International Journal of Enhanced Research in Science, Technology & Engineering
ISSN: 2319-7463, Vol. 10 Issue 5, May-2021, Impact Factor: 7.957

"Common Fixed Points" for OWC Self-Mappings in S- Spaces

K. Prudhvi

Department of Mathematics, University College of Science, Saifabad, Osmania University, Hyderabad, Telangana State, India.

ABSTRACT

In this paper, we prove a unique common fixed point theorem for OWC (Occasionally Weakly Compatible) four self- mappings in S-spaces(Symmetric spaces). This result is a generalization, improvement of some of the well known comparable results existing in the literature.

AMS subject classifications: 47H10, 54H25.

Keywords: Common fixed point theorem, OWC (Occasionally Weakly Compatible), S-space (Symmetric space).

INTRODUCTION

Before 1968, Banach contraction principle is a fundamental concept in fixed point theory. In 1968, R. Kannan [13] obtained a fixed point theorem for a mapping satisfying contractive condition that need not satisfy continuity. Later on many authors has been extended and improved and generalized these result in different ways (see for e.g. [1- 12 &14-15]). Hicks and Rhoades [9] obtained some common fixed point theorems in symmetric spaces and semi metric spaces. And Abbas and Rhoades [6] obtained common fixed point theorems for OWC(Occasionally Weakly Compatible mappings) satisfying a generalized contractive conditions in symmetric spaces. In the present paper, we obtained, a common fixed point theorem for OWC(Occasionally Weakly Compatible) four self- mappings in S-spaces(Symmetric spaces).

.

PRELIMINARIES

The following definitions are useful in our main theorem and which are due to [6].

Definition 2.1.Two maps A and B are said to be weakly compatible if they commute at coincidence points.

Definition 2.2. Let X be a set , f, g self maps of X. A point x in X is called a coincidence point of f and g iff fx = gx. We shall call w = fx = gx a point of coincidence of f and g.

Definition 2.3. Two self maps f, g of a set X. A point x in X are said to be occasionally weakly compatible iff there exists appoint x in X which is a coincidence point of f and g at which f and g are commute.

Lemma 2.4. [10].Let X be a set , f, g are occasionally weakly compatible self -maps of X. If f and g have a unique point coincidence w = fx = gx , then w is a unique common fixed point of f and g.

Our results are proved in symmetric spaces, which are more general than metric spaces.

Definition 2.5. Let X be a set. A symmetric on X is a mapping $\rho : X \times X \to [0, \infty)$ such that $\rho(x, y) = 0$ if and only if x = y, and $\rho(x, y) = \rho(y, x)$ for x, y ϵ X. Let A϶[0,∞), $R_A^+ = [0, A)$. Let F: $R_A^+ \to R$ satisfy

(i) F(0) = 0 and F(t) > 0 for each tϵ(0,A)and

(ii) F is non decreasing on R_A^+.

Define, F{0,A}={F: $R_A^+ \to$ R: F satisfies (i) - (ii)}.

Let Aϵ[0,∞). Let ψ: $R_A^+ \to$ R satisfies

(i) Ψ(t)< t for each t ϵ (0, A) and

(ii) Ψ is non decreasing.

Define, ψ{0,A)={ψ: $R_A^+ \to$ R: F satisfies (i) - (ii) above}.

Some of the examples of mappings F: $R_A^+ \to$ R: F satisfies (i) - (ii) ,we have refer to [15] .

Definition 2.6. A control function Φ is defined by Φ: $\mathbb{R}^+ \to \mathbb{R}^+$ which satisfies Φ (t) = 0 if and only if t = 0.

MAIN RESULTS

In this section, we proved, a unique common fixed point theorem for four self-mappings for OWC(Occasionally Weakly Compatible) self- mappings in S-spaces(symmetric spaces).

Theorem 3.1. Let X be a set, ρ a symmetric on X. Let f, g, M and N be four self- mappings of X satisfying the following conditions:

(i) $F(\Phi(\rho(fu, gv))) \leq \psi(F(K_\Phi(u, v))$,

where, $K_\Phi(u, v) = Max\ \{ \Phi(\rho(Mu, Nv) , \Phi(\rho(fu, Mu)) + \Phi(\rho(gv, Nv))/2 , \Phi(\rho(fu, Nv)) + \Phi(\rho(gv, Mu))/2))\}$,

for each u ,v ϵ X, F ϵ F([0, B) and ψ ϵ ψ [0, F((B-0)),

where B = D if D = ∞ and B > D if D < ∞. And

(ii) (f, M) and (g, N) are OWC.

Then f, g , M and N are having a unique common fixed point in X.

Proof. Given by (ii) (f, M) and (g, N) are OWC, then there exists two points u, v ϵ X such that fu = Mu and gv = Nv. Suppose that fu ≠ gv. Then from (i) we get that

$$K_\Phi(u, v) = Max\ \{ \Phi(\rho(fu, gv) , \Phi(\rho(fu, fu)) + \Phi(\rho(gv, gv))/2 , \Phi(\rho(fu, gv)) + \Phi(\rho(gv, fu))/2))\},$$
$$= Max\ \{ \Phi(\rho(fu, gv) , 0, \Phi(\rho(fu, gv)) + \Phi(\rho(gv, fu))/2))\},$$
$$= Max\{ \Phi(\rho(fu, gv), \Phi(\rho(fu, gv))\}$$
$$= \Phi(\rho(fu, gv)). \quad \ldots \quad (1).$$

Then by (i) and (1) we get that

$$0 < F(\Phi(\rho(fu, gv))) < \psi(F(K_\Phi(u, v))$$
$$= \psi(F(\Phi(\rho(fu, gv)))$$
$$< F(\Phi(\rho(fu, gv))) \text{ , which is a contradiction.}$$

Therefore , Φ(ρ(fu, gv)) = 0 .

⇒ ρ(fu, gv) = 0 .

⇒ fu = gv. That is, fu = Mu = gv = Nv.

Suppose that there exists another point w such that fw = Mw, then using (i) and (1) we get that

fw = Mw = gv = Nv = fu = Mu.

Hence z = fu = Mw is the unique point of f and M.

By symmetry there exists a unique point z_1 ϵ X such that $z_1 = gw = Nz_1$.

Then it follows that $z = z_1$,

z is a common fixed point of f, g, M and N and z is unique from condition (i) and (1).

Therefore, f, g, M and N are having a unique common fixed point in X. And this completes the proof of the theorem.

CONCLUSION

Our results are more general than the results of [6].

ACKNOWLEDGMENT

The author is grateful to the reviewers to review this article and giving the valuable suggestions to improve this article.

REFERENCES

[1]. M.Aamri and D.El. Moutawakil, Some new common fixed point theorems under strict contractive conditions, J.Math.Anal.Appl.270(2002),181-188.

[2]. M.A.Al.Thagafi, N. Shahzad,Generalized I – nonexpensive selfmaps andinvariant approximations, Acta. Math. Sinica 24(2008), 867-876.

[3]. A.Aliouche, A common fixed point theorem for weakly compatible mappings in symmetric spaces satisfying a

contractive condition of integral type, J. Math. Anal. Appl.322(2006), 796-802.
[4]. M. Abbas and G. Jungck, Common fixed point results for non commuting mappings without continuity in cone metric spaces, J. Math. Anal. Appl. 341(2008), 416-420.
[5]. M**.** Abbas, B. E. Rhoades, Fixed and periodic point results in cone metric spaces, Appl. Math. Lett. 21(2008), 511-515.
[6]. M**.** Abbas, B. E. Rhoades, Common fixed point theorems for occasionally weakly compatible mappings satisfying a generalized contractive condition, Mathematical Communications 13(2008), 295-301.
[7]. I. Altun, B. Durmaz, Some fixed point theorems on ordered cone metric spaces, Rend. Circ. Mat. Palermo 58(2009), 319-325.
[8]. Arvind Bhatt and Harish Chandra, Occasionally weakly compatible mappings in cone metric space , Applied Mathematical Sciences, Vol. 6,no. 55,(2012), 2711 – 2717.
[9]. T.L.Hics and B.E. Rhoades, Fixed point theory in symmetric spaces with applications to probabilistic spaces, Non –linear Anal. 36(1999), 331-344.
[10]. G.Jungck and B.E. Rhoades , Fixed point theorems for occasionally weakly compatible mappings , Fixed Point Theory , 7(2006), 286-296.
[11]. G.Jungck and B.E. Rhoades , Fixed point theorems for occasionally weakly compatible mappings, Erratum, Fixed Point Theory, 9(2008), 383-384.
[12]. G.Jungck, Compatible mappings and common fixed points, Int. J. Math & Math. Sci.,9(1986),771-779.
[13]. R. Kannan , Some results on fixed points, Bull. Calcutta. Math.Sci.60(1968),71-76.
[14]. K. Prudhvi, A Study on Fixed Points for " Occasionally Weakly Compatible" Mappings in C- Metric Spaces, International
[15]. Journal of All Research Education and Scientific Methods (IJARESM), Vol.8, Issue 6, (2020), 424-426.
[16]. X. Zhang, Common fixed point theorems for some new generalized contractive type mappings, J. Math. Anal. Appl. 333(2007), 780-786.