel. It is also **arbitrarily** defined as 0.62835182… The second assumption is that numbers in circular form will contain all of the sub-infinitesimal data of π in their real channel, replacing their "regular" real sub-infinitesimal values of 00000… The third assumption is that the specific infinite value is calculated using the sub-infinitesimal data as (k/(for infinite values) fake sub-infinitesimal data of π as a decimal, (for transfinite values) √(fake sub-infinitesimal data), (for finite values) fake sub-infinitesimal data/π, (for the infinitesimal values) as 1/(fake infinitesimal data -1) and sub-infinitesimal data is defined as fake sub-infinitesimal data)*∞

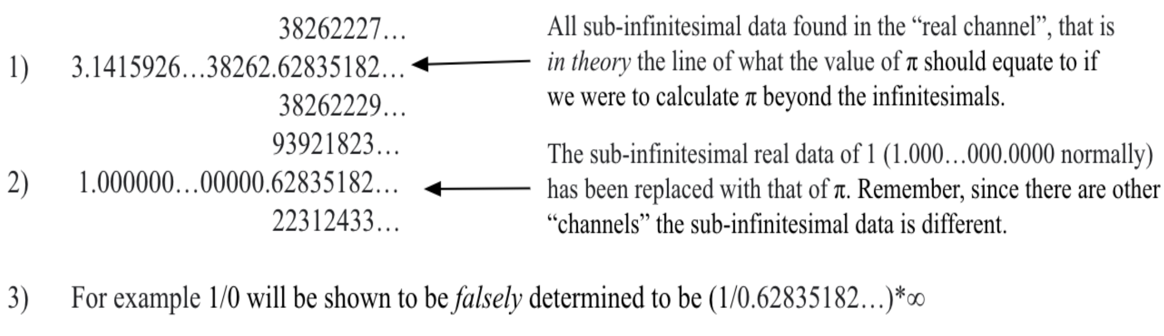
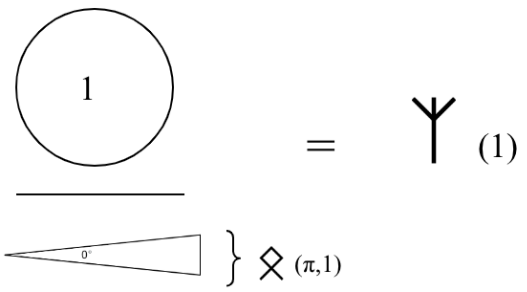


Figure 17: False system of division by zero.

Having established all of that, we can finally define 1/0 as seen below.

1/0 is defined as the following:

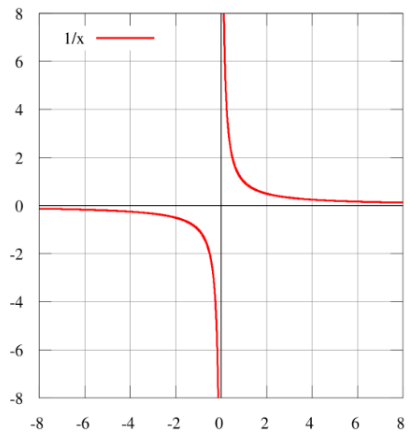
A circle of circumference of 1 being divided by one of it's zero triangles, will equate to the total "amount" of zero triangles in a circle of circumference of 1. That infinite amount is defined as algiz (1). This method allows us to also define 2/0 as well as any number k/0 as the "amount" of zero triangles in a circle of circumference k.



Sub-infinitesimal data of 1 in circular form, (contains some π's sub-infinitesimal data)

Figure 18: The new defined definition of 1/0

And there it is, 1/0. Using the sub-infinitesimal zero-triangles, and "adding" them up with the data present in the specific infinite value of ᛉ(1) will yield a full circle of circumference of 1, meaning we have divided 1 by zero. However, there is still one issue with division by zero that we have not addressed throughout the entire paper and also the reason as to why "amount" is in quotations, and that is the ± discrepancy, since the limit of f(x)=1/x as x approaches zero tends both to a positive and negative infinity. This is a problem, as our explanation of division by zero is the amount of zero-triangles inside a circle, implying it to be positive even though that is not the case. Seen below is an explanation of how it's unsigned, as well as the concept of the specific infinite value more in depth.



Here we have an image of how 1/x tends to both positive and negative infinity as x approaches 0. In regards to our theory, which describes the specific infinite value to be "adding" zero triangles up to form a value, this would imply the zero-triangles to be positive, which is incorrect. In truth, the specific infinite value ᛉ(1) has specific infinite data which when multiplied by zero will cause the values formed to be positive. Below is the explanation in more detail.

Here we can imagine the "adding" process of the zero-triangles for 1/0. The zero-triangle being multiplied by ᛉ (1)

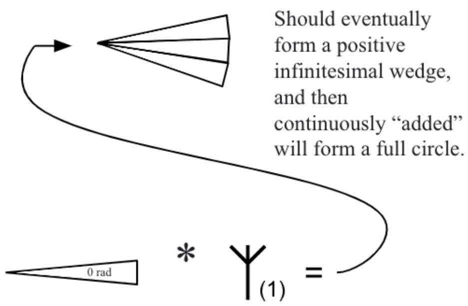


Figure 19: Addressing the ± discrepancy.
Source: (*Function-1 x*, 2006)

However, since the zero-triangles are 0-dimensional, they are **not positive or negative** in magnitude, since they possess no magnitude. The data of ᛉ (1) is "reacting" to the sub-infinitesimal data of 1 to cause a 1 to form. Below is more explanation on how ᛉ (1) reacts to ᛟ (1).

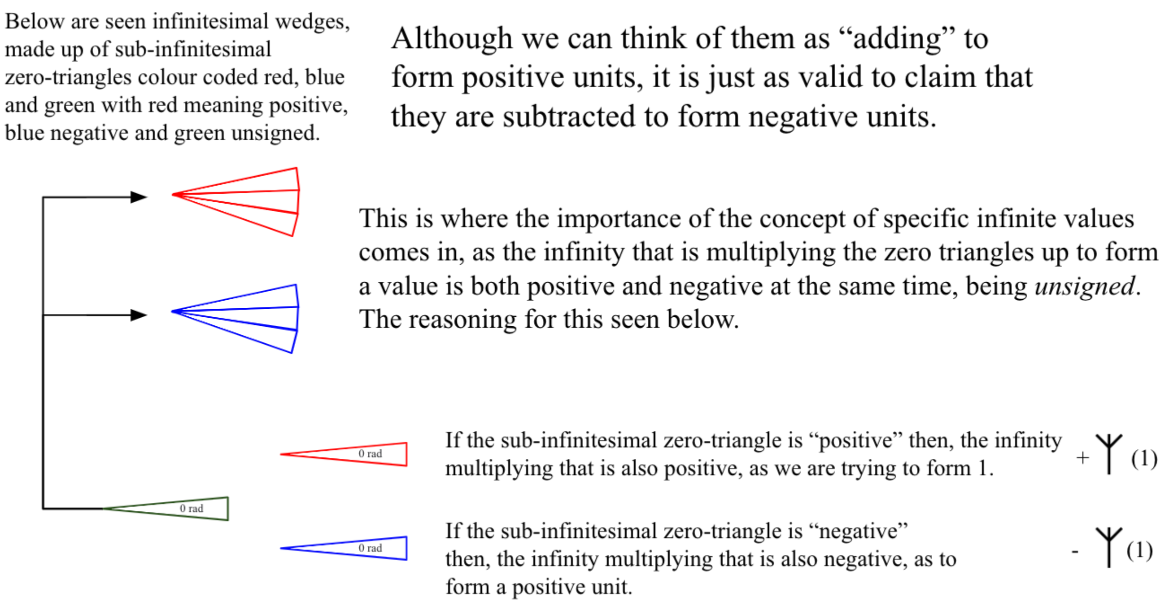


Figure 20: ± discrepancy extended.

In reality however, the zero triangles are neither positive nor negative, the infinite value ᛉ(1) contains specific infinite data that will ensure that the zero-triangles add up to form **positive** infinitesimal wedges. The difference between the multiplicative inverse of 1/0 and -1/0 is seen in the next figure.

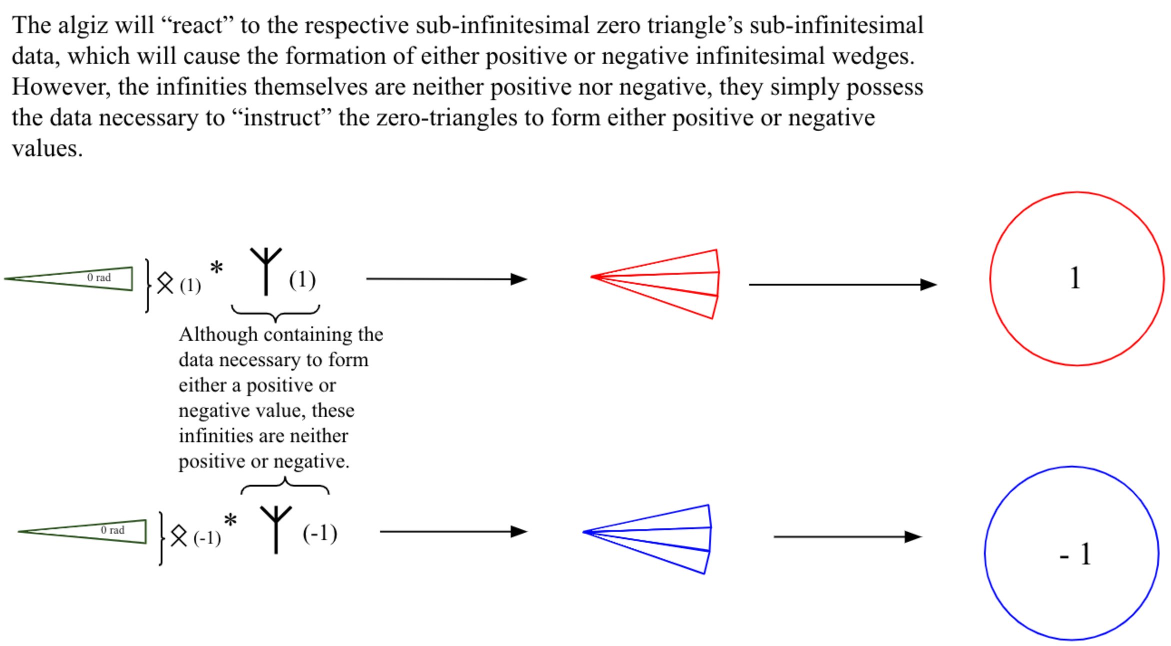
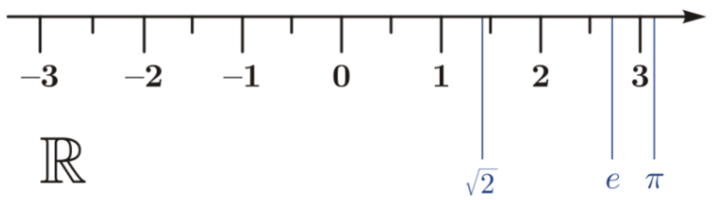


Figure 21: Specific infinite data of ᛉ causing the formation of either positive or negative units.

It is from here, now having defined division by zero we talk about how the ᛉ really *functions* as a multiplicative inverse to 1/0. Here we finally discuss the idea of specific infinite values, and what they mean in the context of sub-infinitesimals. The concept of specific infinite values mirrors many of the same concepts of sub-infinitesimal data, mostly as a means of differentiating identical numbers from each other. Although the difference between identical finite numbers is usually determined by sub-infinitesimal data and nothing more, infinite "numbers" are not really defined numerically, they are more defined contextually.

The concept of specific infinite values arises from the fact that the concept of infinity manifests itself in very different ways, for example, let’s take the amount of natural numbers to that of the amount of real numbers. Although it has been proven that there are more real numbers than natural numbers (Meyer, 2021), both infinities are infinite, so shouldn’t they be equal in magnitude?

Same amount of numbers due to the fact that they’re both infinite?



However in reality, not only is it illogical to claim that there are the same amount natural numbers than real numbers, it has been proven that there are more real numbers than natural numbers. However, both are still infinite, so how can one really be bigger than the other?

Figure 22: Infinities equating to each other.
Source: Natural Numbers (*Natural Numbers Key Stage 2*, n.d.) (*Real Number Line*, 2005)

This is where we explain in more depth the concept of specific infinite values, and how using it allows us to show how one infinity is “bigger” than another in numerical form. Consider these two following infinite numbers, 1000000… and 9999999… One is a sequence of 1 followed by an infinite amount of 0’s whilst the other is an endless sequence of 9’s. Although both are infinite, they differ in appearance. Does this mean they are equal, despite one seemingly being larger than the other? The answer to that question lies within our theory on specific infinite values. We believe that although all infinities are all equal in magnitude, each can carry different “infinite” data, with some possessing more data than others, resulting in “bigger” infinites. Returning back to our natural infinity and real infinity, instead of arguing that one is larger than the other, we describe it as possessing more data than the other. If we were to show it in numerical form, one would possess more data than the other.

From that, we can see how 1/0 is not any “larger” than 2/0, moreso the data that 2/0 is different than 1, causing a circle of circumference of 2 to form instead of a circumference of 1.

Using the false system (rule 3 of false system), we determine ϒ(1) to be:

159146511265…79268645756…2000105963.62835182…97402546158.62835182…

Infinite data values | Transfinite data values | Finite and decimal values. | Infinitesimal values and sub-infinitesimal data.

ϒ(2) for example, according to the false system is equal to:

318293022531…158537291513…4000211927.125670364…194805092317.125670364

Infinite data values | Transfinite data values | Finite and decimal values. | Infinitesimal values and sub-infinitesimal data.

Figure 23: The specific infinite data of ϒ(1) and ϒ(2).

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These specific infinite values, although being equal to one another in terms of magnitude, possess different data, making it so when multiplied by zero they will result in different solutions, those being 1 and 2 respectively. It is important to note that this specific infinite data also "instructs" the 0 to morph into the sub-infinitesimal data needed to undergo multiplication.

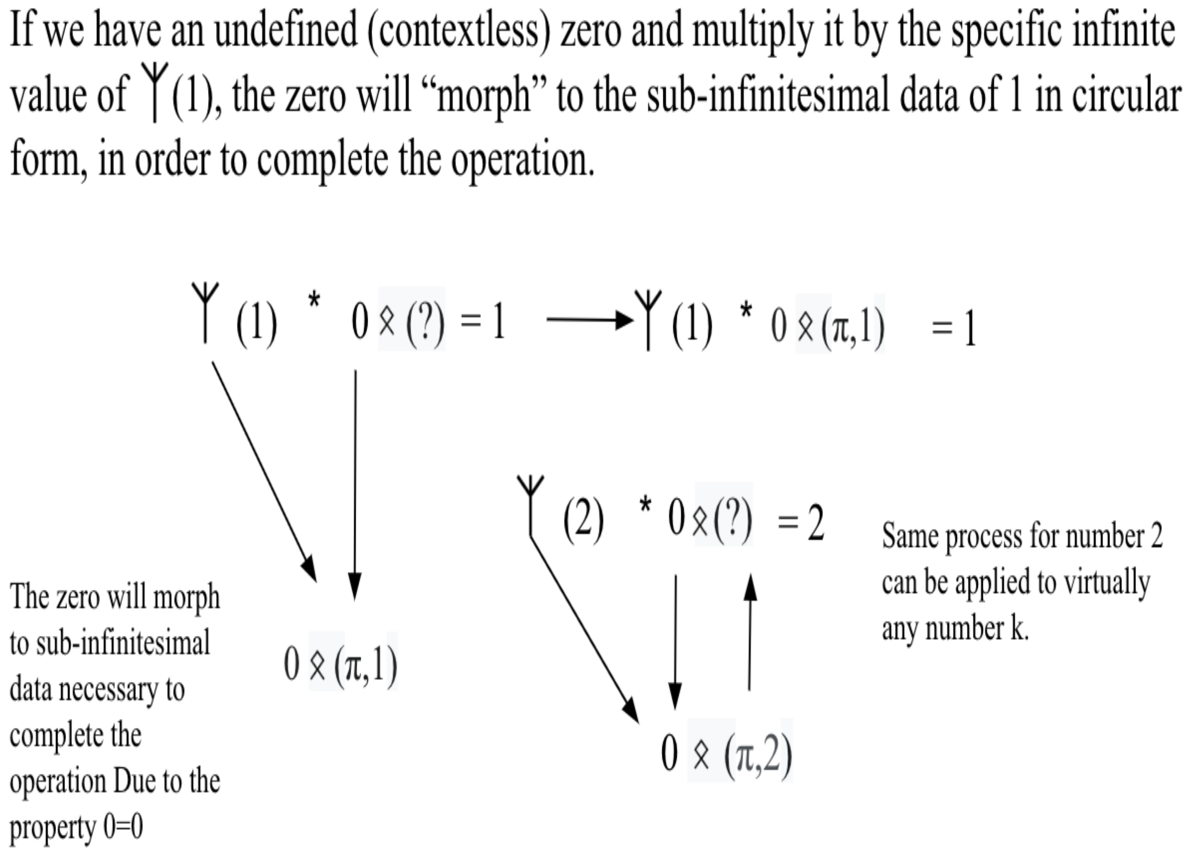

Figure 24: Sub-infinitesimal data morphing in presence of specific infinite data.

It is here, after having established that division by zero is possible for virtually any number, that we finally discuss two more numbers, those being 0 and ε. These two numbers are unique compared to the other numbers as we cannot apply our theory on division by zero to these two. Although we will attempt to define these two, it must be noted that this part of the paper is written much more tentatively than the rest of the paper, so do not take the following as definitive proof to these numbers divided by zero, as our method of dividing by zero cannot be applied to these numbers.

The idea of defining 0/0 is different to that of defining division by zero, because unlike division by zero which is still considered as undefined, some consider 0/0 to be indeterminate, and that is not without reason, as algebraically, attempting to solve this problem yields any solution as correct. Below we shall attempt to solve 0/0, by equating 0/0 to be equal to some number k:

$$0/0=k$$
$$0=0*k$$
$$0=0.$$

The result we get is that 0=0, which is "true" in the sense that the equation "works", however, if we replace k with any number, such as 1, 2 or even 100, the outcome would be the same, and we'd get 0/0=1 and 0/0=2 and even 0/0=100, which would mean that 1=2 and even 1=100, which is false. Although 0/0 can be equal to any number k (except for certain transfinites and infinities, as that would result in the 0 multiplying to a value other than 0), explicitly defining 0/0 to be equal to any number would run us into contradictions. Thus, if we cannot define 0/0 to be a value, what can we define it as? In calculus, the concept of the differentiation is close to that of 0/0, as derivatives are an instantaneous rate of change defined as $\text{Lim}_{h\to 0}((f(x+h)-f(x))/h)$. As h approaches 0, we get the instantaneous rate of change of the function. If we plug in 0 in h we would get 0/0, so can we define 0/0 to be the process of differentiating? Well, not exactly as h does not ever *reach* zero, so it cannot be defined as 0/0, since it is more akin to a infinitesimal ratio between y and x, so we cannot define 0/0 to be differentiation and we have not found a way of explicitly defining 0/0. However, we have found a way to explain how 0/0 can equate any k without contradiction, and it is seen below.

Below we see 0/0, and we will equate it to 5. To the right, we'll equate it to 4

$$\frac{0}{0} = 5 \qquad\qquad \frac{0}{0} = 4$$

Although both equations are technically right, 4≠5, however, the 0/0's sub-infinitesimal data does "allow" for 0/0 to equal to 5 and 4, as seen below. (keep in mind, it's not really 0.5 ε being that's being divided by 0.1 ε, it is more some "certain" sub-infinitesimal data divided by some other allowing for 0/0 to equate to a number, in this case, 4 and 5.

$$\frac{\text{ᛟ}\ (0.5\ \varepsilon)}{\text{ᛟ}\ (0.1\ \varepsilon)} = 5 \qquad\qquad \frac{\text{ᛟ}\ (0.4\ \varepsilon)}{\text{ᛟ}\ (0.1\ \varepsilon)} = 4$$

Think back to the concept of identification data, how different numbers will contain different sub-infinitesimal data depending on how they were obtained (such as 5-4 and e^0 both resulting in 1). 0/0 is one of the instances where such data does have a significant impact on the outcome of the solution, as the 4 and 5 (and any number) will have unique sub-infinitesimal data describing them as the quotients of 0/0.

$$\frac{0}{0} = 5\ \text{ᛟ}\ (0/0) \qquad\qquad \frac{0}{0} = 4\ \text{ᛟ}\ (0/0)$$

Figure 25: Describing 0/0 in terms of sub-infinitesimal data.

Meaning if you were to equate the two, we would get the following:

If 0/0 is true then 5=4, which is a logical fallacy. However, due to the fact that 5 and 4 are quotients to 0/0, the 5 or 4 will seemingly "morph" to either 5 or 4 to satisfy the equation.

$$\frac{0}{0} = \frac{0}{0}$$

$$5\ \text{ᛟ}\ (0/0) = 4\ \text{ᛟ}\ (0/0) \longrightarrow 5\ \text{ᛟ}\ (0/0) = 0/0 \longrightarrow 5\ \text{ᛟ}\ (0/0) = 5\ \text{ᛟ}\ (4,0/0)$$

4 or 5 will redefine itself as 0/0, and then define itself as 5 or 4 to satisfy the equation. This process also occurs vice-versa and will also give a 4=4 with sub-infinitesimal data of (5,0/0).

Figure 26: Numbers redefining themselves due to their sub-infinitesimal data.

The above method of "defining" 0/0 with the use of sub-infinitesimals is not one that is very useful, and only really serves as an explanation as to why 0/0 can equate to any number k without causing any fallacies. This leads us to our next number, that being ε/0. ε/0 is an interesting case as although we know that it is possible for 0 to form values in certain cases, explicitly defining ε/0 to equal to some value is more difficult, on account of the fact that we cannot make use of the same method of dividing by zero on infinitesimals than for finite numbers. We have shown that depending on the sub-infinitesimal data of 0, ε/0 can be defined as practically any number, which is not a very useful solution. In the case of ⅓ , we saw how in that case, 3*0=ε, but by that logic, ε/0=3 which can lead to inconsistencies. Thus, would we resolve such a problem? We believe that although 0*k can equate to ε cannot be defined as ε/0≠k, with k being some finite number. Our reasoning for this is due to the fact that the only way to divide by zero is by making use of the sub-infinitesimal data of π, which would result in an unsigned transfinite value with specific **transfinite** data resulting in ε if multiplied by 0, functioning as it's multiplicative inverse. Although it is possible to multiply zero by a finite value to equal some infinitesimal value, those cases are anomalous themselves, requiring an infinitesimal to be present. The reasoning for this is that finite values do not contain the data necessary to form values under "normal" conditions, while transfinites do consistently contain the data necessary to form values, they are not infinite on account of the fact that infinite values would form finite values from 0.

CONCLUSION

It is here where we conclude our theory on division by zero as well as our theory on sub-infinitesimals. In this paper we have found a method of defining division by zero in a manner that does not cause algebraic inconsistencies, and that is thanks to the "discovery" of sub-infinitesimals as well as the sub-infinitesimal "interpretation" of 0-dimensional space from which the sub-infinitesimal data of π is used to define the "true" sub-infinitesimal geometry of a circle, allowing us to logically conclude that by multiplying the sub-infinitesimal "zero-triangles" with a specific infinite value containing unique infinite data, defined as Ɏ (1), will result in values formed from "nothingness", leading to a proper multiplicative inverse to 1/0, as well as any non-zero k/0.

REFERENCES

[1]. Vsauce. (2015, December 8). Supertasks [Video]. YouTube. https://www.youtube.com/watch?v=ffUnNaQTfZE
[2]. How to Divide by Zero. (2020, November 25). 1 Divided by 0. https://www.1dividedby0.com/
[3]. Riemann sphere. (2022, July 9). In Wikipedia. https://en.wikipedia.org/wiki/Riemann_sphere
[4]. Carlström, J. (2001, September). Wheels — On Division by Zero (No. 11). Department of Mathematics Stockholm University. https://www2.math.su.se/reports/2001/11/2001-11.pdf
[5]. Britannica, T. Editors of Encyclopaedia (2016, October 17). infinitesimal. Encyclopedia Britannica. https://www.britannica.com/science/infinitesimal
[6]. Números hiperreales. (2005, February 24). [Illustration]. https://en.wikipedia.org/wiki/Infinitesimal#/media/File:N%C3%BAmeros_hiperreales.png
[7]. Smooth infinitesimal analysis. (2022, March 13). In Wikipedia. https://en.wikipedia.org/wiki/Smooth_infinitesimal_analysis
[8]. Infinitesimal. (2022, September 27). In Wikipedia. https://en.wikipedia.org/wiki/Infinitesimal
[9]. Why can't you divide by zero? - TED-Ed. (2018, April 23). [Video]. YouTube. https://www.youtube.com/watch?v=NKmGVE85GUU
[10]. Function-1 x. (2006, May 26). [Graph]. Function-1 x. https://commons.wikimedia.org/wiki/File:Function-1_x.svg
[11]. Meyer, J. (2021, September 9). Proof of more Real numbers than Natural numbers. Logic and Language. Retrieved August 3, 2022, from https://www.jamesrmeyer.com/infinite/proof-more-reals-than-naturals.html
[12]. Natural Numbers Key Stage 2. (n.d.). [Graph]. https://www.mathematics-monster.com/images2/natural_numbers_on_number_line.jpg
[13]. Real number line. (2005, September 19). [Graph]. https://upload.wikimedia.org/wikipedia/commons/d/d7/Real_number_line.svg