International Journal of Enhanced Research in Science, Technology & Engineering
ISSN: 2319-7463, Vol. 13 Issue 6, June-2024, Impact Factor: 8.375

The Design and Analysis of Tuned Nonlinear MAG Spring Vibration Absorber

Jasem Alrajhi[1,2], Jasem Alazemi[1], Khaled Alhaifi[1], Nawaf Alhaifi[1]

[1]Autom0tive and Marine Department, College of Technological Studies -PAAET KUWAIT
[2]Corresponding Author

ABSTRACT

In this research paper, the aim is to show how Mag-spring (newly invented Magnetic spring) as a nonlinear absorber could enhance vibration cancellation of the main system especially in a forced vibration in the frequencies around the resonance of the main system. It is shown that, this can be accomplished by keeping the natural frequency of the absorber, $\omega_n = \sqrt{\frac{k}{m}}$ intact with excitation frequency by employing a softening spring attached to the absorber mass. This kind of absorber will enhance the system to perform as a self-regulating system by being stiff when frequency is high and soft when frequency is low. In other words, at high frequency the amplitude of vibration is low and the spring at that point will behave as a hard spring. While, at low frequency the vibration amplitude is high and the at that point will behave as a soft spring. This kind of behaviour is expected when providing a system with a variable k, so, it will keep following the excitation frequency (when frequency is high, k is high, and when frequency is low, k is low as well). The results are compared with an equivalent linear absorber system. The Mag-spring was invented for a purpose other than vibration cancelation. However, the non-linearity that Mag-spring shows brought the idea of investigating a new way that could guide to a novel point of view in enhancing the vibration cancellation in vibrating structures. The results and discussion for the nonlinear spring (Mag-spring) and equivalent linear spring are presented in this paper; including a study of the effect of mass ratio, effect of damping ratio (light damping), and a comparison between linear and nonlinear tuned vibration absorber (TVA).

INTRODUCTION

This paper mainly focuses on analytical discussion how a 2DOF linear system could perform as a tuned vibration absorber (TVA). Also, the analytical solution of an un-damped TVA.The aim here is to study the behaviour of a nonlinear spring instead of linear one in a vibration absorber system. To do so, next section in the research paper presents the softening nonlinear behaviour of the Mag-spring by a polynomial showing relation between force and displacement of spring. Using nonlinear Mag-spring as a vibration absorber gives two coupled nonlinear second order differential equations. Finally paper presents the mathematical process to solve these two nonlinear differential equations for the primary system and the nonlinear absorber is discussed. The Matlab program written to solve governing equations is explained as well.This paper also discusses about an ideal nonlinear stiffness and a few theoretical ideal nonlinear stiffness curves for a TVA. The simulation results for these different curves and the Mag-spring have been compared as well.

Tuned vibration absorber

To design a linear vibration absorber, specific parameters should be properly adjusted. Stiffness and mass of absorber are the two most significant parameters involved in designing an absorber. However, domain of the vibration of the absorber should be maintained in an acceptable range, according to the space provided. Also the design of the effective domain of the absorber (the vibration in this domain remains in an acceptable range), which depends on mass ratio, should be calculated as well.

Figure 1 shows a main system and absorber. Stiffness and mass of the main system and absorber are k, m, k_a, m_a respectively.

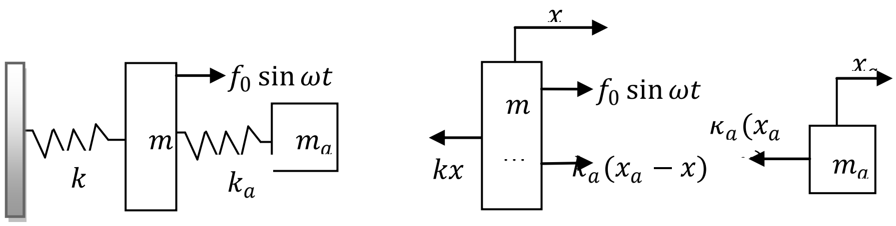

Figure 1: main system (k,m) and absorber (k_a,m_a)

$$X_1 = \frac{F_0(k_a - m_a\omega^2)}{(k + k_a - m\omega^2)(k_a - m_a\omega^2) - k_a^2}$$ **Eqn. 01**

$$X_2 = \frac{k_a F_0}{(k + k_a - m\omega^2)(k_a - m_a\omega^2) - k_a^2}$$ **Eqn. 02**

When $\boldsymbol{\omega_{na}} = \sqrt{\frac{\boldsymbol{k_a}}{\boldsymbol{m_a}}}\,, \boldsymbol{X} = \boldsymbol{0}\,, \boldsymbol{X_a} = -\frac{\boldsymbol{F_0}}{\boldsymbol{k_a}}$

Then, for designing an absorber the following parameters should be considered:

1) $\frac{k_a}{m_a} = \omega_{na}^2 = \omega_n^2 = \frac{k}{m}$
2) X_a<= specified space (half/less of the given space)
3) $\omega_{n1} - \omega_{n2} \geq$ minimum specified

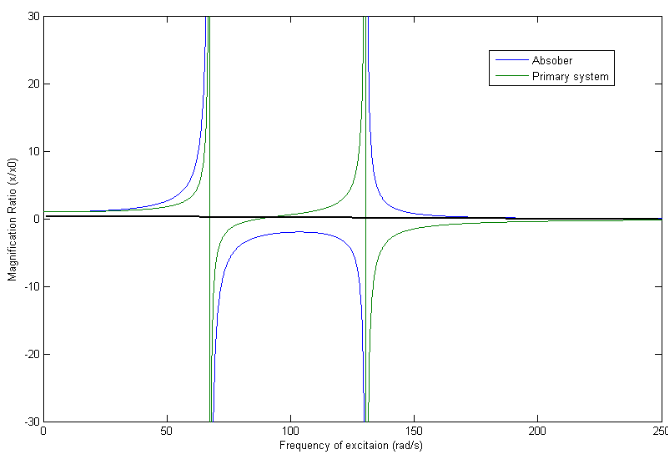

Figure 2: frequency domain for a system with two degree of freedom

Which the last one can be shown in the form of $\vartheta = \frac{m_a}{m}$ *and* $\frac{\omega_{n1} - \omega_{n2}}{\omega_n} = \sqrt{\vartheta}$

However, for an absorber with nonlinear spring, the method (analysis of the absorber) will slightly be different as there is no a straight analytical solution for the equations of motions (depends on the nonlinearity of the absorber). The definition of $\omega_{na} = \sqrt{\frac{k_a}{m_a}}$ will not be straight forward anymore and ω_{na} will change with the amplitude of vibration. In the next,Whenω_{n1} and $\omega_{n2}$are the two natural frequencies of the two degree of freedom system, it can be seen from **Eqn. 1** that when the excitation frequency ω equals either of ω_{n1} or ω_{n2}, resonance takes place.

The following example clarifies the above concepts for a system with linear absorber, assume that:
$k_1 = 10000\frac{N}{m} k_2 = 3850\ \frac{N}{m}$ and $m_1 = 1kg and m_2 = 0.3850Kg$ this gives:

$\omega_{na} = \sqrt{\frac{k_a}{m_a}} = \sqrt{\frac{3850}{0.385}} = 100$ rad/s$and \omega_n = \sqrt{\frac{k_1}{m_1}} = \sqrt{\frac{10000}{1}} = 100$ rad/s

Which means the absorber is tuned to the main system. Therefore, ω_{n1} and ω_{n2} for this system are, 73rad/s and 135rad/s respectively as in

Figure 2. The forced response for this system is shown above:

If m_a and k_a are chosen such that $\omega = \sqrt{\frac{k_a}{m_a}}$, then, $X_1 = 0$, that is, the first mass does not vibrate at all.So, if the main system is a SDOF mass spring (m_1, k_1) system subjected to a vibratory excitation frequency ω, then by connecting it to a second mass spring (m_a, k_a) system with $\omega = \omega_{na} = \sqrt{\frac{k_a}{m_a}} = \sqrt{\frac{k_1}{m_1}}$, the main system will stop oscillating under this excitation. So, the connected mass spring system is serving as a TVA.

$$\boldsymbol{\omega_{na}} = \sqrt{\frac{\boldsymbol{k_a}}{\boldsymbol{m_a}}} \quad \text{and} \quad \boldsymbol{\omega_n} = \sqrt{\frac{\boldsymbol{k_1}}{\boldsymbol{m_1}}} \quad \textbf{Eqn.3}$$

Where, ω_n is the main system's natural frequency and ω_{na} is the absorber's natural frequency.
At $\frac{\omega}{\omega_{na}} = 1$, a zero response can be gained.

The zero response cannot be gained when damping is presented to the main system at $\frac{\omega}{\omega_{na}} = 1$. But, low response will be delivered for a range of frequenciesaround$\frac{\omega}{\omega_{na}} = 1$.

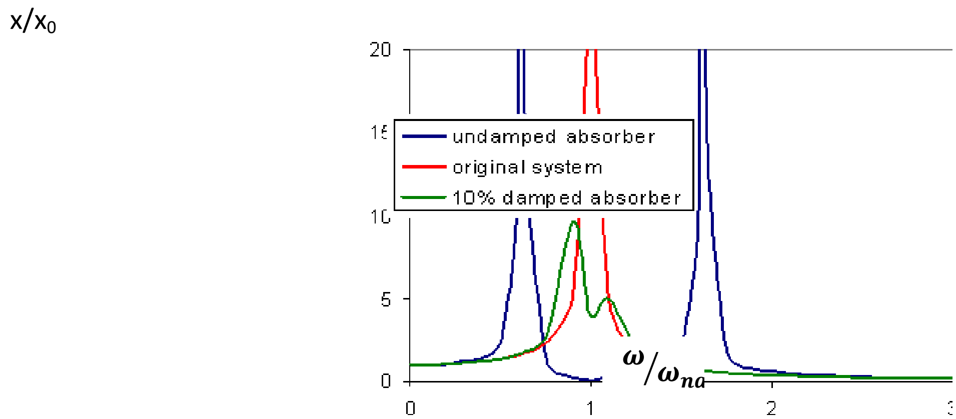

Figure 3: comparison of domain of vibration between a linear absorber system with damping and without damping.

Non-linear spring

The characteristics of non-linear magnetic spring were first calibrated by measuring the maximum amplitude of the system when the load was varied and found that, it is a hardening spring. This kind of spring is ideal to be added in parallel to the spring of a single degree of freedom system (SDOF) to control unwanted vibration. In SDOF system, when the amplitude of vibration is minimal, the nonlinear spring shows low stiffness and barely affects the stiffness of the system. To achieve this goal, it has been decided to use some parts of the working domain of the nonlinear spring which shows the softening behaviour. The non-linearity of the spring was modelled and a graph as in

Figure 4 has been obtained by means of best curve fitting with 11^{th} order.

Table 01: Calibration data for non-linear magnetic spring

X (m)	F (N)	X (m)	F (N)
0	0	0.0138	33
0.00126	1	0.0151	36
0.00217	2	0.0164	38
0.00323	2.5	0.0178	40
0.00445	3	0.019	42
0.00562	5.4	0.02015	42
0.00712	8.2	0.02118	43
0.00853	11	0.0222	43.9
0.00958	17	0.02337	43.5
0.01027	21	0.0247	44

0.0118	25	0.0258	44
0.0128	28.5		

Based on the curve, it is clearly seen that the effective operational amplitude domain of the non-linear system is from 0 to 15mm. The regions of 0-5mm, 5-10mm. 10-15mm represent high, medium and low stiffness respectively.

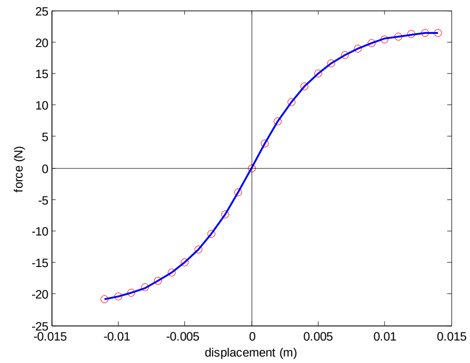

Figure 4: Modified non-linear calibrated curve

The curve fitting on graph

Figure 4, produces a polynomial of order 11th for the nonlinear spring with softening behaviour that its coefficients are documented in

Table2. The mathematical model for the solution is given by:

$$f(x) = A_{11}x^{11} + A_{10}x^{10} + A_9x^9 + A_8x^8 + A_7x^7 + A_6x^6 + A_5x^5 + A_4x^4 + A_3x^3 + A_2x^2 + A_1x + A_0 \quad \textbf{Eqn. 04}$$

Table2: Polynomial solution of modified non-linear data

coefficient	value
A_0	0.002896853
A_1	3874.69756
A_2	-2590.56601
A_3	-47765585.5
A_4	257588759.9
A_5	6.26569E+11
A_6	-7.7201E+12
A_7	-4.93E+15
A_8	8.69E+16
A_9	1.79E+19
A_{10}	-3.24E+20
A_{11}	-1.86E+22

The effective stiffness of the nonlinear Mag-spring can be achieved by running the vibrating system with constant forces in time domain and calculate the related amplitude at each force.

Figure 5 shows how the Mag-spring nonlinearity is changing with displacement. As it can be seen from the graph, the stiffness of the spring is from 3800N/m to 100N/m while the displacement is changing from 0 mm to 12 mm.

The non-linear stiffness obtained is then substituted in the main governing equations of two- dimensional non-linear system, which are:

$$m_1\ddot{x}_1 + k_1 x_1 - f(x_1, x_2) = F_0 \sin(\omega t) \quad \textbf{Eqn. 05}$$
$$m_2\ddot{x}_2 + f(x_1, x_2) = 0 \quad \textbf{Eqn. 06}$$

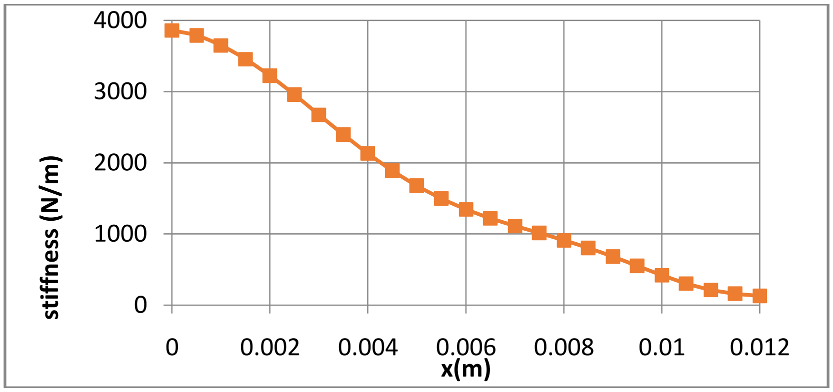

Figure 5: Graph of nonlinear stiffness (Softening behaviour)

Solving two-dimensional non-linear vibration-absorber system in MATLAB

Convergence criteria and maximum permissible time step

The equations are second order derivative of the displacement of the mass in the main system,$x_1(t)$, and of the absorber mass, $x_2(t)$, in time domain. To find the solution for these two differential equations, Matlab ode45 function is used. This function needs three sets of inputs; first, the differential equation itself, secondly, the initial time and final time and lastly, the initial velocity and initial displacement. As $x_2(t)$ and $x_1(t)$ in these equations are dependent on each other, they cannot be integrated separately in time domain. In this case if the time steps are small enough, it is possible to solve each equation separately and assume the other parameter is constant. The initial time t, and the final time is chosen t+dtat ode45. The criterion is chosen in a way that the difference between the integrated parameters in the two consecutive iterations should be less than 1% of the previous value. 1% has been found realistic after several runs. The size of time step itself is an important issue as well. In our cases the values of dt=0.001 for time step was chosen, as the dt=0.01was not converging well. This process is shown in the inner loop of flow diagram of Figure 8.

Steady state solution and light damping

To reach a steady state response, light damping should be added to the differential equations of the system. In the results section, the effect of light damping on a system with two degrees of freedom is depicted. An important issue is to know how many time steps should be taken before reaching to a steady state solution in time domain. To check the steady state conditions in the response of the differential equations, two schemes can be used. In the first scheme, the time domain of the results at extreme parts is checked visually to make sure that the results are converged to a steady state solution before picking the maximum amplitude. In the second scheme a certain criteria is introduced to distinguish the steady state situation with the program itself. In this method, the last three maximum amplitude in time domain is stored in three different variables and as the steady state condition, the difference between the first and third maximum should be less than a one percent of the maximum amplitude (this value could be different if more accurate results required). The position this condition needs to be checked has been shown with "Check steady state" in the program flow diagram on Figure 8. When this criterion is fulfilled, the program stop time marching and will start to simulate the next excitation frequency.

Time domain to frequency domain criteria

The expectation from the solution is to display the result in frequency domain. This expectation can be realized by coding that screens through all the amplitudes obtained for one set frequency to determine the maximum value. By other words, firstly in each frequency of excitation (ω) the converged time domain of the solution for both the absorber and the main system is calculated and secondly the maximum of this solution selected as the amplitude related to each frequency of excitation. Final results will be the amplitude of vibration verses the frequency or the frequency domain results.

Flow diagram of the program

The flow diagram of the steps taken by the program is shown on Figure 8. The aim is to find the amplitude of the vibration of the main system and the absorber for a range of the forced excitation from ω=1 to 500 rad/s. The flow diagram starts by defining initial parameters. There are three types of parameters. The first group are the initial conditions (initial displacement and initial velocity for the main system and absorber), the second group are the constant parameters (damping ratio, mass and stiffness of the main system and absorber and amplitude of the force, F0). The third group are the program counters and controller.

The main part of the flow diagram consists of three loops in each other. The inner loop with the k as the counter checks the convergence of the displacement at each time step, the loop in the middle, time loop, created the time domain response at each frequency of excitation. And it will stop if the response has reached to the steady state response. The outer loop, frequency loop, gives the frequency domain of the system. In this loop for each frequency of excitation the time loop is started from the beginning and the maximum domain of the time response at that frequency will store as the amplitude related to that frequency of excitation. The solutions obtained from the procedure illustrated on Figure 8 and explained in next section are analysed in the MATLAB program.

State space variables

To solve the differential equations in matlab, it is needed to write the equations in the State space form in different functions (file). As for a two degree of freedom system there are two differential equations then two functions have been defined as follows:The first function is related to the main system and is shown as “myfunc_x1”. This function will be called from the main matlab program through the ode45 function. It needs the initial conditions and time domain intervals as inputs. The output of this function will be displacement “y1(1)” and first derivate of the displacement “y1(2)” of the differential equation in time domain.

```
function y1dot=myfunc_x1(t1,y1)
global x_2 x_2dot wn m1 k1 k2 m e w a0 a1 a2 a3 a4 a5 a6 a7 a8 a9 a10 a11 EQN damp1 damp2
F0=2;
y1d1=y1(2);
y1d2=-k1/m1*y1(1)+(((a11*(x_2-y1(1))^11)+(a10*(x_2-y1(1))^10)+(a9*(x_2-y1(1))^9)+(a8*(x_2-y1(1))^8)+...
   (a7*(x_2-y1(1))^7)+(a6*(x_2-y1(1))^6)+(a5*(x_2-y1(1))^5)+(a4*(x_2-y1(1))^4)+...
   (a3*(x_2-y1(1))^3)+(a2*(x_2-y1(1))^2)+((x_2-y1(1))*a1)+a0)/m1)+((1/m1)*F0*sin(w*t1))+...
   -y1(2)*(damp1/m1)+(damp2/m1)*(x_2dot-y1(2));
%////
y1dot=[y1d1;y1d2];
```

The second function is related to the absorber and is shown as “myfunc_x2”. The state space form of the absorber differential equation is as follows:

```
function y2dot=myfunc_x2(t2,y2)
global  x_1 x_1dot m2 k1 k2 m e w a0 a1 a2 a3 a4 a5 a6 a7 a8 a9 a10 a11 EQN damp2
y2d1=y2(2);
y2d2=-((a11*(y2(1)-x_1)^11)+(a10*(y2(1)-x_1)^10)+(a9*(y2(1)-x_1)^9)+(a8*(y2(1)-x_1)^8)+...
   (a7*(y2(1)-x_1)^7)+(a6*(y2(1)-x_1)^6)+(a5*(y2(1)-x_1)^5)+(a4*(y2(1)-x_1)^4)+...
   (a3*(y2(1)-x_1)^3)+(a2*(y2(1)-x_1)^2)+((y2(1)-x_1)*a1)+a0)/m2-(y2(2)-x_1dot)*damp2/m2;
% end
y2dot=[y2d1;y2d2];
```

Results

Nonlinear Mag-spring

The nonlinear Mag-spring is tested as vibration absorber in this research, therefore, it is necessary to defined the system parameters; like proper absorber mass and light damping before any further analysis and comparison with linear absorber. To select a proper light damping for the system, the effect of damping ratio, on a nonlinear vibration absorber is investigated. The absorber mass has been defined for this nonlinear system, with analogy to a linear vibration absorber.

After defining the proper parameters for the nonlinear vibration absorber, a comparison with a linear absorber has been conducted. In the first place, the frequency response of a main system with a linear and nonlinear softening stiffness has been compared at different amplitude of excitation. In the second stage, mass ratio as an important parameter to control the vibration cancellation range of an absorber has been presented for a linear and nonlinear vibration absorber. Finally, a few theoretical methods are suggested to improve the nonlinearity of the spring in a way to have a self-regulating absorber in a wider working cancellation range of the absorber. To achieve this goal, the written MATLAB program was run several times and the results are presented in the following subsections.

Effect of damping ratio

It has been shown that one of the important parameter to define a proper TVA system is the damping ratio. Vibration absorber without damping has a very narrow range of vibration cancellation and a small change in excitation frequency could lead to a very high magnification ratio. Even in the linear absorber finding an optimum damping ratio to design an effective absorber is a complicated issue and has not been fully resolved (Liu and Coppola 2010). However, in this section using damping ratio is limited only to lead the response of the system to a steady state solution; therefore light damping concept has been considered for this issue.

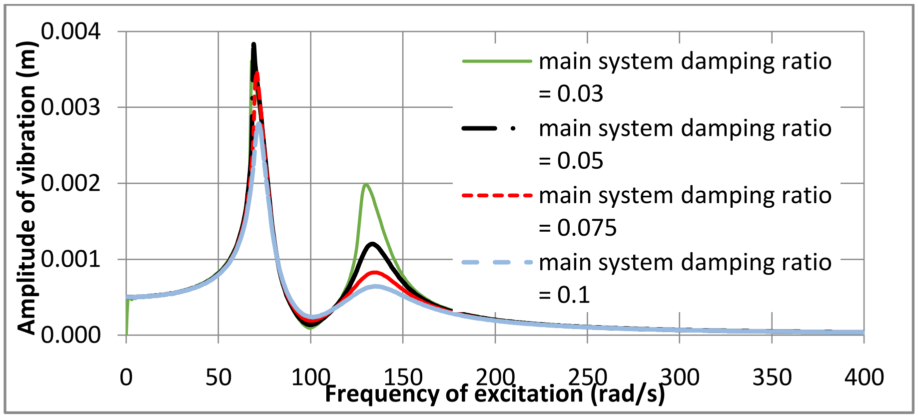

Figure 6: Main system, Amplitude of Vibration in system with a nonlinear absorber verses frequency of excitation, F_0=5N.

Figure 6 shows the amplitude of vibration of the main system verses excitation frequency for various damping ratios. In this figure, the main system stiffness and mass are 10000N/m and 1 Kg respectively; therefore, the natural frequency of the main system is 100rad/s. To make an easier comparison, the same values have been chosen similar to the previous studies.As it can be seen from the

Figure 7; low damping lead to a better cancellation at the natural frequency of the main system and vice versa, however, higher damping ratio reduce the amplitude of the vibration significantly, specially the second peak. As in the linear system, finding an optimum damping ratio to suppress amplitude at resonance with lowest cancellation amplitude could be a potential for another research, which is not the subject of this study (here just light damping has been used to lead the response to a steady state solution).

The amplitude of vibration verses frequency of excitation of the absorber has been shown in

Figure 7 at various damping ratios.

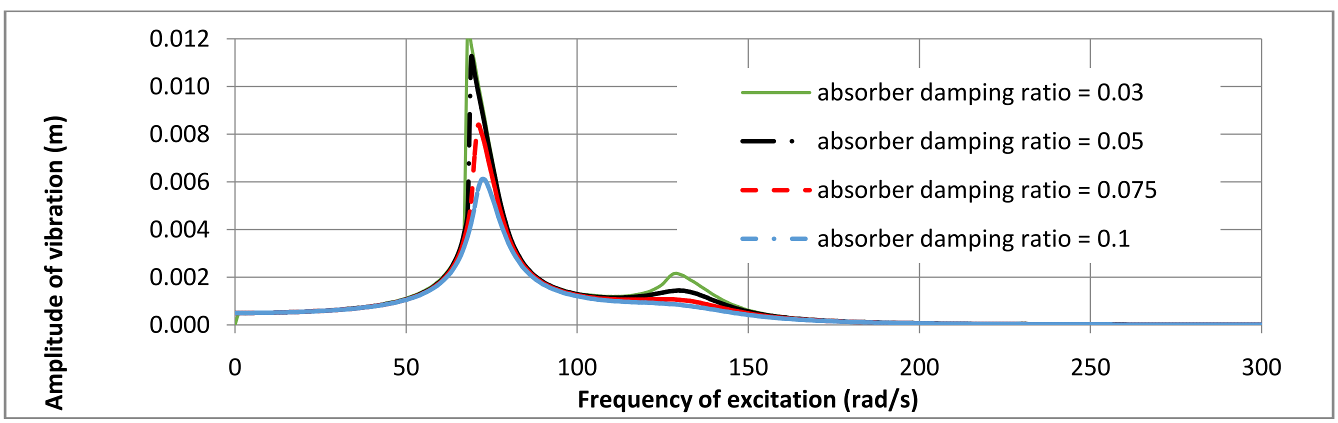

Figure 7: Nonlinear absorber, Amplitude of Vibration verses frequency of excitation, F_0=5N

As it can be seen from the

Figure 7, at higher damping ratio the second peak nearly disappeared. Also, the first peak has been reduced significantly by increasing the damping ratio. It can be concluded, in contrast with the main system; increasing damping ratio just has positive effect on amplification of the absorber at all frequency of excitation and could lead to a more stable situation.

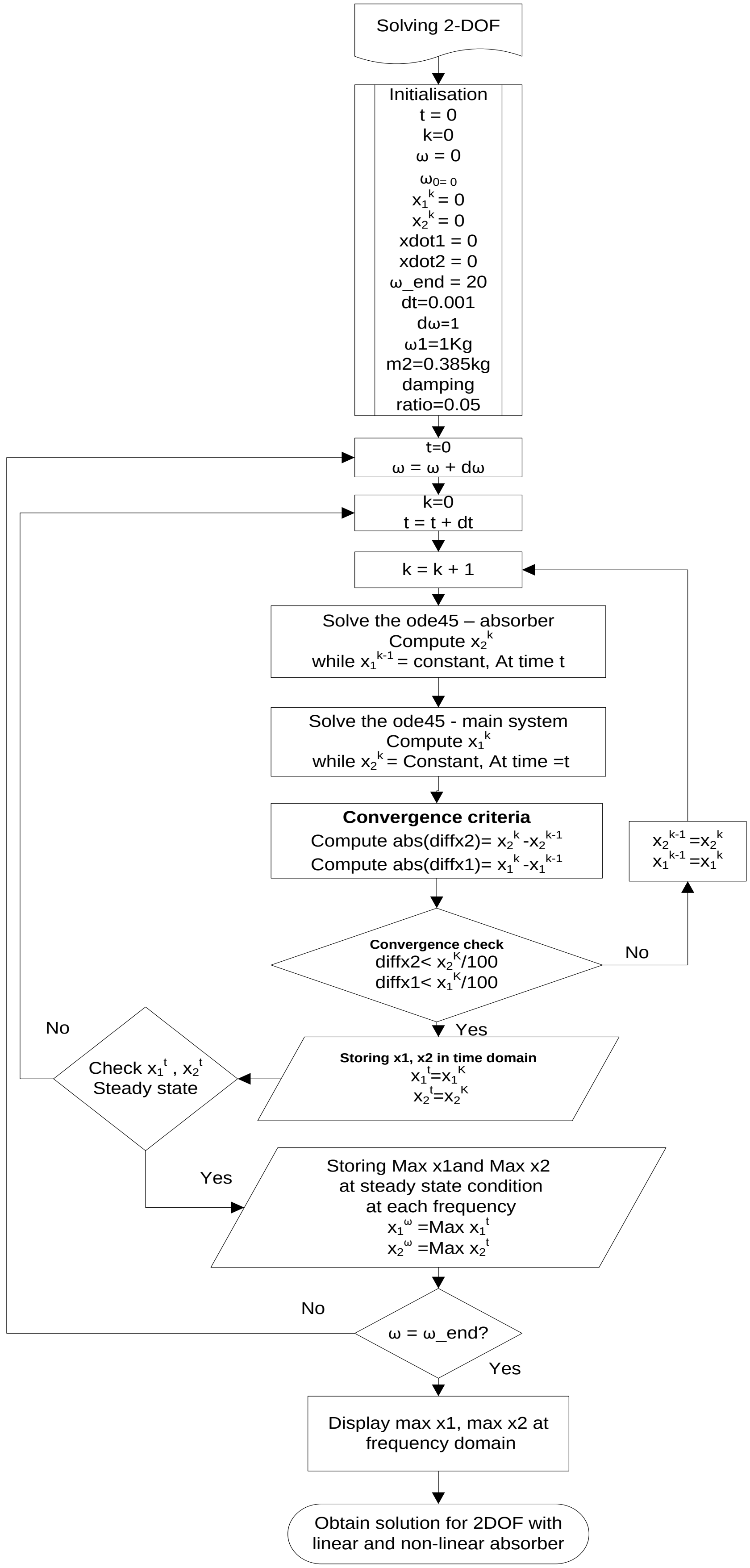

Figure 8: Program flow to solve 2-DOF

International Journal of Enhanced Research in Science, Technology & Engineering
ISSN: 2319-7463, Vol. 13 Issue 6, June-2024, Impact Factor: 8.375

Definition of the absorber mass with nonlinear stiffness

One of the important issues to deal with a nonlinear absorber is tuning the absorber to the natural frequency of the main system.

Figure 9 shows the amplitude of the main system when the Mag-spring is used as a nonlinear absorber. As it can be seen from the figure, different masses have been used for a single nonlinear absorber and the results are showing that none of the cases has tuned 100% to the natural frequency of the main system.

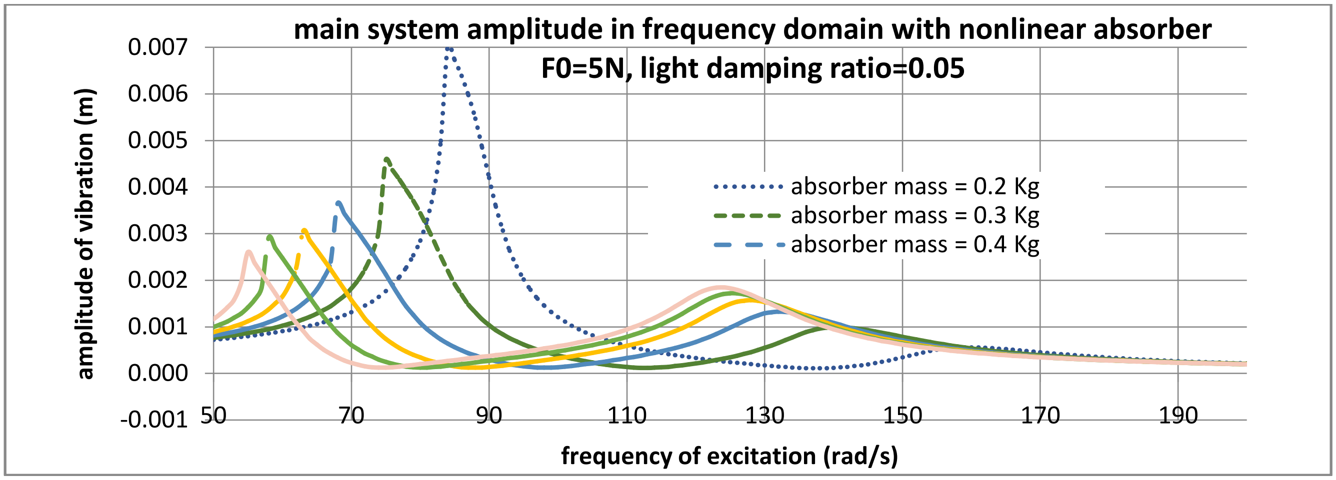

Figure 9: Amplitude of the main system with nonlinear absorber with different mass ratio at frequency domain. F_0=5N and a light damping ratio=0.05

Figure 9 shows that the best mass for the absorber to be tuned with the main system is between mass ratio of 40% and 30% and the lowest amplitude of vibration for these two cases is between 95 rad/s and 110 rad/s respectively.

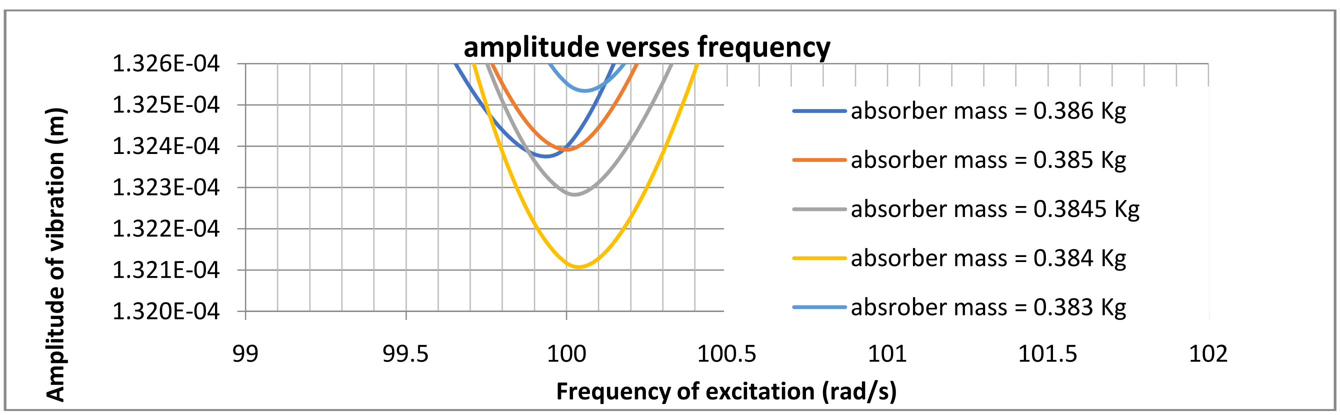

Figure 10: Amplitude of vibration verses the frequency of excitation for different mass ration at natural frequency of the main system (100 rad/s).

It has been found in this case that to tune a nonlinear absorber with the main system natural frequency, The mass ratio value for the nonlinear absorber should be matched with the gradient of force displacement graph (

Figure 4) at the near zero displacement or the stiffness of the origin of nonlinear stiffness graph (

Figure 5). For instance, Mag-spring nonlinearity starts with k = 3850N/m. Therefore, the proper mass for the absorber is 0.3850 to tune the main system with the natural frequency of the 100 rad/s.

Figure 11 depicts the amplitude of vibration verses excitation frequency for the nonlinear absorber with different mass ratios. As it can be seen, the system with lower mass ratio will produce higher amplitude for the absorber and vice versa.

Linear verses nonlinear absorber

In this section, the performance of the Mag-spring as a nonlinear absorber is compared with a linear absorber with m_a=0.385kg and k_a=3850N/m attached to a main system with the stiffness of 10000N/m and mass of m=1 kg. however, about the parameters of the absorber are totally different. As it has been discussed in the previous section, the optimum mass ratio (m_2) for this Mag-spring as a nonlinear absorber is equal to 38.5% of the main system mass. This mass tunes the absorber to the main system natural frequency. The natural frequency of the main system is 100 rad/s.

Figure 10, shows how the nonlinear absorber with m_2= 0.385kg is making the lowest amplitude for the main system at its natural frequency, hence tunes well with the main system. Therefore, for a proper comparison, the mass of the linear absorber is selected to be 38.5% of the main system; hence the stiffness of the linear absorber should be 3850N/m. By selecting these parameters for the linear absorber, the natural frequency of the absorber and the main system become same as 100rad/s.

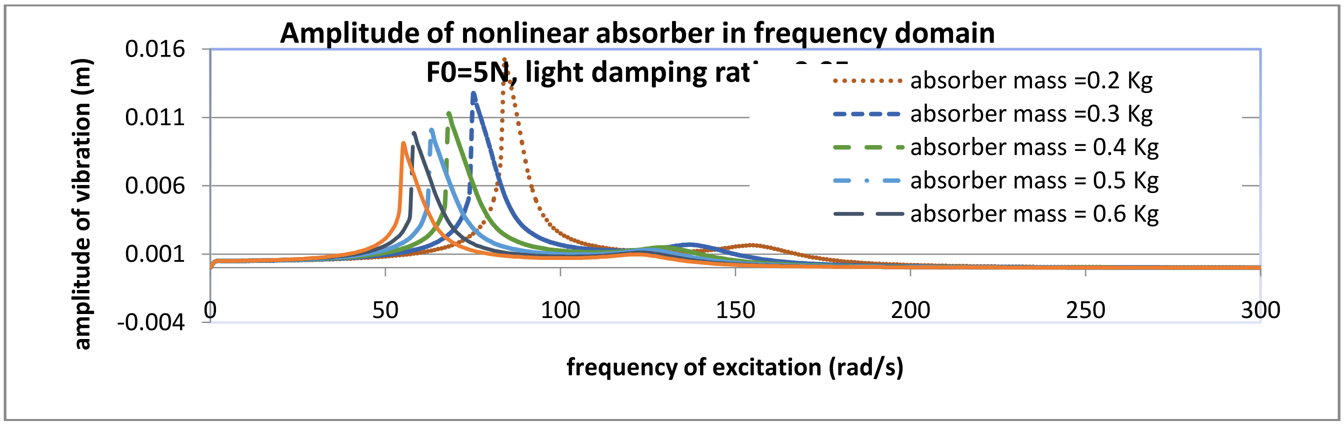

Figure 11: Amplitude of a nonlinear absorber verses frequency at different mass of absorber. F_0=5N and light damping ratio = 0.05

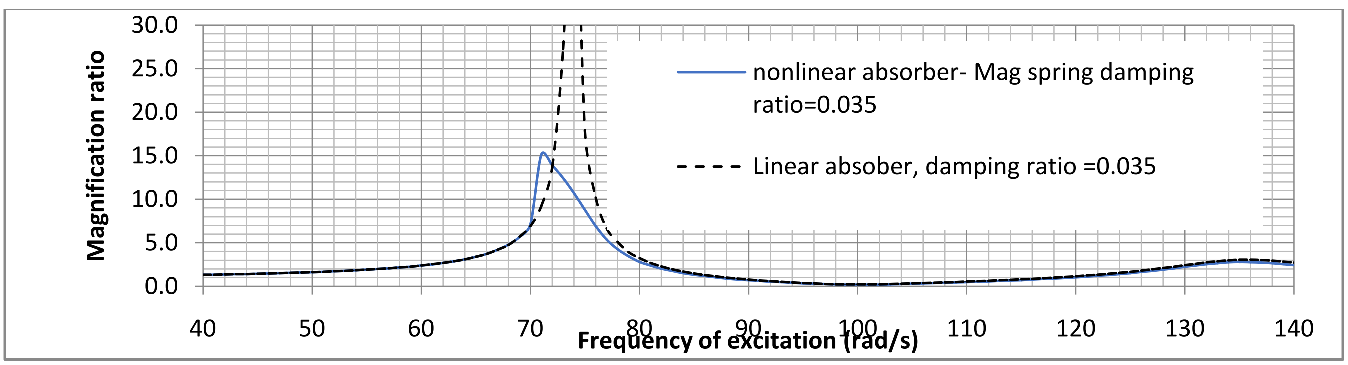

Figure 12: Magnification ratio of the main system (k=10000N/m and m=1Kg) verses frequency of excitation in two cases: first with a linear absorber with k=3850N/m and m=0385Kg; second, with Mag-spring as an absorber with nonlinear stiffness and m=0.385Kg. Damping ratio=0.035 and F_0=2N.

Figure 12 and

Figure 13 show the magnification ratio of the main system response with a linear and nonlinear absorber. Both figures show that the range of vibration cancellation is clearly wider after magnification ratio of one. For instance, at 80 rad/s frequency of excitation (with F_0=5N and damping ratio=0.05), the magnification ratio in a system with linear absorber is 3.2; however, for the system with nonlinear absorber the magnification is 2, which shows about 60% reduction in magnification ratio, which is a significant value. Although this improvement is not a constant improvement, this could be as high as 15times reduction in amplification ratio at excitation frequency of 74 rad/s(natural frequency of the two degree of freedom system), or as low as zero at 100 rad/s (which is the natural frequency of the main system). i.e. these values are different from frequency to frequency and depend on systems parameters.In addition, the maximum amplification of the system with nonlinear absorber has been reduced significantly at the same damping ratio and excitation amplitude. Moreover, the main natural frequency of the system has been shifted away from the natural frequency of the main system between 5% to 10% depending on the system parameters. In other words, if a nonlinear stiffness is designed properly for working conditions, it could work better than a linear absorber.It is worth to notice that in linear tuned vibration absorber, the excitation force is not an important issue, as long as using the non-dimensional parameters (like the magnification ratio). However, in a system with nonlinear absorber, excitation force amplitude is important. In other words, when the force is higher,different level of nonlinearity of the absorber will be used. When high force is applied to the system, it will cause higher amplitude which in turn will deflect the nonlinear spring more. As a result, the nonlinearity will be allowed to play a larger role in the vibration cancellation process. So, when the involvement of the nonlinear effect is enlarged by the high force amplitude, oscillation displacement will be observed of being forced to be suppressed and minimized.It can be concluded, that it is really beneficial to make the nonlinear effect kicks in as soon as possible, which in turn will accelerate the vibration cancellation process. In other words, shortening the displacement required to hit the nonlinearity starting point will make the vibration cancellation faster and better.

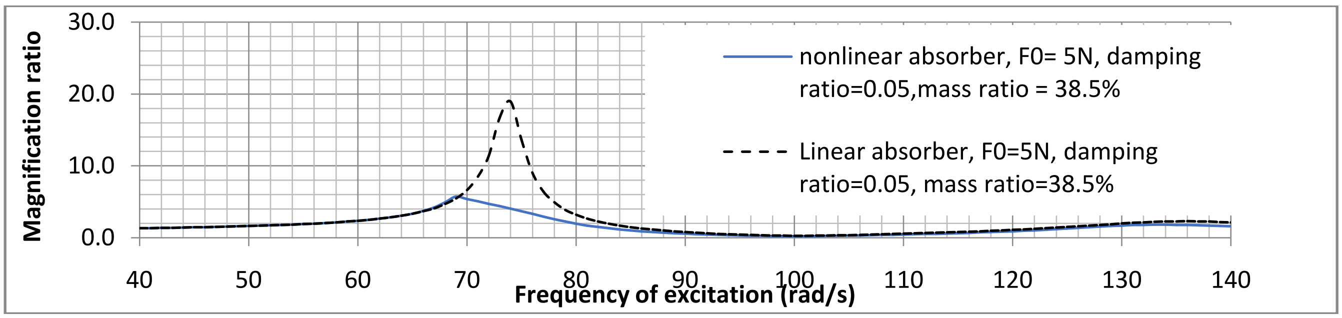

Figure 13: Magnification ratio of the main system (k=10000N/m and m=1Kg) verses frequency of excitation in two cases: first with a linear absorber k=3850N/m and m=0385Kg; second, with Mag-spring as an absorber with nonlinear stiffness and m=0.385Kg. Damping ratio=0.05 and F_0=5N.

Mass ratio study of linear and nonlinear absorber

In this section, the mass ratio as an important parameter in TVA has been discussed. Role of mass ratio in nonlinear absorber is slightly different from a system with linear absorber as stiffness of the nonlinear absorber is not constant and tuning the absorber and main system has a bit more difficulty.

Figure 14shows the effect of damping ratio on the range ofcancelation for a main system with linear tuned vibration absorbers and different mass ratio. It is clear from the graph that all the absorbers has the same natural frequency and hence tune with the main system.However,

Figure 9, shows a nonlinear absorber system with different mass ratio; according to the graph, the absorber natural frequency is different from the main system depends on the mass ratio.

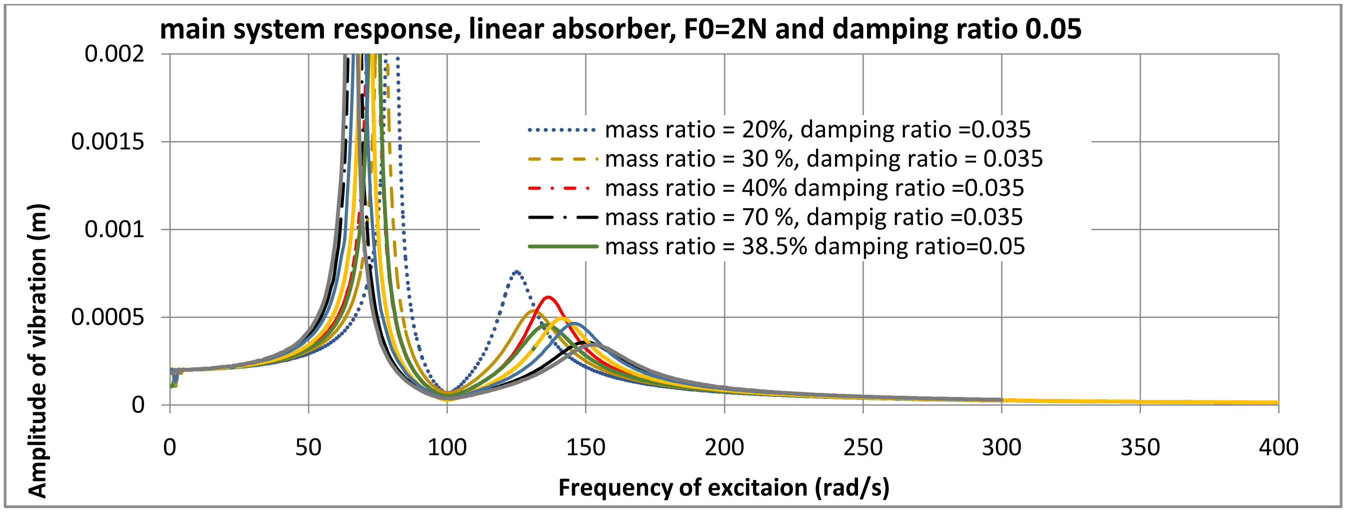

Figure 14: Amplitude of the main system (k=10000N/m, m=1Kg) with linear absorber with different mass ratio (and stiffness) verse the frequency of excitation, F_0=2N and damping ratio is 0.05.

Although the nonlinear absorber could work with different mass ratio in an acceptable range, it is recommended that a nonlinear absorber is to be used with an absorber mass which gives a wider range of cancelation around the natural frequency of the main system depends on expected frequency of excitation.

Figure 15 and

Figure 10 compare the behaviour of the main system with a linear absorber and a nonlinear absorber. In the linear absorber, all the system's responses provided with different mass ratios has minimum amplitudes of vibration around the natural frequency of the main system.To have a fair comparison between vibration absorbers with linear and nonlinear stiffness, the behaviour of the main system (k=10000N/m, m=1Kg) with a linear absorber (k=3850N/m) at different mass ratios is studied.

Figure 16 and

Figure 9 show linear and nonlinear vibration absorber, both have similar behaviour when one single spring has been used with different mass ratio; however, the cancellation range of the system with nonlinear absorber is about 5% wider (

Figure 17), also the amplitude of vibration is significantly lower (at similar conditions) than the system with linear mass ratios, especially at higher values for the absorber mass. That is due to the nonlinearity in the absorber stiffness, which increases when the vibration amplitude get larger. High amplitudes make the nonlinear spring to deflect more, which in turn allows the nonlinear range of the spring to play a larger role in suppressing the high amplitude, which will make the cancellation range wider. This can be noticed when looking at

Figure 17, that at the first peak of vibration, the nonlinear effect shows a better response in widening the cancellation range than the second peak due to the larger amplitude.

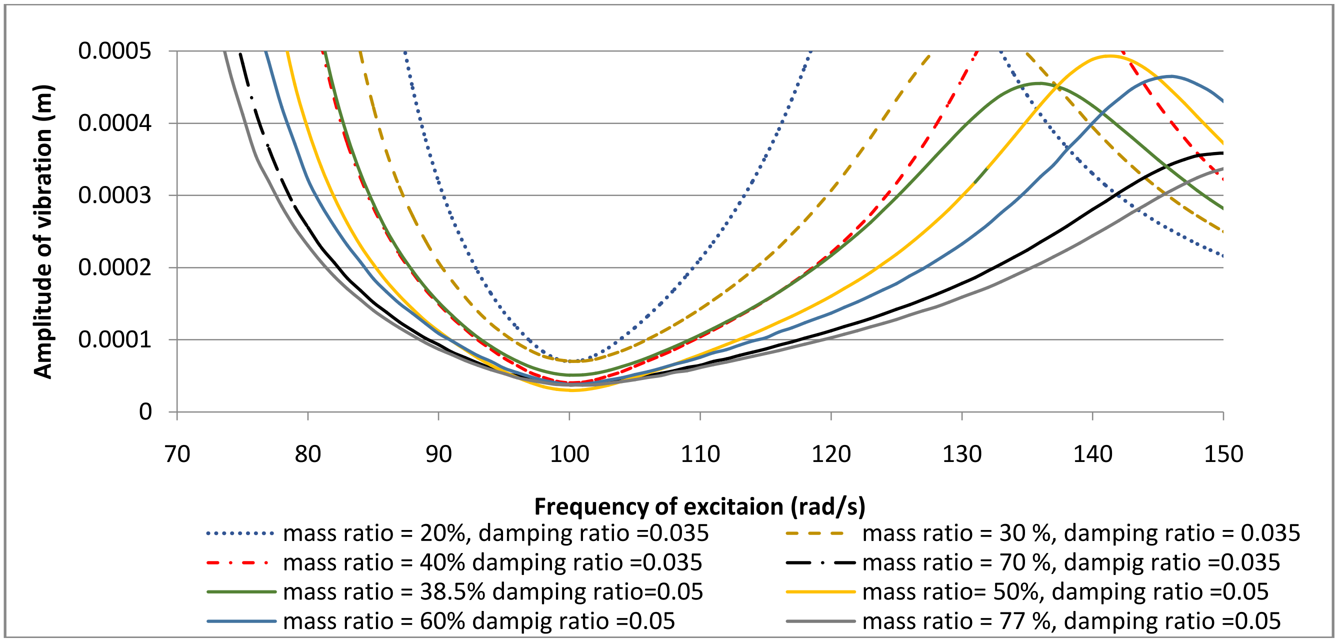

Figure 15: Closer look to the amplitude of the main system (k=10000N/m, m=1Kg) with linear absorber with different mass ratio (and stiffness) verse the frequency of excitation, F_0=2N and damping ratio is 0.05.

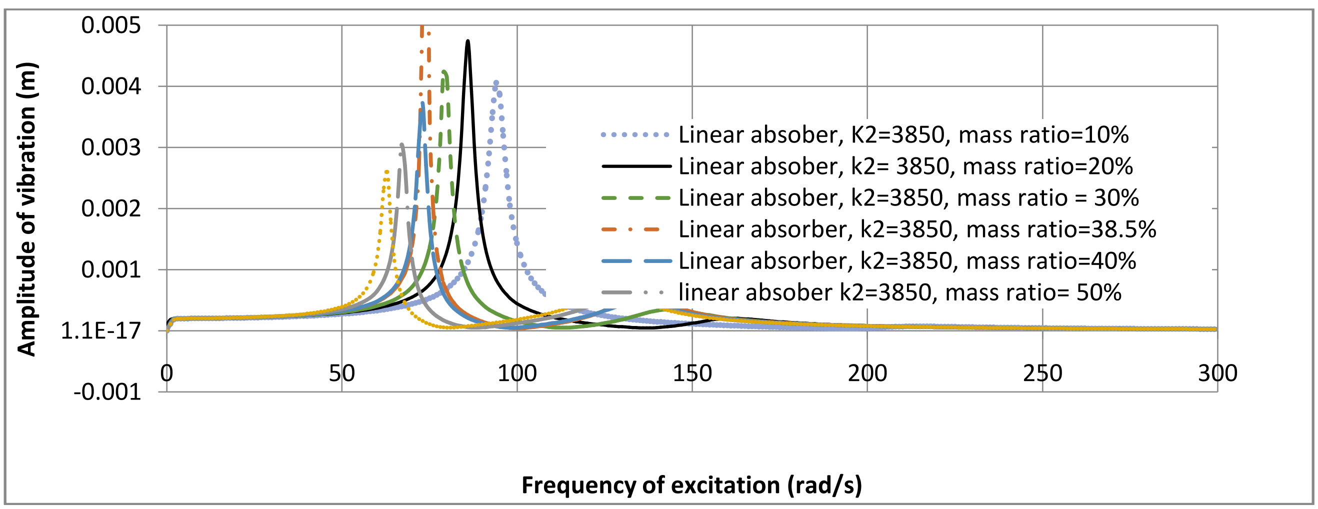

Figure 16: Amplitude of vibration of the main system (k=10000N/m and m=1Kg) with an absorber (k=3850N/m with different mass) verses the frequency of excitation F_0=2N, damping ratio0.05.

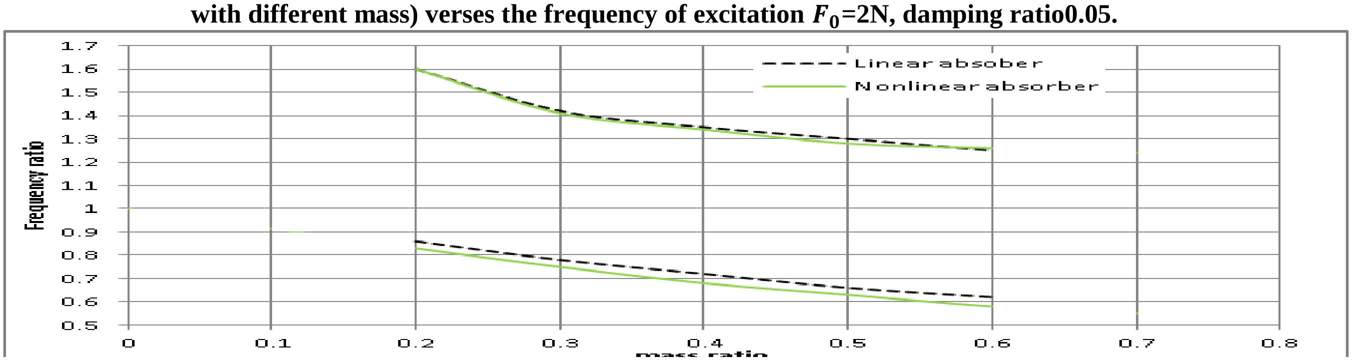

Figure 17: a two degree of freedom system natural frequencies verses the mass ratio of a linear (k=3850N/m) and nonlinear (Mag-spring) vibration absorber. F_0=2N, damping ratio=0.05.

Figure 17 compare the natural frequencies of a system with linear and nonlinear absorber at equivalent conditions, i.e. main system (k=10000N/m, m=1Kg), excitation force (F_0=2N), light damping ratio= 0.05. As it can be seen from the graph, the second natural frequency for a linear and nonlinear absorber is nearly the same.

Amplitude of vibration of the tuned nonlinear vibration absorber verses the frequency of excitation for two different nonlinear absorbers is shown on

Figure 18. To simulate this situation it has been assumed that two nonlinear springs have been used on parallel as an absorber. Comparison of

Figure 14 and

Figure 18 for tuned vibration absorber with linear and nonlinear absorber show that the nonlinear absorber could have at least about 5% improvement in widening the range of natural frequencies (the frequency gap between the two natural frequencies) of the system (

Figure 19), apart from lowering the amplitude of vibration at system natural frequencies.

Figure 19 shows the natural frequency of a tuned vibration absorber system with linear and nonlinear absorber, as it can be seen the nonlinear absorber (especially at higher mass ratio and lower frequency of the system which is most important) is widening the frequency gap between the natural frequencies of the tuned vibration absorber system with two degree of freedom. The lowest natural frequency of the system with nonlinear absorber is about 5% lower than a system with linear absorber in the same conditions, which is a significant improvement.

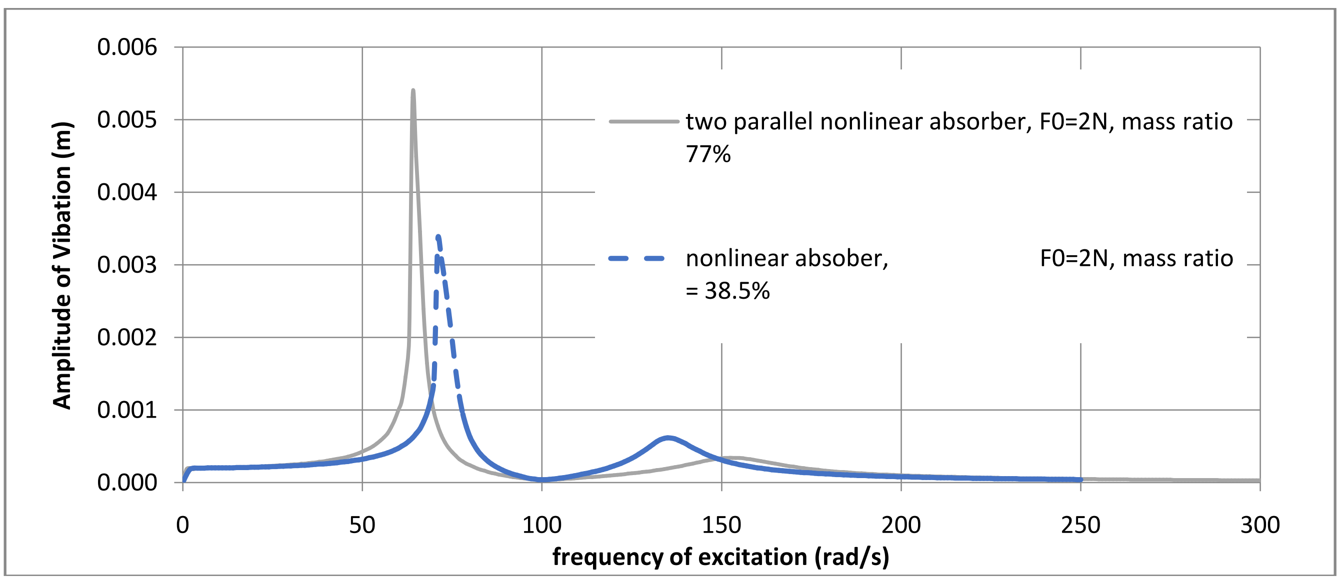

Figure 18: Amplitude of vibration of the main system (K=10000N/m, m=1Kg) verses frequency of excitation, F_0=2N, of a tune vibration absorber for two different nonlinear stiffness; damping ratio=0.05

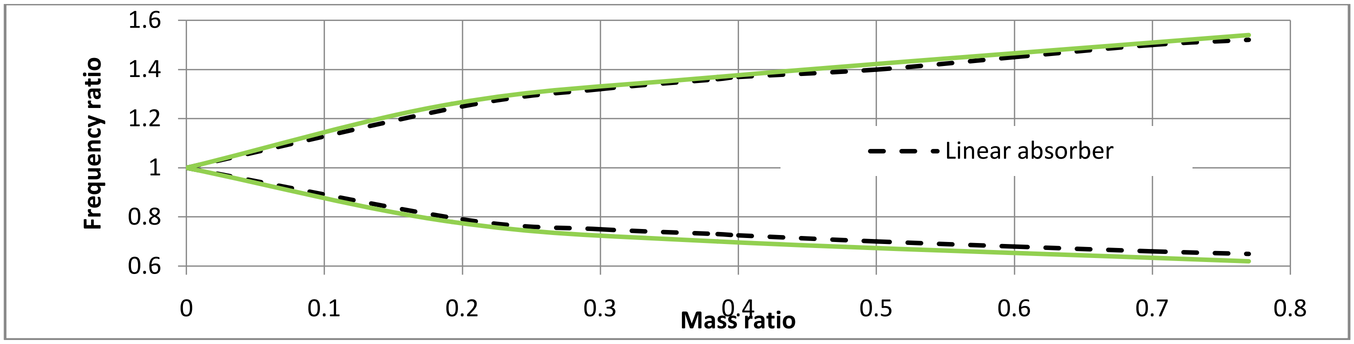

Figure 19: a tuned two degree of freedom system natural frequencies verses the mass ratio of a linear and nonlinear (Mag-spring) vibration absorber. F_0=2N, damping ratio=0.05.

CONCLUSIONS

In this study, Mag-spring as a new application of magnetic spring is introduced as a nonlinear softener absorber for the first time. The coordinate of the mag-spring is shifted to a new point, it will work as a softening spring. The non-linearity of the spring was modelled and observed hat curve fitting is best with 11th order. The lower the amplitude of absorber, the higher is the stiffness and the higher the amplitude of absorber, the lower the stiffness.

The program which was written to simulate the behaviour of a system with two degree of freedom solved the nonlinear differential equation of the primary system and the nonlinear absorber simultaneously. At this iterative program, for all frequency of excitation, the time domain of the amplitude of vibration of the main system and the absorber iteratively was calculated. As the two differential equations are coupled in time domain as well as frequency domain; another inner iterative time loop is used. As a result, time increment is depended to the convergence of the differential equation at the current time. Finally, the maximum amplitude of the response at steady state time domain (at each frequency of excitation) is used to produce the frequency domain of the amplitude of the system.

Finally, the Program was run several times, with different parameters with linear and nonlinear absorber to check the effectiveness of using a nonlinear absorber. It was shown how the light damping coefficient has been used to reach to a steady state response. Also it has been shown how the mass ratio will be defined for a nonlinear absorber to be tuned to the primary system. The nonlinear Mag-Spring behaviour has been compared to an equivalent linear absorber.

Finally, 4 different ideal nonlinear stiffness curves were introduced to be used as vibration absorbers. The response of these various nonlinear verses frequency of excitation is compared with the nonlinear Mag-spring response and a linear response as well. The utilised techniques in providing the stiffness curves were found to give encouraging results in regard of vibration cancellation.

REFERENCES

[1]. Godoy, W. R., & Trindade, M. A. (2020). Design and analysis of a geometrically nonlinear dynamic vibration absorber. Journal of Computational and Nonlinear Dynamics, 15(8).
[2]. Lupini, A., Mitra, M., & Epureanu, B. I. (2019). Application of tuned vibration absorber concept to blisk ring dampers: a nonlinear study. Journal of Engineering for Gas Turbines and Power, 141(10).
[3]. Addessi D., Lacarbonara W., Paolone A., On the linear normal modes of planar pre-stressed curved beams, Journal of Sound and Vibration 284 (2005) 1075–1097.
[4]. Jiao, G. (2020). Design of a Magnetic Interaction-Based Vibration Absorber for Continuous Beam. *Shock and Vibration*, *2020*, 1-18.
[5]. Kakou, P., & Barry, O. (2020, October). Simultaneous Vibration Mitigation and Energy Harvesting for a Nonlinear Oscillator. In *Dynamic Systems and Control Conference* (Vol. 84287, p. V002T38A002). American Society of Mechanical Engineers.
[6]. Casciati, F., Magonette, G., and Marazzi, F. (2006). Technology of semiactive devices and applications in vibration mitigation, Wiley, Chichester, UK.Chu et al. 2005
[7]. Cella G., DeSalvo R., Sannibale V., Tariq H., Viboud N., Takamori A., Seismic attenuation performance of the first prototype of a geometric anti-spring filter, Nuclear Instruments and Methods in Physics Research A 487 (2002) 652–660.

[8]. Cella G., Sannibale V., DeSalvo R., Marka S., Takamori A., Monolithic geometric anti-spring blades, Nuclear Instruments and Methods in Physics Research A 540 (2005) 502–519.
[9]. Deshpande S., Mehta S., Jazar G.N., Jump avoidance conditions for piecewise linear vibration isolator, Proceedings of the ASME International Design Engineering Technical Conferences and Computers and Information in Engineering Conference—DETC2005, 20th Biennial Conference on Mechanical Vibration and Noise, Long Beach, CA, USA, Vol. 1C, 2005, pp. 1955–1962.
[10]. Dogan, H., Sims, N. D., & Wagg, D. J. (2022). Design, testing and analysis of a pivoted-bar inerter device used as a vibration absorber. *Mechanical Systems and Signal Processing*, *171*, 108893.
[11]. Garoi F., Winterflood J., Ju L., Jacob J., Blair D.G., Passive vibration isolation using a Roberts linkage, Review of Scientific Instruments 74 (2003) 3487–3491.
[12]. Habib, G., & Epasto, A. (2023). Towed wheel shimmy suppression through a nonlinear tuned vibration absorber. *Nonlinear Dynamics*, 1-14.
[13]. Harne R.L., Fuller C.R., Modeling of a passive distributed vibration control device using a superposition technique, Journal of Sound and Vibration 331 (2012) 1859–1869
[14]. Yang, F., Sedaghati, R., & Esmailzadeh, E. (2022). Vibration suppression of structures using tuned mass damper technology: A state-of-the-art review. *Journal of Vibration and Control*, *28*(7-8), 812-836.
[15]. Heertjes M., van de Wouw N., Nonlinear dynamics and control of a pneumatic vibration isolator, ASME Journal of Vibration and Acoustics 128 (2006) 439–448.
[16]. Ibrahim R.A., Somnay R.J., Nonlinear dynamic analysis of an elastic beam isolator sliding on frictional supports, Journal of Sound and Vibration 308 (2007) 735–757.
[17]. Ha, S. I., & Yoon, G. H. (2021). Numerical and experimental studies of pendulum dynamic vibration absorber for structural vibration. *Journal of Vibration and Acoustics*, *143*(1).
[18]. Kari L., A nonlinear dynamic stiffness model of a vibration isolator at finite deformation, Materials Science Forum 440–441 (2003) 475–480.
[19]. Kari L., On the dynamic stiffness of preloaded vibration isolators in the audible frequency range: modeling and experiments, Journal of Acoustical Society of America 113 (2003) 1909–1921.
[20]. Chang, W., Jin, X., Huang, Z., & Cai, G. (2021). Random response of nonlinear system with inerter-based dynamic vibration absorber. *Journal of Vibration Engineering & Technologies*, *9*(8), 1903-1909.
[21]. Mizuno T., Vibration isolation using zero-power magnetic suspension, Proceedings of the 15th World Congress of the International Federation of Automatic Control (IFAC), Barcelona, Spain, 2002, p. 955.
[22]. JensenJ.S., Buckling of an elastic beam with added high-frequency excitation, International Journal of Nonlinear Mechanics 35 (2000) 217–227.
[23]. Jiang L., Stredulinsky D., Szabo J., Chernuka M.W., Numerical characterization of nonlinear stiffness properties of pre-stressedvibration isolation mounts, Canadian Acoustics—Acoustique Canadienne 30 (2002) 70–71.
[24]. MizunoT., Proposal of a vibration isolation system using zero-power magnetic suspension, Proceedings of Asia-Pacific Vibration Conference, Seoul, Korea, Vol. 2, 2001, pp. 423–427.
[25]. Duc Phuc, V., & Tran, V. T. (2022). Optimization design for multiple dynamic vibration absorbers on damped structures using equivalent linearization method. *Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multi-body Dynamics*, *236*(1), 41-50.
[26]. ZhangJ.Z., LiD., ChenM.J., DongS., An ultra-low frequency parallel connection nonlinear isolator for precision instruments, Key Engineering Materials 257–258 (2004) 231–236.