

A fuzzy multi-objectives model based on decision tree analysis: Ship replacement decision optimisation

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ABSTRACT

The optimisation problem for ship replacement decisions involves developing a practical and applicable fuzzy multi-objective mathematical model. To formulate the mathematical model, the decision is reformulated by a decision tree, leading to the derivation of the mathematical model. The concept of domination is used to achieve multi-objectives for ship replacement decisions. In this process, the fuzzy sets method plays a crucial role in determining the coefficients' range due to uncertainty in the future, thereby enhancing the model's adaptability and robustness.

INTRODUCTION

Effective maintenance management is crucial for all organisations, particularly those that involve intensive use of ships. Achieving excellence in this area means that ships perform up to their design standards, maintenance costs are monitored and budgeted, service levels remain high, and maintenance personnel are skilled and motivated. A key element of effective maintenance management is the optimisation of maintenance decisions[1]. Mathematical models to optimise ship replacement decisions evaluate costs or profits over finite or infinite planning horizons[2]. However, these models primarily serve as scrap models, focusing on minimising waste rather than improving organisational objectives[3], [4]. These mathematical models need to be improved. They typically optimise replacement decisions based on a single objective and fail to account for potential changes in objective coefficients over time[5]. The fuzzy multi-objective model to optimise the ship replacement decision addresses these limitations; it utilises the concept of domination and the fuzzy sets theory. The concept of domination is used to determine the efficient solution that achieves the ship replacement decision objectives, and the fuzzy sets theory functions similarly to multivariable sensitivity analysis and assesses the range of objective coefficients considering future uncertainties[6].

REFORMULATION

The ship replacement decision is reformulated in a convenient form for analysis[7]. The decision tree support tool is excellent for communicating decision optimisation perception and gives a clear view of the analysis [8].

Decision variables

Table 1: Extended form

$$\sum_K X \sum_K^R X \sum_K^R X \sum_K^R X \sum_K^R X$$

1	K	K	K	K	K	9	K	R	K	K	K
2	K	K	K	K	R	10	K	R	K	K	R
3	K	K	K	R	K	11	K	R	K	R	K
4	K	K	K	R	R	12	K	R	K	R	R
5	K	K	R	K	K	13	K	R	R	K	K
6	K	K	R	K	R	14	K	R	R	K	R
7	K	K	R	R	K	15	K	R	R	R	K
8	K	K	R	R	R	16	K	R	R	R	R

To analyse the ship replacement decision, the compact form is extended for the decision variables of the ship replacement decision and represented by a decision tree[9].The decision variables analysed during finite horizon donatean example of a ship's service life[10].The service life refers to the duration theship is used before it is retired.

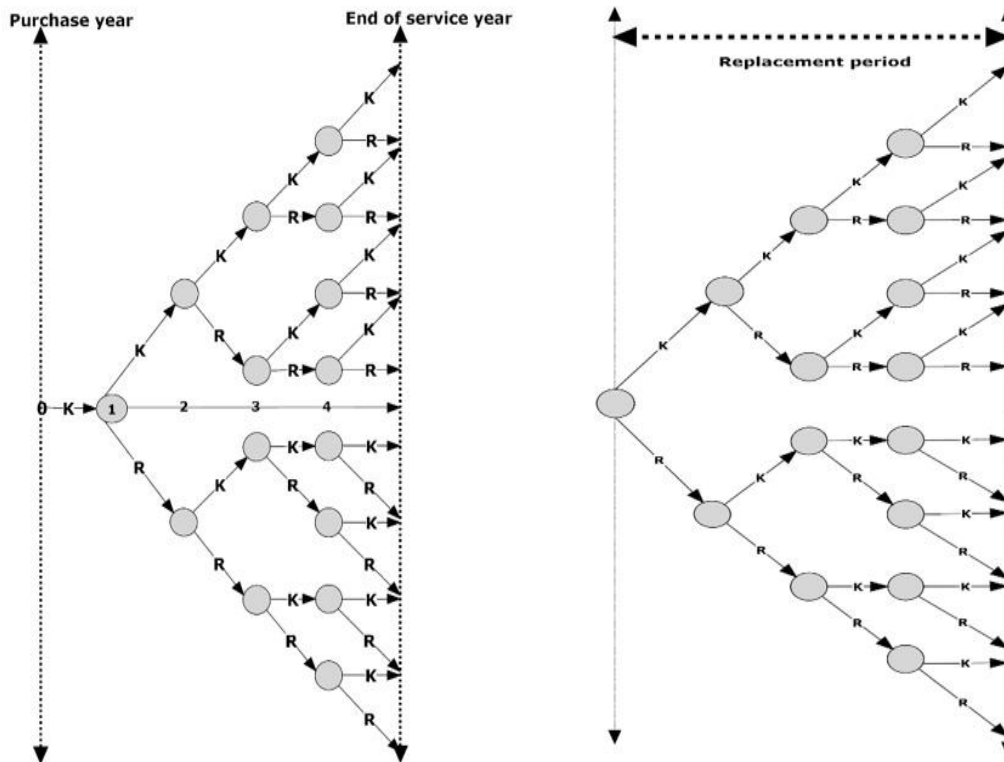


Figure 1: Sequence of decisions and replacement period

The nodes in the sequences of decisions represent replacement years. The arrows (K) and (R) represent the decisions to keep and replace the ship with a new one. The shipreplacement period is part of a sequence of decisions; the replacement period begins one year after purchasing the ship.In the replacement period, the number of decisions is increasing by (n^2) , and there are two main types of sequences of decisions: sequences of decisions that start with the decision (K) and sequences of decisions that start with the decision (R). The sequences of decisions that start with the decision (K) and those that start with the decision (R) are arranged in Table 2.

Table 2: The main types of sequences of decisions

	Sequences of decisions start with (K)					Sequences of decisions start with (R)			
	1	2	3	4		1	2	3	4
1	K	K	K	K	9	R	K	K	K
2	K	K	K	R	10	R	K	K	R
3	K	K	R	K	11	R	K	R	K
4	K	K	R	R	12	R	K	R	R
5	K	R	K	K	13	R	R	K	K
6	K	R	K	R	14	R	R	K	R
7	K	R	R	K	15	R	R	R	K
8	K	R	R	R	16	R	R	R	R

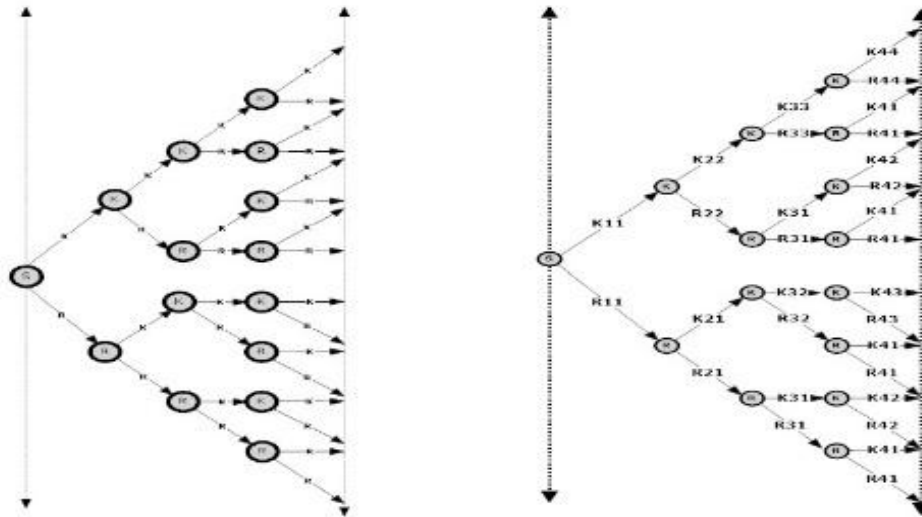


Figure 2: Type of nodes and replacement period with (I) and (J)

There are three types of nodes in the replacement period: (S) source, (K) keep, and (R) replace nodes. Figure 3 shows the type of nodes in the replacement period. The source node is the first node in the replacement period. The decisions that exit from the source node are (K) and (R); for both decisions, the replacement period year (I) is equal to one, and the age that enters the replacement period (J) is equal to one as well. Keep nodes are the nodes that the decision (K) enters. The decisions that exit from the keep nodes are (K) and (R); for both decisions, the replacement period year (I) is increasing by (I+1), and the age from entering the replacement period is increasing by (J+1). Replace nodes are the nodes that the decision (R) enters. The decisions that exit from the replace nodes are (K) and (R); for both decisions, the replacement period year (I) is increasing by (I+1), and the age from entering the replacement period is one. Figure 4 shows the addition of (I) and (J) to the decisions within the replacement period.

Objectives

To optimise the ship replacement decision based on multi-objectives, the concept of domination is used to determine the efficient solution that achieves multi-objectives[11].

Concept of domination

For optimising the ship replacement decision, maximising profit and availability are critical economic and operational objectives [12]. Furthermore, minimising emissions is an essential environmental objective, given the significant impact ship emissions have on the environment and public health [13]. To determine the efficient solution that achieves the multi-objectives for the ship replacement decision by the concept of domination, the values of the objectives need to be calculated separately and compared[14].

Objectives calculation

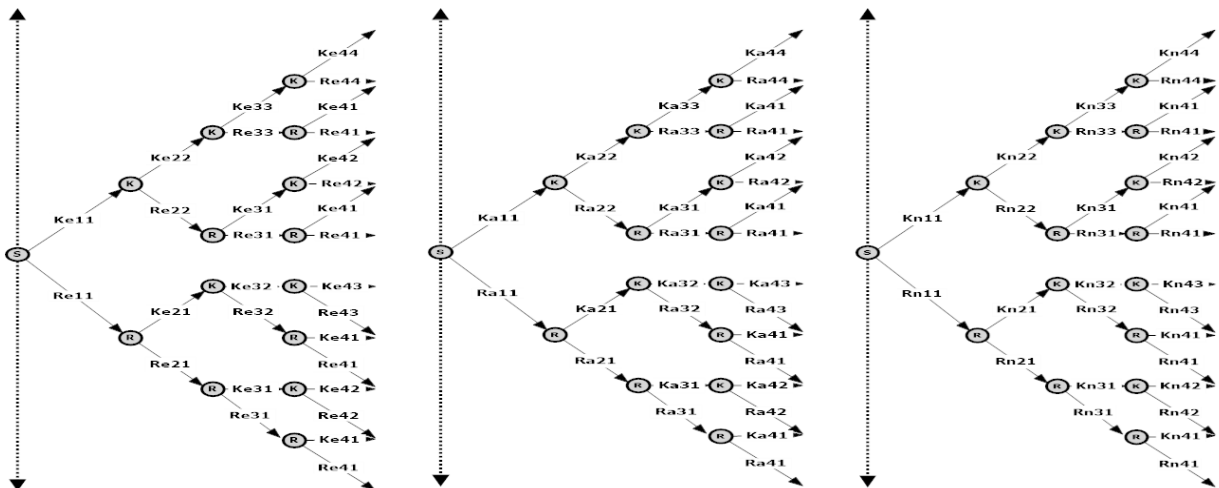


Figure 3: Objectives coefficients

The coefficients (n), (a), and (e) are linked to the sequence decisions in the replacement period to calculate the values of the objective. The net profit coefficients (n) for the decisions (K) are calculated by subtracting the ship cost from its revenue at different ages [15]. Revenue varies throughout the ship's lifespan and correlates with age [16]. The cost is a result of keeping the ship, including direct and indirect costs [17]. The net profit coefficients (n) for the decisions (R) are calculated by subtracting the replacement cost from the net profit for the new ship at age one [18]. The replacement cost is calculated by subtracting the new ship price from the resale value [19]. The net profit for the new ship is calculated by subtracting the new ship's cost from its revenue. The various values for the net profit objective are computed by calculating the values of sequences of decisions (N). In table (3), the net profit coefficients (n) are arranged according to the decisions years (I), age from entering the network (J), and types (K, R).

Table 3: Net profit coefficients

Net profit coefficients for the decisions (K)					Net profit coefficients for the decisions (R)				
I/J	1	2	3	4	I/J	1	2	3	4
1	n11				1	n11			
2	n21	n22			2	n21	n22		
3	n31	n32	n33		3	n31	n32	n33	
4	n41	n42	n43	n44	4	n41	n42	n43	n44
Values	N3	N5	N9	N1		N4	N6	N10	N2
	N7	N13				N8	N14		
	N11					N12			
	N15					N16			

The availability coefficients (a) for the decisions (K) result from the ship's availability in service, computed by dividing the uptime on the operating cycle at different ages [20]. The availability coefficients (a) to the decisions (R) result from the new ship availability and can be calculated by dividing the uptime on the operating cycle at age one. The various values for the availability objective are computed by calculating the values of sequences of decisions (A). In table (4), the availability coefficients (a) are arranged according to the decisions years (I), age from entering the network (J), and type (K, R).

Table 4: Availability coefficients

Availability coefficients for the decisions (K)					Availability coefficients for the decisions (R)				
I/J	1	2	3	4	I/J	1	2	3	4
1	a11				1	a11			
2	a21	a22			2	a21	a22		
3	a31	a32	a33		3	a31	a32	a33	
4	a41	a42	a43	a44	4	a41	a42	a43	a44
Values	A3	A5	A9	A1		A4	A6	A10	A2
	A7	A13				A8	A14		
	A11					A12			
	A15					A16			

The fuel consumption used to calculate the emissions coefficients (e) to the decisions (K) and (R) [21]. The fuel consumption coefficients for the decisions (K) are calculated by (L/100KM) at different ages. The fuel consumption coefficients for the decisions (R) are calculated by (L/100KM) at age one. The various values for the emissions objective are computed by calculating the values of sequences of decisions (E). In table (5), the emissions coefficients (e) are arranged according to the decisions years (I), age from entering the network (J), and type (K, R).

Table 5: Emission coefficients

Emissions coefficients for the decisions (K)					Emissions coefficients for the decisions (R)				
I/J	1	2	3	4	I/J	1	2	3	4
1	e11				1	e11			
2	e21	e22			2	e21	e22		
3	e31	e32	e33		3	e31	e32	e33	
4	e41	e42	e43	e44	4	e41	e42	e43	e44
Values	E3	E5	E9	E1		E4	E6	E10	E2
	E7	E13				E8	E14		
	E11					E12			
	E15					E16			

Objectives comparison

To determine the efficient solution that achieves the multi-objectives of the ship’s replacement decision, the objectives’ values are arranged according to the number of sequences of decisions to be compared.

Table 6: Objectives value

No.	Net profit	Availability	Emissions
1	N1	A1	E1
2	N2	A2	E2
3	N3	A3	E3
4	N4	A4	E4
5	N5	A5	E5
6	N6	A6	E6
7	N7	A7	E7
8	N8	A8	E8
9	N9	A9	E9
10	N10	A10	E10
11	N11	A11	E11
12	N12	A12	E12
13	N13	A13	E13
14	N14	A14	E14
15	N15	A15	E15
16	N16	A16	E16

A sequence of decisions (x) is said to dominate another sequence of decisions (y) if both conditions are true; the values of the sequence of decisions (x) are no worse than the values of the sequence of decisions (y) in all objectives, and the value of the sequence of decisions (x) is better than the value of the sequence of decisions (y) in at least one objective. If one of the conditions is violated, then the sequences of decisions (x) do not dominate the sequences of decisions (y)[11].

COEFFICIENTS

To consider potential changes in objective coefficients over time, the fuzzy sets method is used to determine the coefficients range due to uncertainty in the future [22].

Fuzzy sets

A fuzzy set of coefficients is a class of coefficients in which there is no sharp boundary between those coefficients that belong to the class and those that do not[23]. The upper and lower bounds of the fuzzy set need to be calculated to obtain the class's coefficients. The lower and upper bounds are computed from the maximum and minimum parameters. By calculating the lower and upper bounds, the coefficients that belong to the class can be obtained, and the fuzzy coefficients range can be determined at a membership function equal to one.

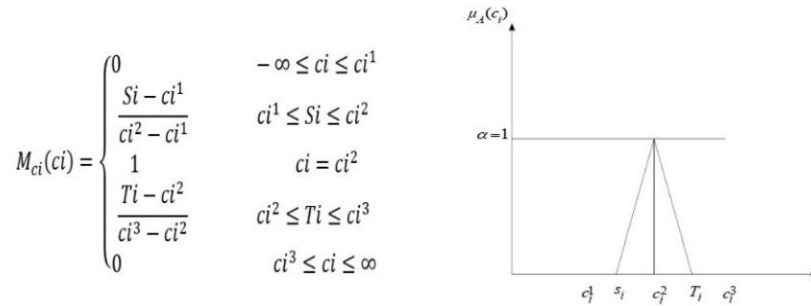


Figure 3: Coefficients range

Deriving

To derive a fuzzy multi-objectives mathematical model, the model's objective function and constraints need to be defined[24]. The decision typedistinguishes the sequences of decisions to define the objective function. The replacement period network is considered a transportation network with one source and multiple destinations to define the constraints.

Objective function

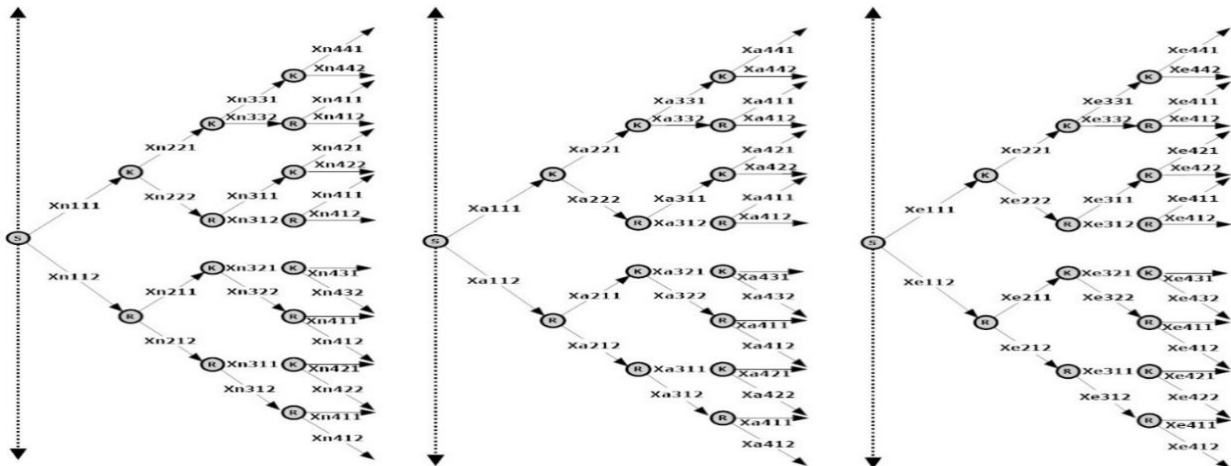


Figure 4: Objectives coefficients distinguished with (p)

To define the objective function, the sequences of decisions are distinguished by the decision type (p). Decision type equals one for (K) and two for (R). The objective function is formulated as a multi-objective function to determine the sequence of decisions that achieves the multi-objectives to the ship's replacement decision. The first objective represents maximising the net profit, and is denoted by:

$$\tilde{f} = \sum_{i=1}^a \sum_{j=1}^i \sum_{p=1}^2 x_{ijp} n_{ijp}$$

Where n_{ijp} are fuzzy coefficients for the net profit, assuming that n_{ijp} are fuzzy numbers whose membership functions $\mu_{n_{ijp}}$. Where:

- a*: actual replacement practice period
- i*: the decisions network years
- j*: the decisions age from entering the network
- p*: decision type
- 1: for decision (K)
- 2: for decision (R)
- n*: net profit coefficient

The second objective represents maximising the availability, and it is denoted by:

$$\tilde{f} = \sum_{i=1}^a \sum_{j=1}^i \sum_{p=1}^2 x_{ijp} a_{ijp}$$

Where a_{ijp} are fuzzy coefficients for the availability, assuming that a_{ijp} are fuzzy numbers whose membership functions $\mu_{a_{ijp}}$.

The third objective represents minimising the emissions, and it is denoted by:

$$\tilde{f} = \sum_{i=1}^a \sum_{j=1}^i \sum_{p=1}^2 x_{ijp} e_{ijp}$$

Where e_{ijp} are fuzzy coefficients for the emissions, assuming that e_{ijp} are fuzzy numbers whose membership functions $\mu_{e_{ijp}}$.

Constraints

The identical decisions merged, and the main sequences of decisions combined to allow for fewer constraints.

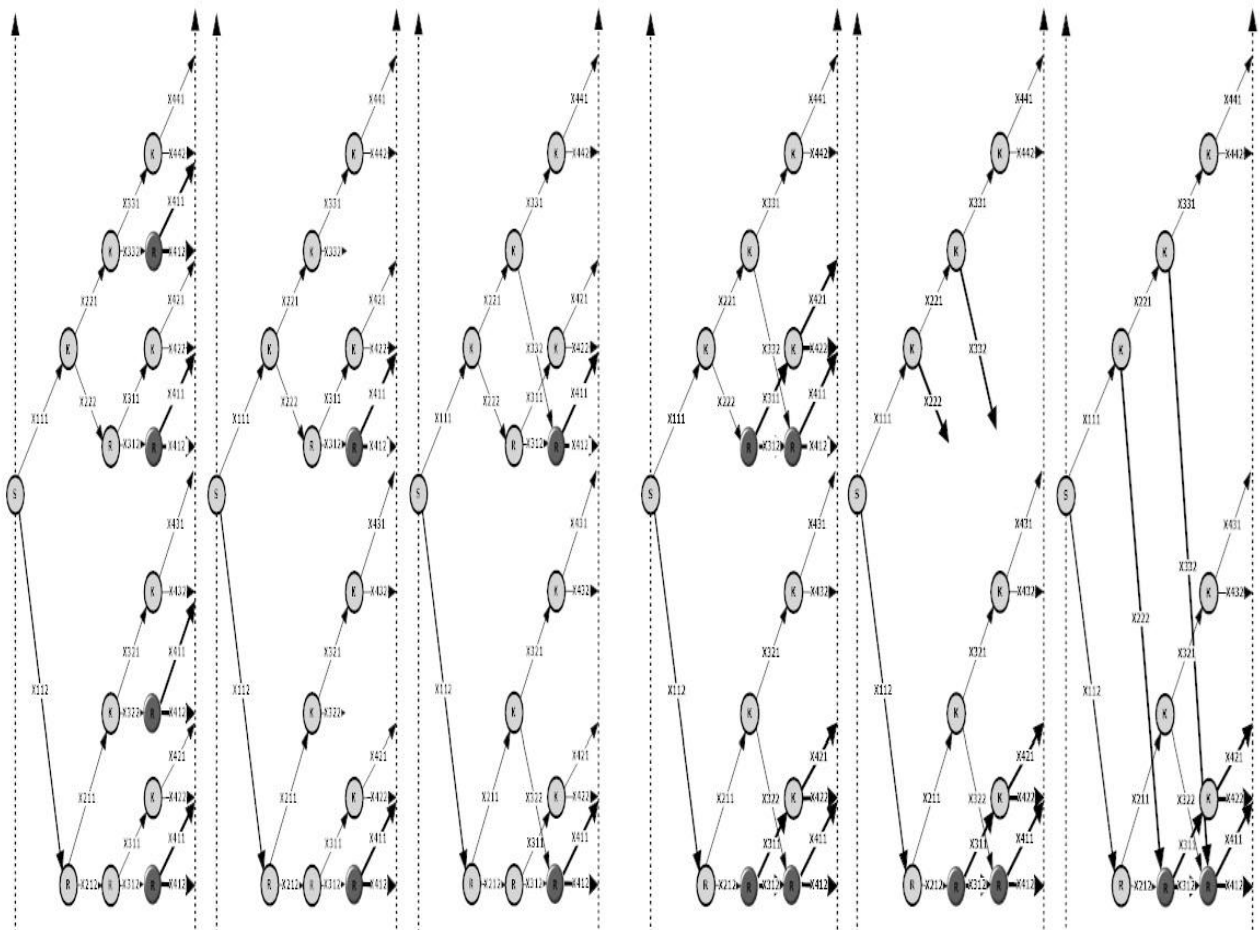


Figure 5: Merging and combining the sequences of decisions

To define the constraints, the replacement period network is considered a transportation network with one source and multiple destinations. The supply at the source node is one unit, and the demand at each destination is one unit. The one unit flows from the source node to the destinations through the admissible sequences of decisions that contain only one replacement decision. The decisions will equal one if in the sequence of decisions that achieve the multiple objectives and will equal zero in all other sequences of decisions.

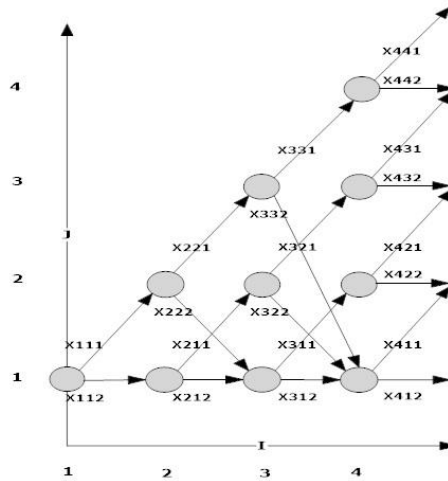


Figure 6: Transportation network

The first constraint produces one unit at the source node.

Equation 1: The one unit

$$\sum_{p=1}^2 x_{ij} = 1$$

The second and third constraints apply the fundamental equation that governs flows in networks (conservation of flow) to allow one unit to flow through sequences of decisions.

Equation 2: For (K) nodes

$$X_{ij1} - \sum_{p=1}^2 X_{(i+1)(j+1)p} = 0, \forall (K) \text{ node}, j \leq i$$

Equation 3: For (R) nodes

$$\sum X_{ij2} - \sum_{p=1}^2 X_{(i+1)jp} = 0, \forall (R) \text{ node}, j \leq i$$

The fourth constraint governs the flow of one unit through the admissible sequences of decisions that contain only one replacement decision.

Equation 4: One unit flow

$$\sum x_{ij2} \leq 1, j \leq i$$

The last constraint expresses that the decisions will equal one if in the sequence of decisions that achieves the objective function and will equal zero in all other sequences of decisions.

Equation 5: Decisions 0 or 1

$$X_{ijp} = 0 \text{ or } 1, \forall X_{ijp}$$

The fuzzy multi- objectives mathematical model

The fuzzy form for the mathematical model to optimise the ship's replacement decision is shown below:

$$Max : f_1^{\square} f_2^{\square} - f_3^{\square}$$

Subject to:

$$\sum_{p=1}^2 x_{ij} = 1$$

$$X_{ij1} - \sum_{p=1}^2 X_{(i+1)(j+1)p} = 0, \forall (K) \text{ node}, j \leq i$$

$$\sum X_{ij}2 - \sum_{p=1}^2 X(i+1)_{jp} = 0, \forall (R) \text{ node}, j \leq i$$

$$\sum x_{ij}2 \leq 1, j \leq i$$

$$X_{ijp} = 0 \text{ or } 1, \forall X_{ijp}$$

Deterministic form

By using the fuzzy sets method with a suitable membership function, the fuzzy form transferred to a deterministic form [23]:

$$\text{Max} : f^{\square}_1 f^{\square}_2 - f^{\square}_3$$

Subject to:

$$\sum_{p=1}^2 x_{ij} = 1$$

$$X_{ij}1 - \sum_{p=1}^2 X(i+1)(j+1)_p = 0, \forall (K) \text{ node}, j \leq i$$

$$\sum X_{ij}2 - \sum_{p=1}^2 X(i+1)_{jp} = 0, \forall (R) \text{ node}, j \leq i$$

$$\sum x_{ij}2 \leq 1, j \leq i$$

$$X_{ijp} = 0 \text{ or } 1, \forall X_{ijp}$$

$$n_{ijp} \in \mu n^{\square}_{ijp}$$

$$a_{ijp} \in \mu a^{\square}_{ijp}$$

$$e_{ijp} \in \mu e^{\square}_{ijp}$$

Equivalent form

The deterministic form can be transformed into the following equivalent form:

$$\text{Max} : f^{\square}_1 f^{\square}_2 - f^{\square}_3$$

Subject to:

$$\sum_{p=1}^2 x_{ij} = 1$$

$$X_{ij}1 - \sum_{p=1}^2 X(i+1)(j+1)_p = 0, \forall (K) \text{ node}, j \leq i$$

$$\sum X_{ij}2 - \sum_{p=1}^2 X(i+1)_{jp} = 0, \forall (R) \text{ node}, j \leq i$$

$$\sum x_{ij}2 \leq 1, j \leq i$$

$$X_{ijp} = 0 \text{ or } 1, \forall X_{ijp}$$

$$n^1_{ijp} \leq n_{ijp} \leq n^2_{ijp}$$

$$a^1_{ijp} \leq a_{ijp} \leq a^2_{ijp}$$

$$e^1_{ijp} \leq a_{ijp} \leq e^2_{ijp}$$

The ε -constraint method

To ease the difficulties the weighted sum method faces in solving multi-objective optimisation problems (MOOP) by having non-convex objective space. The ε -constraint method is used to formulate the fuzzy multi-objectives mathematical model by keeping one of the objectives and restricting the rest within a user-specified value (Haimes, Lasdon et al. 1971).

$$\text{Max} : f^{\square}_1(x)$$

Subject to:

$$f^{\square} m(x) \leq \varepsilon$$

$$\sum_{p=1}^2 x_{ij} = 1$$

$$X_{ij1} - \sum_{p=1}^2 X(i+1)(j+1)p = 0, \forall(K) \text{ node}, j \leq i$$

$$\sum X_{ij2} - \sum_{p=1}^2 X(i+1)jp = 0, \forall(R) \text{ node}, j \leq i$$

$$\sum x_{ij2} \leq 1, j \leq i$$

CONCLUSION

To reformulate the ship replacement decision in a form convenient for analysis, the decision variables are represented by a decision tree during a finite horizon [2], [9]. The objectives are represented by the method that treats multi-objectives, and the objectives' coefficients are represented by the method that treats the coefficients' fuzziness. To represent the decision variables by the decision tree during a finite horizon, the compact form for the sequences of decisions is extended during an example of five years [7]. The five-year example denotes the ship service life, which refers to the duration the ship is used before it is retired [10]. To represent the objectives of the ship replacement decision by the method that treats the multi-objectives, the concept of domination is used to determine the efficient solution that achieves the multi-objectives of the ship replacement decision [11]. To determine the efficient solution that achieves the multi-objectives for the ship replacement decision by the concept of domination, the values for the objectives need to be calculated separately and compared [25]. To represent the objectives' coefficients by the method that treats the coefficients' fuzziness, the fuzzy sets method is used to determine the coefficients range due to uncertainty in the future [22]. To determine the coefficient range due to uncertainty in the future, the fuzzy set of the coefficient must be calculated separately. A fuzzy set of coefficients is a class of coefficients in which there is no sharp boundary between those coefficients that belong to the class and those that do not [23]. The upper and lower bounds of the fuzzy set need to be calculated to obtain the class's coefficients. The lower and upper bounds are computed from the maximum and minimum parameters. By calculating the lower and upper bounds, the coefficients that belong to the class can be obtained, and the fuzzy coefficients range can be determined at a membership function equal to one. The model's objective function and constraints are defined to derive a fuzzy multi-objectives mathematical model [26]. To define the objective function, the sequences of decisions are distinguished by the decision type (p). Decision type equals one for (K) and two for (R). The replacement period network is considered a transportation network with one source and multiple destinations to define the constraints. The supply at the source node is one unit, and the demand at each destination is one unit. The one unit flows from the source node to the destinations through the admissible sequences of decisions that contain only one replacement decision. The decisions will equal one if in the sequence of decisions that achieve the multi-objectives and will equal zero in all other sequences of decisions. To ease the difficulties the weighted sum method faces in solving multi-objective optimisation problems (MOOP) by having non-convex objective space. The ε -constraint method is used to formulate the fuzzy multi-objectives mathematical model by keeping one of the objectives and restricting the rest within a user-specified value [27].

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