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Plane Wave Analysis in Unsaturated Thermoelastic Porous Media and their reduced cases

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ABSTRACT

Medium plays an important role while studying about seismic wave. Medium is a particular type of material having unvarying properties. Medium transfers energy from one location to another location. It works as transporter which transfer any form of energy like signal, light, sound. Any types of waves need a medium to propagate. The properties of wave are highly influenced by the nature of material In this paper, unsaturated poro-thermoelastic medium is reduced in elastic solid by using reduction algorithm. The wave equation is generalized for unsaturated poro-thermoelastic medium, unsaturated porous medium and elastic medium.

Keywords: Unsaturated, porous, wave, velocity.

INTRODUCTION

Seismic wave propagation in fluid unsaturated porous solid has been the subject of explorations by many analyzers in different fields like earth sciences, geophysics, earthquake engineering etc. One of the most prominent ways of information about interior of earth is called seismic activity. Seismic activity is also helpful in the study and prediction of earthquake and tsunamis. The concept of waves is an integral part of our scientific culture and has nourished physicists and mathematicians, pure and applied alike, for centuries. Many important discoveries in physics, including quantum mechanics, have involved wave phenomena. The wave concept owes some of its scientific success to its mathematical tractability. Thermoelasticity is concerned with dynamical systems that interact with their surroundings by exchanging heat in addition to mechanical work and external work. Biot (1956) was the first to develop the thermoelasticity theory, which is based on the standard Fourier heat conduction rule, to explain the relation among variations in temperature and elastic deformation. Because of the parabolic nature of the energy equation, this theory conveyed that thermal signals would move at an infinite speed. Another theory of thermoelasticity presented by Coleman and Noll (1963) and Coleman (1964). To formulate a hyperbolic heat conduction equation, they modified the Fourier law of heat conduction by including two relaxation times. Sharma, M.D. (2008) modified Biot's theory, to demonstrate the presence of three longitudinal and one transverse waves in an isotropic, porous, thermoelastic material that has been saturated with a non-viscous fluid.

The velocities and attenuation of the three longitudinal waves are examined using numerical examples. Kumar et.al. (2018) presented a study that examines the reflection of plane seismic waves at a material's double-porosity dual-permeability plane surface under stress-free conditions. It is taken into account when the P1 and SV waves interact. Three longitudinal and one shear waves are found in the medium as a result of the incident waves' reflections. A non-singular system of linear equations is used to derive the expressions for the reflection coefficients for a given incident wave. An energy matrix is used to calculate the energy shares of reflected waves. Sharma, M.D. (2013) A mathematical model that describes how harmonic plane waves propagate through a material with two types of pores and a viscous fluid inside of it. In the composite porous medium, an Eigen-system of order four implies the presence of three longitudinal waves and one transverse wave. Barak et al. (2018) examines the acoustic wave propagation at the water/double-porosity sediment interface with a uniform elastic solid substrate. This study examines the acoustic wave propagation at the water/double-porosity sediment interface with a uniform elastic solid substrate. They constituted a mathematical model through three layers with distinct elastic_properties. Theoretically, under suitable boundary conditions, the closed form analytical equations for the coefficients of reflection and transmission are derived. This set of non-singular linear algebraic equations is used to calculate these expressions. The non-singular system is dependent on many different types of material parameters. Therefore, a numerical example is used to determine the effects of various properties of sandwiched layer on reflection and transmission coefficients. It proves that the existence of a double-porosity layer significantly affects the coefficients of reflection and transmission. Kumari, Manjeet(2022) investigated a theory which incorporates LS (Lord–Shulman) and GL (Green–Lindsay) theories.

The four dilatational waves and one shear wave are predicted by the generalized equations of motion when they are solved using the potential functions approach .The incidence of the P1 (or *SV*) wave generates the five reflected waves. Based on the permeable and impermeable boundary constraints, a system of five linear non-homogeneous equations is used to calculate the reflection coefficients. These reflection coefficients are then used to compute the distribution of incident energy. The mathematical derivations introduced in this study can investigate the impact of subsurface features (liquid saturation, porosity, surface pores characteristics, thermal expansion coefficients, and wave frequency) on the propagation characteristics (propagation and attenuation directions, phase shift, energy ratio) of reflected waves. Additionally, energy conservation is demonstrated at the unsaturated porothermoelastic media's stress-free surface. Sharma, M. D. (2018) presented a study of a reflection–refraction problem at an interface between two dissimilar dissipative media. Complex amplitude coefficients for reflected/refracted waves that are resolved to determine their amplitude ratios and phase shifts satisfy boundary conditions at the interface.

These complex coefficients are then used to determine how incident energy is distributed between reflected and refracted waves. The propagation properties of waves that are reflected and refracted from the inhomogeneous incidence of attenuated waves are explored by using a numerical example. Sharma, M. D(2006) developed a mathematical model for wave propagation in an anisotropic generalized thermoelastic media .A comparison between poroelastic and thermoelastic propagation in the general anisotropic media is made. Kumar, Manjeet et al. (2022) purposed a study to study the propagation of inhomogeneous waves in a partially saturated poro-thermoelastic media through the examples of the free surface of such media. The Helmholtz decomposition theorem is used to solve the mathematical model developed by Zhou et al. Using the potential, the propagation velocities of bulk waves in partially saturated poro-thermoelastic media are calculated.. Here we are studied about the different type of medium such as unsaturated thermoelastic, unsaturated elastic, and elastic mediums. One medium can be reduced in other medium by eliminating some particular parameter. In present study, a reduction algorithm is used to reduce unsaturated porothermoelastic medium to elastic solid. Unsaturated porothermoelastic medium is reduced to unsaturated porous elastic medium by removing thermal effect further it is reduced to elastic solid by removing porosity. For each of the medium equation of motion, constitution relation and wave equations are explained.

Unsaturated porothermoelastic medium

The unsaturated porothermoelastic medium contains the three component pore liquid, pore gas and solid grains. The void volume fraction preoccupied by liquid and gas phase are measured by the degree of saturation of liquid and gas respectively.

The Basic Equations

The equation of motion for unsaturated porothermoelastic medium following Wang at al. (2021) in the absence of external forces

$\sigma_{ij,j} = \rho\ \ddot{u}_i^s + \rho_l \ddot{u}_i^l + \rho_g \ddot{u}_i^g$

$(-p_l)_{,j} = \rho_l \ddot{u}_j^s + \frac{\rho_l}{\Phi S^l} \ddot{u}_j^l + \frac{\mu_l}{k_r^l\, k} \dot{u}_j^l$

$(-p_g)_{,j} = \rho_g \ddot{u}_j^s + \frac{\rho_g}{\Phi S^l} \ddot{u}_j^g + \frac{\mu_g}{k_r^g\, k} \dot{u}_j^g$

$\mathrm{K}\, T_{,jj} + \omega^2 \mathrm{cT} = -\omega^2 \check{\tau}_q [d_2 u_{\boldsymbol{j,j}}^{\boldsymbol{s}} + d_3 u_{\boldsymbol{j,j}}^{\boldsymbol{l}} + d_4 u_{\boldsymbol{j,j}}^{\boldsymbol{g}}]$

Constitutive Relation

Following Wang et al. (2021), in the absence of external forces, and ε =1 the constitutive relations for unsaturated porothermoelastic media are:

$\sigma_{ij} = \{ \bar{\lambda}\, u_{\boldsymbol{k,k}}^{\boldsymbol{s}} + D_1 u_{\boldsymbol{k,k}}^{\boldsymbol{l}} + D_2 u_{\boldsymbol{k,k}}^{\boldsymbol{g}} + D_3 \mathrm{T} \} \delta_{ij} + \mu (u_{\boldsymbol{i,j}}^{\boldsymbol{s}} + u_{\boldsymbol{j,i}}^{\boldsymbol{s}})$

$(-p_l)_j = B_1 u_{\boldsymbol{k,kj}}^{\boldsymbol{s}} + B_2 u_{\boldsymbol{k,kj}}^{\boldsymbol{l}} + B_3 u_{\boldsymbol{k,kj}}^{\boldsymbol{g}} + B_4 \boldsymbol{T}_{\boldsymbol{,j}}$

$(-p_g)_j = B_5 u_{\boldsymbol{k,kj}}^{\boldsymbol{s}} + B_6 u_{\boldsymbol{k,kj}}^{\boldsymbol{l}} + B_7 u_{\boldsymbol{k,kj}}^{\boldsymbol{g}} + B_8 \boldsymbol{T}_{\boldsymbol{,j}}$

Where δ_{ij} denotes knocker symbol. Dot over a variable implies partial derivative with time and comma before an index implies partial space differentiation.

Where u^s, u^l, u^g define the component of the particle displacement of the frame liquid and gas relative to solid respectively. μ is the shear modulus of the drained matrix. σ_{ij} is the component of the total stress tensor, p_l and p_g are the liquid and gas pressures, and **T** is the increment of temperature above a reference absolute temperature T_0 and δ_{ij} is the knocker symbol. Where u^s,u^l, u^g define the component of the particle displacement of the frame liquid and gas relative to solid respectively. σ_{ij} is the component of the total stress tensor, p_l and p_g are the liquid and gas pressures. ρ_l and ρ_g denotes the density of iquid and gas and ρ denotes the density of composite. Porosity is denoted by Φ. S^l and S^g identifies the saturation of liquid and gas respectively. μ_l and μ_g identifies the dynamical viscosity of liquid and gas. k_r^l, k_r^g are used for relative permeability of iquid and gas, and k denotes the intrinsic permeability. $\check{\tau}_q$ is heat flux phase lag. c is specific heat per unit volume . K is thermal conductivity

The other elastic coefficients [Wang et al. 2021] used in the above equations are:

$\bar{\lambda} = \lambda + \alpha\,[\Upsilon a_{11} + (1 - \Upsilon)a_{21}]$,
$D_1 = \alpha\,[\Upsilon a_{12} + (1 - \Upsilon)a_{22}]$,
$D_2 = \alpha\,[\Upsilon a_{13} + (1 - \Upsilon)a_{23}]$,
$D_3 = \alpha\,[\Upsilon a_{14} + (1 - \Upsilon)a_{24}]-\beta_s$
$d_1=T_0\beta_T[\Upsilon a_{14} + (1 - \Upsilon)a_{24}]$
$d_2=T_0[\beta_s + \beta_T(\Upsilon a_{11} + (1 - \Upsilon)a_{21})]$
$d_3=T_0\beta_T[\Upsilon a_{12} + (1 - \Upsilon)a_{22}]$
$d_4=T_0\beta_T[\Upsilon a_{13} + (1 - \Upsilon)a_{23}]$
$B_1 = a_{11}$, $B_2= a_{12}$, $B_3= a_{13}$, $B_4= a_{14}$,
$B_5 = a_{21}$, $B_6 = a_{22}$, $B_7 = a_{23}$, $B_8= a_{24}$
$a_{11} = A_{22}/G$,
$a_{12}=(A_{22}A_{14} - A_{12}A_{24})/\Phi S^l G$
$a_{23}=(A_{22}A_{15} - A_{12}A_{25})/\Phi S^g G$
$a_{14}=(A_{22}A_{16} - A_{12}A_{26})/G$
$a_{21}= -A_{21}/G$
$a_{22}=(A_{11}A_{24} - A_{21}A_{14})/\Phi S^l G$
$a_{23}=(A_{11}A_{25} - A_{21}A_{15})/\Phi S^g G$
$a_{24}=(A_{11}A_{16} - A_{21}A_{26})/G$
$G = A_{11}A_{22} - A_{21}A_{12}$
$A_{11} = \Phi S^l\beta_{\omega p}$
$A_{12} = \Phi S^g M_a/R\rho_g T_b$
$A_{13}=1-\Phi$, $A_{14} = \Phi S^l$, $A_{15} = \Phi S^g S^l$
$A_{16}= -[(1-\Phi)\beta_{sT} + \Phi S^l\beta_{\omega T} + (\Phi S^g M_a p_g{}^*/R\rho_g T_b{}^2)]$
$A_{21} = \Phi\,[S^g S^l\beta_{\omega p} - A_s]$
$A_{22} = \Phi[A_s - S^g S^l M_a/R\rho_g T_b]$
$A_{23} = 0$, $A_{24} = -A_{25} = \Phi S^g S^l$
$A_{26} = \Phi\beta_\Psi A_s\,\chi^{-1}(S_e{}^{-1/m} - 1)^{1/d} + \Phi S^g S^l(\frac{M_a p_g{}^*}{R\rho_g T_b{}^2} - \beta_{\omega T})$
$m=1-1/d$
$S_e=\frac{S^l - S^l{}_{res}}{S^l{}_{sat} - S^l{}_{res}}$,
$A_s = -m\,\chi d(S^l{}_{sat} - S^l{}_{res})\,S_e{}^{m+1/m}(S_e{}^{-1/m} - 1)^{1-1/d}$.

Where Υ denotes the amount that constitutes the proportion of matrix suction that contributes the effective stress which is equivalent to saturation of liquid. β_T be assign of coefficient of thermal. M_a represents the dry air molar mass. Universal gas constant is represented by R. T_b marks the internal temperature of gas which is expounded by Abed and Solowski's ideal gas law [1] . χ , m , d stand for independent variables of Van Genuchten Model[2]. The pressure of gas marks by $p_g{}^*$. $\beta_{\omega p}$, $\beta_{\omega T}$ are stand for the compressibility and thermal expansion coefficient of liquid. The thermal coefficient of solid is identified by β_{sT}. S_e is effective water saturation. β_Ψ is stand for the coefficient of surface tension which is effective by temperature.

Wave Equation

The wave equations of unsaturated thermoelastic medium are

$(\bar{\lambda} + \mu)\ \nabla(\nabla.u^s) + D_1\nabla(\nabla.u^l) + D_2\nabla(\nabla.u^g) + \mu\,\nabla^2 u^s + D_3\nabla\,T = \rho\,\ddot{u}^s + \rho_l\ddot{u}^l + \rho_g\ddot{u}^g$
$B_1\nabla(\nabla.u^s) + B_2\nabla(\nabla.u^l) + B_3\nabla(\nabla.u^g) + B_4\nabla\,T = \rho_l\,\ddot{u}^s + \nu_l\ddot{u}^l + \tilde{\upsilon}_l\dot{u}^l$
$B_5\nabla(\nabla.u^s) + B_6\nabla(\nabla.u^l) + B_7\nabla(\nabla.u^g) + B_8\nabla\,T = \rho_g\,\ddot{u}^s + \nu_g\ddot{u}^g + \tilde{\upsilon}_g\dot{u}^g$

Velocity

Solving equations we obtained an eight order partial differential equation for the propagation of longitudinal waves in unsaturated porothermoelastic media, given by

$[\gamma_0 \nabla^8 + \gamma_1\omega^2 \nabla^6 + \gamma_2\omega^4 \nabla^4 + \gamma_3\omega^6 \nabla^2 + \gamma_4\omega^8]\, \theta_s=0$

Where

$\gamma_0 = f_0 a_0 + g_0 b_0 + h_0 c_0$
$\gamma_1 = f_0 a_1 + f_1 a_0 + g_1 b_0 + g_0 b_1 + h_1 c_0 + h_0 c_1$
$\gamma_2 = f_1 a_1 + f_2 a_0 + f_0 a_2 + g_2 b_0 + g_1 b_1 + g_0 b_2 + h_2 c_0 + h_1 c_1 + h_0 c_2$
$\gamma_3 = f_2 a_1 + f_1 a_2 + g_2 b_1 + g_1 b_2 + h_2 c_1 + h_1 c_2$
$\gamma_4 = f_2 a_2 + g_2 b_2 + h_2 c_2$
$f_0 = \mathrm{K}\,(\bar{\lambda} + 2\mu)$, $f_1 = \mathrm{K}\,\rho + \mathrm{c}\,(\bar{\lambda} + 2\mu) - D_3 \check{\tau}_q d_2$ $\quad f_2 = \mathrm{c}\,\rho$,
$g_0 = \mathrm{K}D_1$, $g_1 = \mathrm{K}\rho_l + \mathrm{c}D_1 - D_3\check{\tau}_q d_3$, $\quad g_2 = \mathrm{c}\rho_l$,
$h_0 = \mathrm{K}D_2$, $h_1 = \mathrm{K}\rho_g + \mathrm{c}D_2 - D_3\check{\tau}_q d_4$, $\quad h_2 = \mathrm{c}\rho_g$,
$a_0 = \tilde{A}_2\tilde{C}_3 - \tilde{A}_3\tilde{C}_2$, $a_1 = \tilde{A}_2\tilde{D}_3 - \tilde{A}_3\tilde{D}_2 + \tilde{B}_2\tilde{C}_3 - \tilde{B}_3\tilde{C}_2$, $a_2 = \tilde{B}_2\tilde{D}_3 - \tilde{B}_3\tilde{D}_2$
$b_0 = \tilde{A}_3\tilde{C}_1 - \tilde{A}_1\tilde{C}_3$, $b_1 = \tilde{A}_3\tilde{D}_1 - \tilde{A}_1\tilde{D}_3 + \tilde{B}_3\tilde{C}_1 - \tilde{B}_1\tilde{C}_3$, $a_2 = \tilde{B}_3\tilde{D}_1 - \tilde{B}_1\tilde{D}_3$
$c_0 = \tilde{A}_1\tilde{C}_2 - \tilde{A}_2\tilde{C}_1$, $c_1 = \tilde{A}_1\tilde{D}_2 - \tilde{A}_2\tilde{D}_1 + \tilde{B}_1\tilde{C}_2 - \tilde{B}_2\tilde{C}_1$, $c_2 = \tilde{B}_1\tilde{D}_2 - \tilde{B}_2\tilde{D}_1$
$\tilde{A}_1 = B_1 D_3 - (\bar{\lambda} + 2\mu)B_4$, $\tilde{A}_2 = B_2 D_3 - D_1 B_4$, $\tilde{A}_3 = B_3 D_3 - D_2 B_4$,
$\tilde{B}_1 = \rho_l D_3 - \rho B_4$, $\tilde{B}_2 = \rho_l D_3 - \rho_l B_4$, $\tilde{B}_3 = -\rho_g B_4$
$\tilde{C}_1 = B_5 D_3 - (\bar{\lambda} + 2\mu)B_8$, $\tilde{C}_2 = B_6 D_3 - D_1 B_8$, $\tilde{C}_3 = B_7 D_3 - D_2 B_8$,
$\tilde{D}_1 = \rho_G D_3 - \rho B_8$, $\tilde{D}_2 = -\rho_l B_8$, $\tilde{D}_3 = \rho_g D_3 - \rho_g B_8$,

By decomposing the partial differential equation we have obtained four Helmholtz equations given by

$(\nabla^2 + \frac{\omega^2}{v_i^2})\, \Phi_i = 0$, i=1,2,3,4

This implies that existence of four compressional waves, each with its own set of scalar potentials Φ_i and complex velocities v_i

$\gamma_4 v^8 - \gamma_3 v^6 + \gamma_2 v^4 - \gamma_1 v^2 + \gamma_0 = 0$

The velocities v_i are defined by the four roots of the above eight order equation with positive real components.

Unsaturated Porous Elastic Solid

The unsaturated thermoelastic medium reduced to unsaturated porous elastic solid if we make this medium free from temperature effect**.** To remove temperature effect we put $T_0=0$, $T_b=0$,K=0

Then dynamic equations of unsaturated elastic medium are given by

$\sigma_{ij,j} = \rho\, \ddot{u}_i^s + \rho_l\, \ddot{u}_i^l + \rho_g\, \ddot{u}_i^g$
$(-p_l)_{,j} = \rho_l\, \ddot{u}_j^s + \frac{\rho_l}{\Phi S^l}\, \ddot{u}_j^l + \frac{\mu_l}{k_r^l\, k}\, \dot{u}_j^l$
$(-p_g)_{,j} = \rho_g\, \ddot{u}_j^s + \frac{\rho_g}{\Phi S^l}\, \ddot{u}_j^g + \frac{\mu_g}{k_r^g\, k}\, \dot{u}_j^g$

The constitutive equation are given by

$\sigma_{ij} = \{ \bar{\lambda}\, u^{\boldsymbol{s}}_{\boldsymbol{k},\boldsymbol{k}} + D_1 u^{\boldsymbol{l}}_{\boldsymbol{k},\boldsymbol{k}} + D_2 u^{\boldsymbol{g}}_{\boldsymbol{k},\boldsymbol{k}} \}\, \delta_{ij} + \mu\, (u^{\boldsymbol{s}}_{\boldsymbol{i},\boldsymbol{j}} + u^{\boldsymbol{s}}_{\boldsymbol{j},\boldsymbol{i}})$
$(-p_l)_j = B_1 u^{\boldsymbol{s}}_{\boldsymbol{k},\boldsymbol{kj}} + B_2 u^{\boldsymbol{l}}_{\boldsymbol{k},\boldsymbol{kj}} + B_3 u^{\boldsymbol{g}}_{\boldsymbol{k},\boldsymbol{kj}}$
$(-p_g)_j = B_5 u^{\boldsymbol{s}}_{\boldsymbol{k},\boldsymbol{kj}} + B_6 u^{\boldsymbol{l}}_{\boldsymbol{k},\boldsymbol{kj}} + B_7 u^{\boldsymbol{g}}_{\boldsymbol{k},\boldsymbol{kj}}$

Coefficients in terms of measurable quantities are given by

$\bar{\lambda} = \lambda + \alpha\, [\Upsilon a_{11} + (1 - \Upsilon) a_{21}]$,
$D_1 = \alpha\, [\Upsilon a_{12} + (1 - \Upsilon) a_{22}]$,
$D_2 = \alpha\, [\Upsilon a_{13} + (1 - \Upsilon) a_{23}]$,
$B_1 = a_{11}$, $B_2 = a_{12}$, $B_3 = a_{13}$,
$B_5 = a_{21}$, $B_6 = a_{22}$, $B_7 = a_{23}$,
$a_{11} = A_{22}/\mathrm{G}$,
$a_{12} = A_{22} A_{14} / \Phi S^l \mathrm{G}$,
$a_{13} = A_{22} A_{15} / \Phi S^g \mathrm{G}$
$a_{21} = -A_{21}/\mathrm{G}$, $a_{22} = (A_{11} A_{24} - A_{21} A_{14}) / \Phi S^l \mathrm{G}$
$a_{23} = (A_{11} A_{25} - A_{21} A_{15}) / \Phi S^g \mathrm{G}$,
$\mathrm{G} = A_{11} A_{22}$
$A_{11} = \Phi S^l \beta_{\omega p}$
$A_{13} = 1 - \Phi$, $A_{14} = \Phi S^l$, $A_{15} = \Phi S^g S^l$

$A_{21} \;=\; \Phi\,[S^g S^l \beta_{\omega p} - A_s\,]$
$A_{22} = \Phi A_s$
A_{24} =-$A_{25} = \Phi S^g S^l$
m=1-1/d
$S_e = \frac{S^l - S^l{}_{res}}{S^l{}_{sat} - S^l{}_{res}}$,
$A_s \;= -m\,\chi d(S^l{}_{sat} - S^l{}_{res})\,S_e{}^{m+1/m}(S_e{}^{-1/m} - 1)^{1-1/d}$.
The wave equations of unsaturated porous elastic solid medium are given by:
$(\bar{\lambda} + \mu)\;\nabla(\nabla.u^s) + D_1\nabla(\nabla.u^l) + D_2\nabla(\nabla.u^g) + \mu\,\nabla^2 u^s \;= \rho\,\ddot{u}^s + \rho_l \ddot{u}^l + \rho_g \ddot{u}^g$
$B_1\nabla(\nabla.u^s) \;+ B_2\nabla(\nabla.u^l) \;+ B_3\nabla(\nabla.u^g) \;= \rho_l\,\ddot{u}^s + \nu_l \ddot{u}^l + \tilde{\upsilon}_l \dot{u}^l$
$B_5\nabla(\nabla.u^s) \;+ B_6\nabla(\nabla.u^l) \;+ B_7\nabla(\nabla.u^g) \;\;= \rho_g\,\ddot{u}^s + \nu_g \ddot{u}^g + \tilde{\upsilon}_g \dot{u}^g$
By using the displacement expressions, the system of equations is resolved into two subsystems. The first one relates the scalar potentials (Φ_s, Φ_l, Φ_g) **.** This is a coupled system which connects the dilatation ($\theta_s = \nabla^2\,\Phi_i$, i = s, l,g) and temperature T . for the time harmonic variations ($\sim e^{-i\omega t}$) of Φ_s , Φ_l , Φ_g with angular frequency ω, this system is obtained as follows
$[(\bar{\lambda} \;+ 2\mu)\nabla^2 + \rho\,\omega^2]\,\theta_s + [\,D_1\nabla^2 + \rho_l\omega^2]\,\theta_l + \;[\,D_2\nabla^2 + \rho_g\omega^2]\,\theta_g$ =0
$[\,B_1\nabla^2 + \rho_l\omega^2]\,\theta_s + [\,B_2\nabla^2 + \rho_l\omega^2]\,\theta_l + [\,B_3\nabla^2]\,\theta_g$ =0
$[\,B_5\nabla^2 + \rho_l\omega^2]\,\theta_s + [\,B_6\nabla^2]\,\theta_l + [\,B_7\nabla^2 + \rho_g\omega^2]\,\theta_g$ =0
Solving equations we obtained an eight order partial differential equation for the propagation of longitudinal waves in unsaturated porous media, given by
$[P_1\nabla^6 \;+ P_2\omega^2\,\nabla^4 \;+ P_3\omega^4\,\nabla^2 \;+ P_4\omega^6\,]\,\theta_s$=0
Where
$P_1 \;= (\bar{\lambda} \;+ 2\mu)K_1 + D_1 L_1 + D_2 M_1$
$P_2 = K_1\rho + (\bar{\lambda} \;+ 2\mu)K_2 + D_1 L_2 + L_1\rho_l + D_2 M_2 + M_1\rho_l$
$P_3 = \;K_2\rho + (\bar{\lambda} \;+ 2\mu)K_3 + D_1 L_3 + L_2\rho_l + D_2 M_3 + M_2\rho_g$
$P_4 = \;K_3\rho + (\bar{\lambda} \;+ 2\mu)K_3 + L_3\rho_l + M_3\rho_g$
$K_1 = B_2 B_7$ - $B_6 B_3$
$K_2 \;= B_2\,\rho_g + B_7\,\rho_l$
$K_3 = \rho_l\rho_g$
$L_1 = B_3 B_5$ - $B_1 B_7$
$L_2 = B_3\rho_g$ - $\rho_g B_1$ -$\rho_l B_7$
$L_3 = -\rho_l\rho_g$
$M_1 = B_1 B_6$ - $B_2 B_5$
$M_2 = B_6\rho_l$ - $\rho_l B_5$ -$\rho_g B_2$
$M_3 = -\rho_g\rho_l$
By decomposing the partial differential equation we have obtained four Helmholtz equations given by
$(\,\nabla^2 + \frac{\omega^2}{v_i^2})\,\Phi_i$ =0 , i=1,2,3,
This implies that existence of three compressional waves, each with its own set of scalar potentials Φ_i and complex velocities v_i
$P_4\,v^6$ - $P_3\,v^4 \;+ P_2 v^2$ - $P_1 = 0$
The velocities v_i are defined by the four roots of the above six order equation with positive real components.

Elastic Medium

The unsaturated porous elastic medium reduced to elastic medium if we remove porosity. The medium which changed its shape when any force is applied and when force is removed it come back its original form.
The dynamic equation of elastic medium is given by
$\sigma_{ij\,,j} = \rho\;\;\ddot{u}_i^s$
The constitutive equation are given by
$\sigma_{ij} \;= \;\bar{\lambda}\,u^{s}_{k,k} + \mu\,(u^{s}_{i,j} + u^{s}_{j,i})$
The wave equation of elastic solid medium is
$(\bar{\lambda} + \mu)\;\nabla(\nabla.u^s) + \mu\,\nabla^2 u^s \;\;= \rho\,\ddot{u}^s$
By using the displacement expressions, the system of equation is resolved into two subsystems. The first one relates the scalar potential Φ_s **.** This is a coupled system which connects the dilatation ($\theta_s = \nabla^2\,\Phi_s$,). For the time harmonic variations ($\sim e^{-i\omega t}$) of Φ_s with angular frequency ω, this system is obtained as follows

$[(\bar{\lambda} + 2\mu)\nabla^2 + \rho\,\omega^2]\,\theta_s = 0$

Solving equation we obtained second order partial differential equation for the propagation of longitudinal waves in elastic solid medium, given by

$\rho\, v^2 - (\bar{\lambda} + 2\mu) = 0$

Concluding Observations

In the presented study, we have reduced unsaturated poro-thermoelastic medium to elastic medium.. The wave equation in every medium is established by mass balance equation and momentum balance equation. As the medium reduced, the degree of equation for the propagation of longitude waves is also decreased.

REFERENCES

[1]. Carcione, J.M., Cavallini, F., Wang, E., Ba, J. and Fu, L. (2019a), "Physics and simulation of wave propagation in linear thermoporoelastic media", Journal of Geophysical Research: Solid Earth,Vol. 124 No. 8, pp. 8147-8166.

[2]. Carcione, J.M., Wang, Z., Ling, W., Salusti, E., Ba, J. and Fu, L. (2019b), "Simulation of wave propagation in linear thermoelastic media", GEOPHYSICS, Vol. 84 No. 1

[3]. Kumar, M., Barak, M.S. and Kumari, M. (2019), "Reflection and refraction of plane waves at the boundary of an elastic solid and double-porosity dual-permeability materials", Petroleum Science, Vol. 16 No. 2, pp. 298-317.

[4]. Kumar, M., Singh, A., Kumari, M. and Barak, M.S. (2021b), "Reflection and refraction of elastic waves at the interface of an elastic solid and partially saturated soils", Acta Mechanica, Vol. 232 No. 1,pp. 33-55.

[5]. Sharma, M.D. (2008), "Wave propagation in thermoelastic saturated porous medium", Journal of Earth System Science, Vol. 117 No. 6, pp. 951-958.

[6]. Zhou, F., Liu, H. and Li, S. (2019), "Propagation of thermoelastic waves in unsaturated porothermoelastic media", Journal of Thermal Stresses, Vol. 42 No. 10, pp. 1256-1271..

[7]. Carcione, J.M., Cavallini, F., Wang, E., Ba, J. and Fu, L. (2019a) Physics and simulation of wave propagation in linear thermoporoelastic media. Journal of Geophysical Research: Solid Earth, Vol. 124 No. 8, pp. 8147-8166

[8]. Barak, M.S., Kumar, M., Kumari, M., & Singh, A. (2020). Inhomogeneous wave propagation in partially saturated soils. Wave Motion, 93, 102470.

[9]. Carcione, J. M., Cavallini, F., Santos, J. E., Ravazzoli, C. L., & Gauzellino, P. M. (2004). Wave propagation in partially saturated porous media: simulation of a second slow wave. Wave Motion, 39(3), 227-240

[10]. Wang, E., Carcione, J. M., Yuan, Y., & Ba, J. (2021). Reflection of inhomogeneous plane waves at the surface of a thermo-poroelastic medium. Geophysical Journal International, 224(3), 1621-1639.

[11]. Sharma, M. D., & Kumar, M. (2011). Reflection of attenuated waves at the surface of a porous solid saturated with two immiscible viscous fluids. Geophysical Journal International, 184(1), 371-384.

[12]. Kumar, M., & Saini, R. (2012). Reflection and refraction of attenuated waves at boundary of elastic solid and porous solid saturated with two immiscible viscous fluids. *Applied Mathematics and Mechanics*, *33*(6), 797-816.

[13]. Kumar, Manjeet, Manjeet Kumari, and Mahabir Singh Barak. "Reflection of plane seismic waves at the surface of double-porosity dual-permeability materials." *Petroleum Science* 15.3 (2018): 521-537..

[14]. Sharma, M. D. "Effect of local fluid flow on reflection of plane elastic waves at the boundary of a double-porosity medium." *Advances in water resources* 61 (2013): 62-73.

[15]. Barak, M. S., M. Kumari, and M. Kumar. "Effect of local fluid flow on the propagation of plane waves at an interface of water/double-porosity solid with underlying uniform elastic solid." *Ocean Engineering* 147 (2018): 195-205.

[16]. Kumari, Manjeet, et al. "Reflection of inhomogeneous plane waves at the surface of an unsaturated porothermoelastic media." *The European Physical Journal Plus* 137.6 (2022): 729.

[17]. Sharma, M. D. "Reflection–refraction of attenuated waves at the interface between a thermo-poroelastic medium and a thermoelastic medium." *Waves in Random and Complex Media* 28.3 (2018): 570-587.

[18]. Sharma, M. D. "Wave propagation in anisotropic generalized thermoelastic media." *Journal of Thermal Stresses* 29.7 (2006): 629-642.

[19]. Kumar, Manjeet, et al. "Inhomogeneous wave reflection from the surface of a partially saturated thermoelastic porous media." *International Journal of Numerical Methods for Heat & Fluid Flow* 32.6 (2022): 1911-1943

[20]. Biot, M.A. (1956), "Thermoelasticity and irreversible thermodynamics", Journal of Applied Physics, Vol. 27 No. 3, pp. 240-253.

[21]. Coleman, B.D. (1964), "Thermodynamics of materials with memory", Archive for Rational Mechanics and Analysis, Vol. 17 No. 1, pp. 1-46

[22]. Coleman, B.D. and Noll, W. (1963), "The thermodynamics of elastic materials with heat conduction and viscosity", Archive for Rational Mechanics and Analysis, Vol. 13 No. 1, pp. 167-178.