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Fixed Points for Two Self- Mappings in Cone Metric Spaces

K. Prudhvi

Department of Mathematics, University College of Science, Saifabad, Osmania University, Hyderabad, Telangana State, India

ABSTRACT

In this paper, we obtain a unique common fixed point theorem for Two self-mappings in cone metric space with using occasionally weakly compatible condition. Our results are generalizing, modifying and improving some the well known and comparable results existing in the present literature.

Keywords: Fixed point theorem, common fixed point theorem, occasionally weakly compatible, cone metric space.

INTRODUCTION

The study of fixed point theorems are important area of the non-linear analysis. Recently the concept of cone metric space was introduced by the authors Huang and Zhang [1]. And they have generalized the concept of a metric space into a cone metric space and the authors replacing the real numbers by an ordered Banach space, and also proved some of the fixed point theorems in this cone metric spaces. Subsequently many authors have been extending, improving, and modifying these cone metric space results in many ways. (for e.g., see, [3-11]). And also recently Bhatt and Chandra [2] proved fixed point theorems in occasionally weakly compatible mappings in cone metric spaces. In this paper, we proved a unique common fixed point theorem for two self-mappings in cone metric spaces with using the condition occasionally weakly compatible mappings.

PRELIMINARIES

The following are useful in our main results which are due to [1],[9].

Definition 2.1. Let N be a real Banach space. A subset F of N is said to be a cone if and only if
(i) F is closed, non-empty and $F \neq \{0\}$;
(ii) $\alpha, \beta \in \mathbb{R}$, $\alpha ,\beta \geq 0$, $u, v \in F \Rightarrow \alpha u + \beta v \in F$;
(iii) $F \cap (-F) = \{0\}$.
Given a cone $F \subset N$, we define a partial ordering $\leq$ with respect to F by $\alpha \leq \beta \Leftrightarrow \beta - \alpha \in F$. A cone F is called normal if $\exists$ a number $B > 0 \ni \forall\ \alpha, \beta \in F$,

$$0 \leq \alpha \leq \beta \Rightarrow \| \alpha \| \leq B \| \beta \| .$$

The least positive number B satisfying the above inequality is called the normal constant of F, while $\alpha \ll \beta$ stands for $\beta - \alpha \in$ interior of F.

Definition 2.2. Let X be a nonempty set of N. Suppose that the map ρ: $X \times X \rightarrow N$ satisfying the following conditions:
(1). $0 \leq \rho(\alpha, \beta)$ for all $\alpha, \beta \in X$ and $\rho(\alpha, \beta) = 0$ if and only if $\alpha = \beta$;
(2). $\rho(\alpha, \beta) = \rho(\beta, \alpha)$ for all $\alpha, \beta \in X$;
(3). $\rho(\alpha, \beta) \leq \rho(\alpha, \gamma) + \rho(\gamma, \beta)$ for all $\alpha, \beta, \gamma \in X$.

Then ρ is called a cone metric on X and (X,ρ) is called a cone metric space.

Definition 2.3. Let (X, ρ) be a cone metric space .We say that $\{x_n\}$ is
(i) a convergent sequence if for any $h \gg 0$, there exists a natural number N such that for all $n>N$,
$\rho(x_n, x) \ll h$, for some fixed x in X .We denote this by $x_n \rightarrow x$ (as $n \rightarrow \infty$) .

(ii) a Cauchy sequence if for every h in N with h>>0, there exists a natural number N such that for all n, m>N, $\rho\ (x_{n,} x_m)$<< h.

Definition 2.4. A cone metric space (X, ρ) is said to be complete if every Cauchy sequence is a convergent sequence.

Definition 2.5 [9]. Let K and L be self-mappings of a set X. If h = Kx = Lx for some x in X, then x is called a coincidence point of K and L, and h is said to be a point of coincidence of K and L.

Proposition 2.1. Let K and L be occasionally weakly compatible self-mappings of a set X if and only if there exists a point x in X which is coincidence point of K and L at which K and L are commute.

Lemma 2.1. Let X be a set, K, L are occasionally weakly compatible self-mappings of X. If K and L have a unique point of coincidence h = Kx = Lx, then h is the unique common fixed point of K and L.

MAIN RESULTS

In this section we prove a unique common fixed point theorem for two self -mappings in cone metric space with using the condition occasionally weakly compatible. The following result is an extension, modifying and improving the results of [2].

Theorem 3.1: Let (X, ρ) be a cone metric space and S be a normal cone . Suppose that g and h are two self-mappings of X .Then the following conditions are hold.

(i) $\rho(gx, gy) < \text{Max}\{ \rho(hx, hy) + \rho(hx, gy) /2 , \rho(hy, gx), \rho(hy, gy) \}$, for all x,y ϵ X and x ≠ y.

(ii) g and h are OWC.

Then g and h have a unique common fixed point.

Proof: From (ii) given that g and h are occasionally weakly compatible. Then there exists a point αϵX such that gα = hα, ghα = hgα. We claim that: gα is a unique common fixed point of g and h. Now First we ascertain that gα is a fixed point of g. For if ggα ≠ gα. Then from (i) we get that

$$\begin{aligned}
\rho(g\alpha, gg\alpha) &< \text{Max}\{ \rho(h\alpha, hg\alpha) + \rho(h\alpha, gg\alpha) /2 , \rho(hg\alpha, g\alpha) , \rho(hg\alpha, gg\alpha) \},\\
&= \text{Max}\{ \rho(g\alpha, hg\alpha) + \rho(g\alpha, gg\alpha) /2 , \rho(gh\alpha, g\alpha) , \rho(gh\alpha, gg\alpha) \},\\
&= \text{Max}\{ \rho(g\alpha, gg\alpha) + \rho(g\alpha,gg\alpha) /2 , \rho(gg\alpha, g\alpha) , \rho(gg\alpha, gg\alpha) \},\\
&= \text{Max}\{2 \rho(g\alpha, gg\alpha) /2 , \rho(gg\alpha, g\alpha) , 0\}\\
&= \rho(gg\alpha, g\alpha),\\
&< \rho(gg\alpha, g\alpha) , \text{ which is a contradiction.}
\end{aligned}$$

Hence ggα = gα and ggα = ghα = hgα = gα. Thus gα is a common fixed point of g and h.

Now we prove that Uniqueness: for suppose that there exists α,β ϵX such that gα = hα = α and gβ = hβ = β and α≠β. Then from (i) we get that

$$\begin{aligned}
\rho(\alpha, \beta) = \rho(g\alpha, g\beta) &< \text{Max}\{ \rho(h\alpha, h\beta) + \rho(h\alpha, g\beta) /2 , \rho(h\beta, g\alpha), \rho(h\beta, g\beta) \},\\
&= \text{Max}\{ \rho(\alpha, \beta) + \rho(\alpha, \beta) /2 , \rho(\beta,\alpha) , \rho(\beta, \beta) \},\\
&= \text{Max}\{2 \rho(\alpha, \beta) /2 , \rho(\beta,\alpha) ,0\},\\
&= \rho(\alpha, \beta),\\
&< \rho(\alpha, \beta), \text{ which is a contradiction.}
\end{aligned}$$

Therefore, α = β. Therefore , g and h are having a unique common fixed point.
Hence proved.

CONCLUSION

Our results are more general then the results of [2].

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