

RLBP Texture Features extracted from Shearlet Face for Facial Expression Recognition

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ABSTRACT

In this article, we proposed a technique for representation of facial expression based on extraction of Robust Local Binary Pattern (RLBP) features in shearlet domain. The shearlet transform exhibit improved directional capability, better ability to represent edges and other singularities along curves as compared to traditional multiscale transform such as wavelet transform. Hence, we transform original face images to frequency domain at a specific scale and orientation using shearlet transform. Noise and illumination invariant texture features are extracted from robust local binary pattern, which forms the feature vector of facial expression. The proposed method is evaluated based on facial expression recognition carried out using a benchmark database such as JAFFE. The facial expression recognition is performed using a chi-square distance measure with a nearest neighbor classifier. Experimental results show that our approach outperforms other popular LBP based approaches.

Keywords: RLBP, Facial Expression, Shearlet, Texture Features

INTRODUCTION

Facial expression is one of the most important communications for human beings to express their emotions and intentions. Facial expression recognition is very interesting and challenging topic in Digital Image Processing and Computer Vision. It is the task of identifying mental activity, facial motion and facial feature deformation from still images, Facial expression recognition from static images is more challenging than from image sequences due to less information obtained from expression actions [1]. The basic facial expression recognition system mainly consists of three stages: face pretreatment, expression feature extraction and expression classification. As an important step, feature extraction of facial expression has recently received increased attention. We have seen, facial expressions have been studied by clinical and social psychologists, medical practitioners, actors and artists. However, in the end of 20th century, with the advances in the fields of robotics, computer graphics and computer vision, animators and computer scientists started showing interest in the study of facial expressions. Fasel and Luetttin [2] conducted survey on automatic facial expression analysis. In this survey, they introduce the most prominent automatic expression analysis methods and systems presented in the literature. Facial motion and deformation extraction approaches as well as classification methods are discussed with respect to issues such as face normalization, facial expression dynamics and facial expression intensity and also with regard to their robustness towards environmental changes. Tian et al. [3] developed an automate face analysis system to analyze facial expressions based on both permanent and transient features. The system can recognize six upper face action units and ten lower face action units with good success rate. For their feature extraction system, they developed a multi-state face component model for example, a three-state lip model can describe open lip state, closed lip state or tightly closed lip state. Similarly, eyes, brow, cheek all have different multi-state models of their own.

Oliver et al. [4] present a method based on 2D blob features, which are spatially compact clusters of pixels similar in terms of low-level image properties, and the hidden Markov model (HMM) was adopted for facial expression and head movement classification. Lyons et al. [5] proposed a method for classifying facial images automatically based on labelled elastic graph matching, 2D Gabor wavelet representation, and linear discriminant analysis (LDA). Abboud et al. [6] used an AAM for facial expression recognition and synthesis, which can normalize the facial expression of a given face and artificially synthesize novel expressions on the same face. Xie and Lam [7] present a method for facial expression recognition based on shape and texture. They propose spatially maximum occurrence model (SMOM) to representation for facial expressions, which is based on the statistical characteristics of training facial images. Then elastic shape-texture matching algorithm is used to measure the similarity between images based on the shape and texture information. X. Feng et al [1] divide the face area of the face image into small regions, from which the LBP histograms are extracted and concatenated into a single feature histogram that represents facial expression descriptor. In

[8] region based local descriptors are used to recognize facial expressions in image sequences using spatiotemporal LBP. C. Shan et al. [9] extract most discriminant LBP features from Boosted-LBP and achieve good recognition rate using support vector machine classifier. S. Zhang et al. [10] proposed a method for facial expression recognition based on LBP and local fisher discriminant analysis. Initially, LBP features are extracted from the original images and reduced the feature dimension of LBP features using local fisher discriminant analysis. Support vector machines classifier is used for recognition of facial expression. In [11] LBP is applied to face image and then LBP image is divided into 3x5 non overlapping blocks, calculate the LBP histogram of each block and concatenating it. Laplacian Eigenmaps is used for feature dimensionality reduction and support vector machine classifier is used for classification. X. Wu et al. [12] used to curvelet transform to extract features of face images for face and facial expression recognition. Recently many researchers proposed curvelet based facial expression recognition [13] [14]. The literature survey reveals that, the combination of curvelet transform with LBP yields good feature descriptor than using curvelet transform alone. The curvelet based LBP texture operator is a good feature extractor. A. saha et al. [15] proposed the combination of curvelet transform and LBP for recognizing the facial expression from still images. Wavelets have been successfully applied to the facial expression recognition due to their time-frequency and multiresolution [16], [17], [18]. However, it is known that wavelets have the limited ability in expressing directional information [19]. To overcome these limitations, a great number of Multiscale Geometric Analysis (MGA) algorithms, owned good characteristic, such as locality, multiresolution, directionality and anisotropy, are employed for facial expression feature extraction to offer more discrimination information in the past few years [20], [21], [22]. Multiscale methods based on shearlets [23] not only have good localization and compactly support in the frequency domain, but also have directionality and anisotropy. With these properties, shearlets can accurately efficiently represent image geometrical information of edges and texture, which are very essential in facial expression recognition. In contrast to other MGA methods such as contourlets [24], ridgelets [25], and curvelets [26], the shearlet framework could provide optimal efficiency and computational efficiency when addressing edges [19].

In [27] our previous work, we presented facial expression recognition based on curvelet transform with complete local binary pattern. The original LBP texture operator has two demerits, i.e., sometimes it produces same binary code for different structural patterns and sensitive to noise. In order to overcome these two drawbacks of LBP, Robust Local Binary Pattern (RLBP) was introduced by Y.Zhao et al. [28] for texture classification. In RLBP, the image local differences are decomposed into three components i.e., signs, magnitudes and central information, in which the gray value of centre pixel in a 3x3 local area is replaced by its average local gray value.

SHEARLET TRANSFORM

Continuous Shearlet

The shearlet transform is a new multiscale geometric analysis tool which has been widely used in image approximation, edge analysis and other fields[19][29][30].

The continuous shearlet transform f is defined by

$$SH_f(a, s, t) = \langle f, \Psi_{ast} \rangle, \quad a \in R^+, s \in R, t \in R^2 \quad (1)$$

where $\Psi_{ast}(x) = a^{-3/4} \Psi(M_{as}^{-1}(x - t))$ of three variables, the scale $a \in R^+$, the shear $s \in R$, the transform $t \in R^2$, is called a continuous shearlet system. $M_{as} = (a, s; 0, \sqrt{a})$ is the composition of the shear matrices $B = (1, s; 0, 1)$ and anisotropic matrices $A = (a, 0; 0, a^{1/2})$. For any $\xi = (\xi_1, \xi_2) \in \hat{R}^2$, $\xi_1 \neq 0$, let

$$\hat{\Psi}(\xi) = \hat{\Psi}(\xi_1, \xi_2) = \hat{\Psi}_1(\xi_1) \hat{\Psi}_2\left(\frac{\xi_2}{\xi_1}\right)$$

where $\hat{\Psi}_1 \in C^\infty(R)$ with $\text{supp} \hat{\Psi}_1 \in [-2, -1/2] \cup [1/2, 2]$, $\hat{\Psi}_2 \in C^\infty(R)$ with $\text{supp} \hat{\Psi}_2 \in [-1, 1]$ and $\hat{\Psi}_2 > 0$ on $(-1, 1)$. Thus, each function $\hat{\Psi}_{ast}$ has frequency support

$$\text{supp} \hat{\Psi}_{ast} \subset \left\{ (\xi_1, \xi_2) : \xi_1 \in \left[-\frac{2}{a}, -\frac{1}{2a}\right] \cup \left[\frac{1}{2a}, \frac{2}{a}\right], \left|\frac{\xi_2}{\xi_1} - s\right| \leq \sqrt{a} \right\} \quad (2)$$

Each element Ψ_{ast} is supported on a pair of trapezoids, oriented along lines of slope s . The support becomes increasingly thin as $a \rightarrow 0$. That is say the scale of the shearlets controlled by the anisotropic scaling matrices A , while the shear matrices B only control the orientation of the shearlets. Those matrices lead to windows which can be elongated along arbitrary directions and the geometric structures for singularities in images can be efficiently represented and analyzed by using them.

In [31] shows that shearlets are localized well and are compactly supported in the frequency domain. Shearlets show highly directional sensitivity and anisotropy. In fact, for two-dimension signal, the band limited shearlets can detect all singular points and track the direction of singular curve adaptively. Furthermore, along with the parameter changes,

shearlets can completely analyze the singular structures of 2-D piecewise smooth functions. Those properties of shearlets are useful especially in image edge and detail information processing.

Discrete Shearlet Transform

The elements of the traditional shearlet can't be separated in the spatial domain, and this property often leads to the difficulty in practically relevant discrete implementation. Based on the above discussion, Lim (W.Q. Lim, 2010) [29] constructed compactly supported shearlets generated by separable functions which are constructed using multiresolution analysis, this lead to a fast discrete shearlet transform implementation. The fast discrete shearlet transform of the image f . Specifically, the shear matrix B_0^s and B_1^s corresponded to the horizontal cone C_0 and vertical cone C_1 dimensions respectively, while the anisotropic scaling matrix A_0 and A_1 were offered to construct the anisotropic discrete wavelet basis along the shear direction and complete the multiscale decomposition. The fast discrete shearlet transform is also computationally very efficient and it requires $O((2^{M+2} + 2)N)$ operations where N is the size of the input image and $2^{M+2} + 2$ is the number of directions, while the 2D-FRFT costs $O(N(\log N))$, the curvelet transform costs $O(N(\log \sqrt{N})^2)$, the ridgelets transform costs $O(N(\log \sqrt{N}))$ [32].

Using the shearlets, a given image can be analyzed at various resolutions for each direction. The low frequency components is the upper left corner of the shearlet coefficients matrix, which concentrate most important information and discard the influence of noises and irrelevant parts [22], will be adopted for further analysis in our approach. Thus, the dimensionality of the data is reduced effectively for computation at the next stage.

ROBUST LOCAL BINARY PATTERN (RLBP)

Y. Zhao et al. [28] proposed Completed Robust Local Binary Pattern (CRLBP), which is invariant to monotonic gray scale transformation and insensitive to noise. The gray value of centre pixel in 3x3 local area is replaced by its average local gray value of the neighbourhood pixel values instead of the gray value of centre pixel value, in which the RLBP is calculated. The Average Local Gray value (ALG) is defined as

$$ALG = \frac{\sum_{i=1}^8 g_i + g}{9}, \quad (3)$$

where g is the gray value of the centre pixel and g_i ($i=0,1,\dots,8$) represents the gray value of the neighbor pixels. ALG is the average gray level of local area, which is obviously more robust to noise than the gray value of the centre pixel. The LBP process is applied by using ALG as the threshold instead of the gray value of central pixel, named as Robust Local Binary pattern (RLBP). This can be defined as

$$RLBP_{p,R} = \sum_{p=0}^{P-1} s(g_p - ALG_c) 2^p = \sum_{p=0}^{P-1} s\left(g_p - \frac{\sum_{i=1}^8 g_{ci} + g_c}{9}\right) 2^p, \quad (4)$$

where g_c is the gray value of central pixel and g_p ($p=0,1,\dots,P-1$) represents the gray value of the neighbor pixel on 3x3 local area of radius R , P is the number of neighbors and g_{ci} ($i=0,1,\dots,8$) is the gray values of the neighbor pixel of g_c . Average local gray level of pixel is used as threshold, therefore RLBP is insensitive to noise and also two different patterns with same LBP code may have different RLBP code, because that neighbors of each neighbor pixel are considered. The RLBP can overcome mentioned demerits of LBP.

Sometimes specific information of the central pixel is needed, but ALG ignores the specific information of individual pixel. In order to define Weighted Local Gray Value (WLG) to balance between anti-noise and information of individual pixel. The WLG is defined as follows

$$WLG = \frac{\sum_{i=1}^8 g_i + \alpha g}{8 + \alpha}, \quad (5)$$

where g and g_i are defined in Eq. (3), α is a parameter set by user. If α is set to 1, WLG is equivalent to ALG. The RLBP is calculated as follows

$$RLBP_{p,R} = \sum_{p=0}^{P-1} s(g_p - WLG_c) 2^p = \sum_{p=0}^{P-1} s\left(g_p - \frac{\sum_{i=1}^8 g_{ci} + \alpha g_c}{8 + \alpha}\right) 2^p, \quad (6)$$

where g_p , g_c and g_{ci} are defined Eq. (4), α is a parameter of WLG.

A Local Binary Pattern is called uniform if it contains at most two bitwise transitions from 0 to 1 or 1 to 0, when the binary string is considered circular. For example, 00000001, 00011111 and 11000111 are uniform patterns. We extend the RLBP to uniform RLBP, the notation used for the RLBP operator: $RLBP_{P,R}^{u2}$ or $U(RLBP_{P,R})$ as defined below:

$$U(RLBP_{P,R}) = |s(g_{P-1} - WLG_c) - s(g_0 - WLG_c)| + \sum_{p=1}^{P-1} |s(g_p - WLG_c) - s(g_{p-1} - WLG_c)| \quad (7)$$

The subscript represents using the operator in a (P, R) neighbourhood. Superscript $u2$ stands for using only uniform patterns and labelling all remaining patterns with a single label.

The $LBP_{P,R}$ operator produces 2^P different output values, corresponding to the 2^P different binary patterns that can be formed by the P pixels in the neighbour set. When the image is rotated, the gray values g_p will correspondingly move along the perimeter of the circle around g_0 . Since g_0 is always assigned to be the gray value of element $(0, R)$ to the right of g_c rotating a particular binary pattern naturally results in a different $LBP_{P,R}$ value. This does not apply to patterns comprising on only 0s (or 1s) which remain constant at all rotation, i.e., to assign a unique identifier to each rotation invariant local binary patterns. We extend the rotation invariant local binary pattern to RLBP as defined below:

$$RLBP_{P,R}^{ri} = \min\{ROR(RLBP_{P,R}, i) \mid i = 0, 1, \dots, P-1\}. \quad (8)$$

Where $ROR(x, i)$ performs a circular bit-wise right shift on the P -bit number x , i times. In terms of pixels, simply corresponds to rotating the neighbour set clockwise so many times that a maximal number of the most significant bits, starting from g_{P-1} , is 0. $RLBP_{P,R}^{ri}$ quantifies the occurrence statistics of individual rotation invariant patterns corresponding to certain micro features in the image:

$$RLBP_{P,R}^{riu2} = \begin{cases} \sum_{p=0}^{P-1} s(g_p - WLG_c) & \text{if } U(RLBP_{P,R}) \leq 2 \\ P + 1 & \text{otherwise} \end{cases} \quad (9)$$

superscript $riu2$ reflects the use of rotation invariant uniform patterns. A histogram of the labelled image $f_l(x, y)$ can be defined as

$$H_i = \sum_{x,y} I\{f_l(x, y) = i\}, \quad i = 0, \dots, n-1 \quad (10)$$

in which n is the number of different labels produced by the LBP operator and

$$I\{A\} = \begin{cases} 1, & A \text{ is true} \\ 0, & A \text{ is false} \end{cases} \quad (11)$$

This histogram contains information about the distribution of the local micropatterns, such as edges, spots and flat areas, over the whole image.

FACIAL EXPRESSION RECOGNITION

The images are initially cropped to extract face from the image. Before extracting the facial features from the image, normalization is done using histogram equalization to increase the contrast of the image, because shearlet transform is not independent of illumination changes. After that shearlet transform is applied to the normalized image.

The shearlet transform decomposes the normalized image into two levels. It is observed that the approximate sub-band as more energy than other sub-bands. Therefore, we apply RLBP on the approximate sub-band by varying the value of weighing parameter α in the range of 1 to 8. Finally, we extract robust and noise and illumination invariant features from approximation sub-band obtained from shearlet transform using RLBP, which represents the expression of the face. The RLBP histogram of 255 labels is calculated based on the class labels of the face images. Feature vectors of the same class labels are grouped to form the training template Z^c for a particular class of facial expression. Thus,

$$Z^c = \{z_1^c, z_2^c, z_3^c, \dots, z_n^c\}, \quad (12)$$

where n denotes number of training samples available for the corresponding class. The representative feature set of the class c is the cluster centre of template Z^c and is calculated as

$$M^c = \frac{1}{n} \sum_{i=1}^n z_i^c. \quad (13)$$

Nearest neighbor classifier is used for classification with Chi-Square metric.

$$\chi^2(S, M^c) = \sum_{i=1}^N \frac{(S_i - M_i^c)^2}{S_i + M_i^c}, \quad (14)$$

where S is the feature vector of length N extracted from the test image.

EXPERIMENTAL RESULTS

In order to evaluate the effectiveness of our proposed technique, we carried out experiments on JAFFE [33] database, which includes 3 or 4 samples for each of the six basic facial expressions and a neutral face image for each subject or person. A total 213 images of 10 subjects and each image size is 256x256 pixels.

Initially all the images of JAFFE database are cropped with size 110x150 pixels to extract the face from the image. Then we normalization is done using histogram equalization to increase the contrast of the face image. Normalized face images are divided into ten sets to carry out 10-fold cross validation i.e., ten rounds of testing carried out and each time, a different combination of nine sets are used for training and the remaining one set is used for testing. Final recognition rates are average of the recognition rates of ten tests. We have conducted several experiments on JAFFE database to identify the optimal value of scale and orientation for shearlet transform and optimal parameters such as α , R and P for RLBP using Nearest Neighbourhood (NN) classifier. In all tests the histogram is calculated, which is the facial feature vector of the face expressions. The shearlet and RLBP(8,1) combination yields good results for 6-class expression compare to other combinations. Similarly same combination yields good results for 7-class facial expressions.

Table 1: Recognition rate of our approach for different combination of shearlet with RLBP

Shearlet and RLBP combination	Recognition Rate (%)
Shearlet + RLBP(8,1)	98.03
Shearlet + RLBP(8,2)	95.26
Shearlet + RLBP(16,2)	93.25
Shearlet + RLBP(16,3)	90.65

The confusion matrices calculated using the combination of Sharelet and RLBP(8,1) for 6-class and 7-class facial expression. The confusion matrix shows the proportion in percentage, any expression shown in a row is falsely detected as another expression in the column. The confusion matrix of 6-class and 7-class expression recognition for the above said combination is shown in the Table 3 and Table 4 respectively. It is observed that Happiness and Surprise expressions can be recognized with high accuracy, while Anger, Fear and Sad are easily confused with others in 6-class. For 7-class observe that, Surprise, Happy, Fear can be recognized with high accuracy, while the recognition rates for Anger, Disgust, Neutral and Sad are less accurate.

Table 2: Comparison result of our approach for JAFFE database

Methods	Recognition Rate (%)
LBP [34]	85.57
Curvelet + LBP [15]	93.69
Curvelet + CLBP[27]	95.56
Our Approach	98.03

The proposed approach is compared with LBP based approach [34], curvelet with LBP approach [15] and curvelet with CLBP approach [27], the results are shown in the Table 2. Chi-square based nearest neighbour classifier is used for all the experiments. Our approach yields 98.03% recognition rate, whereas LBP method performs 85.57%, Curvelet based LBP approach yields 93.69% and curvelet with CLBP yields 95.56%. Therefore, our approach outperforms against LBP and curvelet based LBP and CLBP approach. This is due to fact that, the curvelet transform preserves the crucial edge information and other variations occurred in the face during expression. Further, the RLBP extracts noise and illumination invariant features from faces images.

Table 3: Confusion matrix of 6 class facial expression recognition using shearlet+RLBP on JAFFE Database

Expressions	Anger %	Disgust %	Fear %	Happy %	Sad %	Surprise %
Anger	96.67	3.33	0	0	0	0
Disgust	0	100	0	0	0	0
Fear	3.33	0	96.67	0	0	0
Happy	0	0	0	100	0	0

Sad	3.33	0	0	0	96.67	0
Surprise	0	0	0	0	0	100

Table 4: Confusion matrix of 7 class facial expression recognition using shearlet+RLBP on JAFFE Database

Expressions	Anger %	Disgust %	Fear %	Happy %	Sad %	Surprise %	Neutral %
Anger	88.23	2.94	2.94	0	2.94	0	2.94
Disgust	5.88	88.23	5.88	0	0	0	0
Fear	2.94	2.94	91.17	0	2.94	0	0
Happy	0	0	2.94	91.17	0	0	5.88
Sad	2.94	2.94	8.82	0	85.29	0	0
Surprise	0	0	0	2.94	0	94.11	2.94
Neutral	0	0	5.88	2.94	0	0	91.17

CONCLUSION

In this article, we introduced a novel method for identifying facial expressions by combining shearlet transform and RLBP. Shearlet based RLBP is utilized to extract the features from still images. Our method's performance was assessed using the JAFFE database, and the results demonstrated that it achieves a higher recognition rate compared to LBP, curvelet-based LBP, and curvelet with CLBP methods. This is because the shearlet transform maintains the edges and other changes that happen on the face when expressing emotions. In addition, the RLBP is able to capture features from facial images that are not affected by noise and lighting variations.

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