Automorphisms of non-abelian metabelian groups of order upto 24

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ABSTRACT

In this paper, G be the non-abelian metabelian group and $Aut(G)$ denotes the automorphism group of group G. A group is metabelian group if its commutator group is abelian or if it has an abelian normal subgroupin which the factor group is also abelian.There are 35 non-Abelian metabelian groups of order less than or equal to 24. In this study, we investigate the automorphism groups of all non-abelian metabelian groups of order less than or equal to 24 andthe verification has beenmade through GAP(Groups Algorithm Programming) software.

Keywords: Metabelian Groups, Automorphism, Order.

INTRODUCTION

Let $\boldsymbol{G}$ be a finite metabelian group, $\boldsymbol{Z_n}$ denotes the cyclic group of order $\boldsymbol{n}$, $\boldsymbol{S_n}$ denotes the permutation group of degree $\boldsymbol{n}$, $\boldsymbol{D_n}$ denotes the dihedral group of order $\boldsymbol{2n}$, $\boldsymbol{Q_n}$ denotes quatornion group. A group $\boldsymbol{G}$ is said to be metabelian if $\boldsymbol{G'}$, the derived subgroup of $\boldsymbol{G}$ is abelian. Much has been investigated about the properties of metabelian groups in the literature. In paper [3], structure of metabelian groups of order upto $\boldsymbol{24}$ has been described. In paper [4], the authors studied about the conjugacy classes of metabelian groups of order less than $\boldsymbol{24}$.In paper [1], [2], automorphisms of some non-abelian groups of order $\boldsymbol{p^4}$are computed. In the present paper, we shall find the automorphisms of metabelian groups of order less than equal to $\boldsymbol{24}$.

In [3], Rehman Abdul described the metabelian groups of order less than or equal to 24.

$$(1)D_3 \cong S_3 \cong < a, b; a^3 = b^2 = 1, b\ a\ b = a^{-1} >.$$
$$(2)D_4 \cong < a, b; a^4 = b^2 = 1, b\ a\ b = a^{-1} >.$$
$$(3)Q_3 \cong < a, b: a^4 = 1, b^2 = a^2, a\ b\ a = b >.$$
$$(4)D_5 \cong < a, b: a^5 = b^2 = 1, b\ a\ b = a^{-1} >.$$
$$(5)Z_3 \rtimes Z_4 \cong < a, b: a^3 = b^4 = 1, b^{-1}a\ b = a^2 >.$$
$$(6)A_4 \cong < a, b, c: a^2 = b^2 = c^3 = 1, b\ a = a\ b, c\ a = a\ b\ c, c\ b = a\ c >.$$
$$(7)D_6 \cong < a, b: a^6 = b^2 = 1, b\ a\ b = a^{-1} >.$$
$$(8)D_7 \cong < a, b: a^7 = b^2 = 1, b\ a\ b = a^{-1} >.$$
$$(9)D_8 \cong < a, b: a^8 = b^2 = 1, b\ a\ b = a^{-1} >.$$
$$(10)G \cong < a, b: a^8 = b^2 = 1, b\ a\ b = a^3 >.$$
$$(11)Q_{16} \cong < a, b: a^8 = 1, \quad a^4 = b^2, \quad a\ b\ a = b >.$$
$$(12)D_4 \times Z_2 \cong < a, b, c: a^4 = b^2 = c^2 = 1, a\ c = c\ a, b\ c = c\ b, b\ a\ b = a^{-1} >.$$
$$(13)Q_3 \times Z_2 \cong < a, b, c: a^4 = b^4 = c^2 = 1, b^2 = a^2, b\ a = a^3 b, a\ c = c\ a, b\ c = c\ b >.$$
$$(14)Modular - 16 = G \cong < a, b: a^8 = b^2 = 1, a\ b = b\ a^5 >.$$
$$(15)B \cong < a, b: a^4 = b^4 = 1, a\ b = b\ a^3 >.$$
$$(16)K \cong < a, b, c: a^4 = b^2 = c^2 = 1, a\ b = b\ a, a\ c = c\ a, c\ b = a^2 b\ c >.$$
$$(17)G_{4,4} \cong < a, b: a^4 = b^4 = (a\ b\ )^2 = 1, a\ b^3 = b\ a^3 >.$$
$$(18)D_9 \cong < a, b: a^9 = b^2 = 1, b\ a\ b = a^{-1} >.$$
$$(19)S_3 \times Z_3 \cong < a, b, c: a^3 = b^2 = c^3 = 1, b\ a = a^{-1} b, a\ c = c\ a, b\ c = c\ b >.$$
$$(20)(Z_3 \times Z_3) \rtimes Z_2 \cong < a, b, c: a^2 = b^3 = c^3 = 1, b\ c = c\ b, b\ a\ b = a, c\ a\ c = a >.$$
$$(21)D_{10} \cong < a, b: a^{10} = b^2 = 1, b\ a\ b = a^{-1} >.$$
$$(22)Fr_{20} \cong Z_5 \rtimes Z_4 \cong < a, b: a^4 = b^5 = 1, b\ a = a\ b^2 >.$$
$$(23)Z_5 \rtimes Z_4 \cong < a, b: a^4 = b^5 = 1, b\ a\ b = a >.$$

$$(24) Fr_{21} \cong Z_7 \rtimes Z_3 \cong < a, b: a^3 = b^7 = 1, b\ a = a\ b^2 >.$$
$$(25) D_{11} \cong < a, b: a^{11} = b^2 = 1, b\ a\ b = a^{-1} >.$$
$$(26) S_3 \times Z_4 \cong < a, b, c: a^3 = b^2 = c^4 = 1, a^b = a^{-1}, a\ c = c\ a, b\ c = c\ b >.$$
$$(27) S_3 \times Z_2 \times Z_2 \cong < a, b, c, d: a^3 = b^2 = c^2 = d^2 = 1, a^b = a^{-1}, a\ c = c\ a, a\ d = d\ a, b\ c = c\ b, b\ d = d\ b, c\ d = d\ c >.$$
$$(28) D_4 \times Z_3 \cong < a, b, c: a^3 = b^4 = c^2 = 1, b^c = b^{-1}, a\ b = b\ a, a\ c = c\ a >.$$
$$(29) Q \times Z_3 \cong < a, b, c: a^4 = c^3 = 1, a^2 = b^2, a^b = a^{-1}, b\ c = c\ b, a\ c = c\ a >.$$
$$(30) A_4 \times Z_2 \cong < a, b, c, d: a^2 = b^2 = c^3 = d^2 = 1, a\ b = b\ a, c\ a = a\ b\ c, a\ d = d\ a, a\ c = c\ b, b\ d = d\ b, c\ d = d\ c >.$$
$$(31) Q_{12} \cong < a, b: a^{12} = 1, a^6 = b^2, b^{-1} a\ b = a^{-1} >.$$
$$(32) D_{12} \cong < a, b: a^{12} = b^2 = 1, b\ a\ b = a^{-1} >.$$
$$(33) Z_6 \rtimes Z_4 \cong < a, b: a^4 = b^6 = 1, b\ a\ b = a >.$$
$$(34) Z_3 \rtimes Z_8 \cong < a, b: a^3 = b^8 = 1, b\ a\ b^{-1} = a^{-1} >.$$
$$(35) Z_3 \rtimes Q \cong < a, b, c: a^2 = b^6 = c^2 = 1, ab = ba, \quad ac = ca, (cb)^2 = 1 >.$$

Automorphisms of all non-abelian metabelian groups of order less than equal to 24

Automorphisms of groups of order 6

Automorphisms of D_3:

$$D_3 \cong S_3 \cong < a, b; a^3 = b^2 = 1, bab = a^{-1} >$$
Any automorphism $\psi \in Aut(D_3)$ sends the generators of D_3 as
$$a \to a^i; (i, 3) = 1,$$
$$b \to a^i b; 1 \le i \le 3.$$
Thus,
$$|Aut(D_3)| = 6.$$

Automorphisms of Q_3:

Automorphisms of groups of order 8

Automorphisms of D_4:

$$D_4 \cong < a, b; a^4 = b^2 = 1, bab = a^{-1} >$$
Any automorphism $\psi \in Aut(D_4)$ sends the generators of D_4 as
$$a \to a^i; (i, 2) = 1, 1 \le i \le 4,$$
$$b \to a^j b; 1 \le i \le 4.$$
Hence
$$|Aut(D_4)| = 8.$$

$$Q_3 \cong < a, b: a^4 = 1, b^2 = a^2, aba = b >$$
Any automorphism $\psi \in Aut(Q_3)$ sends the generators of Q_3 as
$$a \to a^i b^j; \quad i \in \{0,1,3\}, \ j \in \{0,1,3\}, \quad i + j \ne 0,$$
$$b \to a^i b^j; \quad i \in \{0,1,3\}, \ j \in \{0,1,3\}, \quad i + j \ne 0.$$
Therefore,
$$|Aut(Q_3)| = 24.$$

Automorphisms of groups of order 10

Automorphisms of D_5:

$$D_5 \cong < a, b: a^5 = b^2 = 1, bab = a^{-1} >$$
Any automorphism $\psi \in Aut(D_5)$ sends the generators of D_5 as
$$a \to a^i; (i, 5) = 1,$$
$$b \to a^j b; 1 \le j \le 5.$$
Thus,
$$|Aut(D_5)| = 20.$$

Automorphisms of groups of order 12

Automorphisms of $Z_3 \rtimes Z_4$:

$$Z_3 \rtimes Z_4 \cong < a, b: a^3 = b^4 = 1, b^{-1}ab = a^2 >$$

Any automorphism $\psi \in Aut(Z_3 \rtimes Z_4)$ sends the generators of $Z_3 \rtimes Z_4$ as

$$a \to a^i; 1 \leq i < 3,$$
$$b \to a^i b^j; 2 \nmid j, 0 \leq i \leq 2, 1 \leq j \leq 3.$$

Thus,

$$|Aut(Z_3 \rtimes Z_4)| = 12.$$

Automorphisms of A_4:

$$A_4 \cong < a, b, c: a^2 = b^2 = c^3 = 1, ba = ab, ca = abc, cb = ac >$$

Any automorphism $\psi \in Aut(A_4)$ sends the generators of A_4 as

$$a \to a^i b^j; 0 \leq i, j \leq 1, \quad i + j \neq 0,$$
$$b \to a^i b^j; 0 \leq i, j \leq 1, \quad i + j \neq 0,$$
$$c \to a^i b^j c^k; 0 \leq i, j \leq 1, \quad k \in \{1,2\}.$$

So

$$|Aut(A_4)| = 24.$$

Automorphisms of D_6:

$$D_6 \cong < a, b: a^6 = b^2 = 1, bab = a^{-1} >$$

Any automorphism $\psi \in Aut(D_6)$ sends the generators as

$$a \to a^i; \quad i \in \{1,5\},$$
$$b \to a^i b; 0 \leq i \leq 5.$$

So

$$|Aut(D_6)| = 12.$$

Automorphisms of groups of order 14

Automorphisms of D_7:

$$D_7 = < a, b: a^7 = b^2 = 1, bab = a^{-1} >$$

Any automorphism $\psi \in Aut(D_7)$ sends the generators of D_7 as

$$a \to a^i; 1 \leq i \leq 6,$$
$$b \to a^j b; 0 \leq j \leq 6.$$

So

$$|Aut(D_7)| = 42.$$

Automorphisms of groups of order 16

Automorphisms of D_8:

Any automorphism $\psi \in Aut(D_8)$ sends the generators of D_8 as

$$a \to a^i; (i, 2) = 1, 1 \leq i < 8,$$
$$b \to a^j b; 0 \leq i \leq 7.$$

So

$$|Aut(D_8)| = 32.$$

Automorphisms of G:

$$G \cong < a, b: a^8 = b^2 = 1, bab = a^3 >$$

Any automorphism $\psi \in Aut(G)$ sends the generators of G as

$$a \to a^i; (2, i) = 1, 1 \leq i \leq 8,$$
$$b \to a^i b; 2/i, 0 \leq i < 8.$$

Thus,

$$|Aut(G)| = 16.$$

Automorphisms of Q_{16}:

$$Q_{16} \cong < a, b: a^8 = 1, a^4 = b^2, \quad aba = b >$$

Any automorphism $\psi \in Aut(Q_{16})$ sends the generators of Q_{16} as

$$a \to a^i; (2, i) = 1, 1 \leq i \leq 8,$$

$$b \to a^i b^j; 0 \le i \le 3, \quad j \in \{1,3\}.$$

Thus,

$$|Aut(Q_{16})| = 32.$$

Automorphisms of $D_4 \times Z_2$:

$$D_4 \times Z_2 \cong < a,b,c: a^4 = b^2 = c^2 = 1, ac = ca, bc = cb, bab = a^{-1} >$$

Any automorphism $\psi \in Aut(D_4 \times Z_2)$ sends the generators of $D_4 \times Z_2$ as

$$a \to a^i c^j; \quad i \in \{1,3\}, \quad j \in \{0,1\},$$
$$b \to a^k b c^l; 0 \le k \le 3, \quad l \in \{0,1\},$$
$$c \to a^m c; \quad m \in \{0,2\}.$$

Hence

$$|Aut(D_4 \times Z_2)| = 64.$$

Automorphisms of $Q_3 \times Z_2$:

$$Q_3 \times Z_2 \cong < a,b,c: a^4 = b^4 = c^2 = 1, b^2 = a^2, ba = a^3 b, ac = ca, bc = cb >$$

Any automorphism $\psi \in Aut(Q_3 \times Z_2)$ sends the generators of $Q_3 \times Z_2$ as

$$a \to a^i b^j c^k; \quad i,j \in \{0,1,3\}, \quad k \in \{0,1\}, \quad i + j \ne 0,$$
$$b \to a^l b^m c^n; \quad l,m \in \{0,1,3\}, \quad n \in \{0,1\}, \quad i + j \ne 0,$$
$$c \to a^q c; \quad q \in \{0,2\}.$$

So

$$|Aut(Q_3 \times Z_2)| = 96.$$

Automorphisms of $Modular - 16$:

$$Modular - 16 = G \cong < a,b: a^8 = b^2 = 1, ab = ba^5 >$$

Any automorphism $\psi \in Aut(G)$ sends the generators of G as

$$a \to a^i b^j; 2 \nmid i, 1 \le i \le 8, \quad j \in \{0,1\},$$
$$b \to a^r b; \quad r \in \{0,4\}.$$

So

$$|Aut(G)| = 16.$$

Automorphisms of B:

$$B \cong < a,b: a^4 = b^4 = 1, ab = ba^3 >$$

Any automorphism $\psi \in Aut(B)$ sends the generators of B as

$$a \to a^i b^j; \quad i \in \{1,3\}, \quad j \in \{0,2\},$$
$$b \to a^l b^m; 0 \le l \le 3, \quad m \in \{1,3\}.$$

Thus,

$$|Aut(B)| = 32.$$

Automorphisms of K:

$$K \cong < a,b,c: a^4 = b^2 = c^2 = 1, ab = ba, ac = ca, cb = a^2 bc >$$

Any automorphism $\psi \in Aut(K)$ sends the generators of K as

$$a \to a^i; \quad i \in \{1,3\},$$
$$b \to b, c, a^2 b, a^2 c, abc, a^3 bc,$$
$$c \to b, c, a^2 b, a^2 c, abc, a^3 bc.$$

So

$$|Aut(K)| = 48.$$

Automorphisms of $G_{4,4}$:

$$G_{4,4} \cong < a,b: a^4 = b^4 = (ab)^2 = 1, ab^3 = ba^3 >$$

Any automorphism $\psi \in Aut(G_{4,4})$ sends the generators of $G_{4,4}$ as

$$a \to a, a^3, b, b^3, ab^2, a^2 b, a^2 b^3, a^3 b^2,$$
$$b \to a, a^3, b, b^3, ab^2, a^2 b, a^2 b^3, a^3 b^2.$$

Thus,

$$|Aut(G_{4,4})| = 32.$$

Automorphisms of groups of order 18
Automorphisms of D_9

$$D_9 \cong < a, b: a^9 = b^2 = 1, bab = a^{-1} >$$

Any automorphism $\psi \in Aut(D_9)$ sends the generators as

$$a \to a^i; 3 \nmid i, 1 \le i \le 8,$$
$$b \to a^j b; 0 \le j \le 8,$$

So

$$|Aut(D_9)| = 54.$$

Automorphisms of $S_3 \times Z_3$:

$$S_3 \times Z_3 \cong < a, b, c: a^3 = b^2 = c^3 = 1, ba = a^{-1}b, ac = ca, bc = cb >$$

Any automorphism $\psi \in Aut(S_3 \times Z_3)$ sends the generators of $S_3 \times Z_3$ as

$$a \to a^i; 1 \le i \le 2,$$
$$b \to a^j b; 0 \le j \le 2,$$
$$c \to c^k; 1 \le k \le 2.$$

So

$$|Aut(S_3 \times Z_3)| = 12.$$

Automorphisms of $(Z_3 \times Z_3) \rtimes Z_2$:

$$(Z_3 \times Z_3) \rtimes Z_2 \cong < a, b, c: a^2 = b^3 = c^3 = 1, bc = cb, bab = a, cac = a >$$

Any automorphism $\psi \in Aut((Z_3 \times Z_3) \rtimes Z_2)$ sends the generators of $(Z_3 \times Z_3) \rtimes Z_2$ as

$$a \to ab^i c^j; 0 \le i, j \le 2,$$
$$b \to b^k c^l; 0 \le k, l \le 2, \quad k + l \ne 0,$$
$$c \to b^m c^n; 0 \le m, n \le 2, \quad m + n \ne 0.$$

So

$$|Aut((Z_3 \times Z_3) \rtimes Z_2)| = 432.$$

Automorphisms of groups of order 20

Automorphisms of D_{10}:

$$D_{10} \cong < a, b: a^{10} = b^2 = 1, bab = a^{-1} >$$

Any automorphism $\psi \in Aut(D_{10})$ sends the generators as

$$a \to a^i; 2 \nmid i, 5 \nmid i, 1 \le i < 10,$$
$$b \to a^j b; 1 \le j \le 10.$$

So

$$|Aut(D_{10})| = 40.$$

Automorphisms of Fr_{20}:

$$Fr_{20} \cong Z_5 \rtimes Z_4 \cong < a, b: a^4 = b^5 = 1, ba = ab^2 >$$

Any automorphism $\psi \in Aut(Fr_{20})$ sends the generators of Fr_{20} as

$$a \to ab^i; 0 \le i \le 4,$$
$$b \to b^j; 1 \le j \le 4.$$

Thus,

$$|Aut(Fr_{20})| = 20.$$

Automorphisms of $Z_5 \rtimes Z_4$:

$$Z_5 \rtimes Z_4 \cong < a, b: a^4 = b^5 = 1, bab = a >$$

Any automorphism $\psi \in Aut(Z_5 \rtimes Z_4)$ sends the generators of $Z_5 \rtimes Z_4$ as

$$a \to a^i b^j; \quad i \in \{1,3\} 0 \le j \le 4,$$
$$b \to b^k; 1 \le k \le 4.$$

Thus,

$$|Aut(Z_5 \rtimes Z_4)| = 40.$$

Automorphisms of groups of order 21

Automorphisms of Fr_{21}:

$$Fr_{21} \cong Z_7 \rtimes Z_3 \cong < a, b: a^3 = b^7 = 1, ba = ab^2 >$$

Any automorphism $\psi \in Aut(Fr_{21})$ sends the generators of Fr_{21} as

$$a \to ab^i; 0 \leq i \leq 6,$$
$$b \to b^j; 1 \leq j \leq 6.$$

Thus,

$$|Aut(Fr_{21})| = 42.$$

Automorphisms of groups of order 22

Automorphisms of D_{11}:

$$D_{11} =< a, b: a^{11} = b^2 = 1, bab = a^{-1} >$$

Any automorphism $\psi \in Aut(D_{11})$ sends the generators of D_{11} as

$$a \to a^i; 1 \leq i \leq 10,$$
$$b \to a^j b; 0 \leq j \leq 10.$$

So

$$|Aut(D_{11})| = 110.$$

Automorphisms of groups of order 24

Automorphisms of $S_3 \times Z_4$:

$$S_3 \times Z_4 \cong < x, y, z: x^3 = y^2 = z^4 = 1, x^y = x^{-1}, xz = zx, yz = zy >$$

Any automorphism $\psi \in Aut(S_3 \times Z_4)$ sends the generators of $S_3 \times Z_4$ as

$$x \to x^i; 1 \leq i \leq 2,$$
$$y \to x^j yz^k; 0 \leq j \leq 2, \quad k \in \{0,2\},$$
$$z \to z^l; \quad l \in \{1,3\}.$$

So

$$|Aut(S_3 \times Z_4)| = 24.$$

Automorphisms of $S_3 \times Z_2 \times Z_2$:

$$S_3 \times Z_2 \times Z_2 \cong < x, y, z, w: x^3 = y^2 = z^2 = w^2 = 1, x^y = x^{-1}, xz = zx, xw = wx, yz = zy, yw = wy, zw = wz >$$

Any automorphism $\psi \in Aut(S_3 \times Z_2 \times Z_2)$ sends the generators of $S_3 \times Z_2 \times Z_2$ as

$$x \to x^i; 1 \leq i \leq 2,$$
$$y \to x^j yz^k w^l; 0 \leq j \leq 2, 0 \leq k, l \leq 1,$$
$$z \to z^u w^v; 0 \leq u, v \leq 1, \quad u + v \neq 0.$$

So

$$|Aut(S_3 \times Z_2 \times Z_2)| = 144.$$

Automorphisms of $D_4 \times Z_3$:

$$D_4 \times Z_3 \cong < x, y, z: x^3 = y^4 = z^2 = 1, y^z = y^{-1}, xy = yx, xz = zx >$$

Any automorphism $\psi \in Aut(D_4 \times Z_3)$ sends the generators of $D_4 \times Z_3$ as

$$x \to x^i; 1 \leq i \leq 2,$$
$$y \to y^j; \quad j \in \{1,3\},$$
$$z \to y^k z; 0 \leq k \leq 3.$$

So

$$|Aut(D_4 \times Z_3)| = 16.$$

Automorphisms of $Q \times Z_3$:

$$Q \times Z_3 \cong < x, y, z: x^4 = z^3 = 1, x^2 = y^2, x^y = x^{-1}, yz = zy, xz = zx >$$

Any automorphism $\psi \in Aut(Q \times Z_3)$ sends the generators of $Q \times Z_3$ as

$$x \to x^i y^j; 0 \leq i \leq 3, 0 \leq j \leq 1,$$
$$y \to x^l y^m; 0 \leq l \leq 3, 0 \leq m \leq 1,$$
$$z \to z^k; 1 \leq k \leq 2.$$

So

$$|Aut(Q \times Z_3)| = 48.$$

Automorphisms of $A_4 \times Z_2$:

$$A_4 \times Z_2 \cong < x, y, z, w: x^2 = y^2 = z^3 = w^2 = 1, xy = yx, zx = xyz, xw = wx, xz = zy, yw = wy, zw = wz >$$

Any automorphism $\psi \in Aut(A_4 \times Z_2)$ sends the generators of $A_4 \times Z_2$ as

$$x \to x^i y^j; 0 \le i, j \le 1, \quad i + j \ne 0,$$
$$y \to x^k y^l; 0 \le k, l \le 1, k + l \ne 0,$$
$$z \to x^l y^m z^n; 0 \le l, m \le 1, 1 \le n \le 2,$$
$$w \to w.$$

So

$$|Aut(A_4 \times Z_2)| = 24.$$

Automorphisms of D_{12}:

$$D_{12} = < a, b: a^{12} = b^2 = 1, bab = a^{-1} >$$

Any automorphism $\psi \in Aut(D_{12})$ sends the generators of D_{12} as

$$a \to a^i; 1 \le i \le 12, (i, 12) = 1,$$
$$b \to a^j b; 0 \le j \le 11.$$

So

$$|Aut(D_{12})| = 48.$$

Automorphisms of Q_{12}:

$$Q_{12} = < a, b: a^{12} = 1, a^6 = b^2, b^{-1}ab = a^{-1} >$$

Any automorphism $\psi \in Aut(Q_{12})$ sends the generators of Q_{12} as

$$a \to a^i; 1 \le i \le 12, (i, 12) = 1$$
$$b \to a^j b; 0 \le j \le 11$$

So

$$|Aut(Q_{12})| = 48.$$

Automorphisms of $Z_6 \rtimes Z_4$:

$$Z_6 \rtimes Z_4 \cong < x, y: x^4 = y^6 = 1, yxy = x >$$

Any automorphism $\psi \in Aut(Z_6 \rtimes Z_4)$ sends the generators of $Z_6 \rtimes Z_4$ as

$$x \to x^i y^j; \quad i \in \{1,3\}, 0 \le j \le 5,$$
$$y \to x^k y^l; \quad k \in \{0,2\}, \quad l \in \{1,5\}.$$

So

$$|Aut(Z_6 \rtimes Z_4)| = 48.$$

Automorphisms of $Z_3 \rtimes Z_8$:

$$Z_3 \rtimes Z_8 \cong < x, y: x^3 = y^8 = 1, yxy^{-1} = x^{-1} >$$

Any automorphism $\psi \in Aut(Z_3 \rtimes Z_8)$ sends the generators of $Z_3 \rtimes Z_8$ as

$$x \to x^i; 1 \le i \le 2,$$
$$y \to x^k y^l; 0 \le k \le 2, (2, l) = 1, 1 \le l \le 8.$$

So

$$|Aut(Z_6 \rtimes Z_4)| = 24.$$

Automorphisms of $Z_3 \rtimes Q$:

$$Z_3 \rtimes Q \cong < x, y, z: x^2 = y^6 = z^2 = 1, xy = yx, xz = zx, (zy)^2 = 1 >$$

Any automorphism $\psi \in Aut(Z_3 \rtimes Q)$ sends the generators of $Z_3 \rtimes Q$ as

$$x \to x,$$
$$y \to x^l y^m; 0 \le l \le 1, \quad m \in \{1,5\},$$
$$z \to x^i y^j z; 0 \le i \le 1, 0 \le j \le 5.$$

So

$$|Aut(Z_3 \rtimes Q)| = 48.$$

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