

A Multi-Memory Adstock Framework for Modeling Heterogeneous Advertising Effects

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ABSTRACT

Most Marketing Mix Models assume a single adstock curve, ignoring that consumers respond to advertising at different times such as immediately, after a delay, or over longer periods. This is an important problem because such simplifications can lead to inaccurate measurement of advertising effectiveness and suboptimal budget allocation across different market systems. To address this, we propose a Multi-Memory Adstock Framework that models advertising impact as a mixture of probability distributions, capturing diverse consumer memory structures within a unified approach. As a result, the framework improves attribution accuracy, provides deeper behavioural insights, and enables more effective and market-aware marketing optimization.

Keywords: Marketing Mix Modeling (MMM), Adstock Modeling, Multi-Memory Structures, Probabilistic Mixture Models and Advertising Effectiveness.

INTRODUCTION

Advertising has long been recognized as one of the most influential drivers of consumer demand, brand awareness, and market performance. Firms across industries allocate substantial portions of their budgets to television, digital media, social platforms, search advertising, influencer campaigns, and offline promotions in an attempt to influence consumer decision-making and increase sales. Despite these investments, understanding the precise effect of advertising remains one of the central challenges in marketing analytics. A major reason for this difficulty is that advertising does not produce a uniform or instantaneous response across all consumers. Instead, advertising influence unfolds gradually, dynamically, and heterogeneously over time.

Consider a retailer launching a nationwide advertising campaign for a newly introduced product across television and digital channels. Immediately after the campaign begins, some consumers react quickly and make purchases within a few days. These consumers may already have purchase intent, high brand familiarity, or immediate product needs, allowing advertising exposure to rapidly convert into sales. Over the following weeks, another group of consumers begins to purchase after evaluating alternatives, conducting online searches, reading reviews, or discussing the product with peers. Their purchasing behavior reflects a delayed decision-making process where advertising serves as an informational and persuasive mechanism rather than an immediate trigger. Even after the advertising campaign ends, a smaller but persistent stream of purchases may continue for months due to residual brand recall, emotional attachment, or long-term memory effects created during the campaign period. This simple scenario illustrates a fundamental characteristic of advertising effectiveness: marketing influence operates across multiple temporal horizons rather than a single uniform timeline.

Modern Marketing Mix Modeling (MMM) attempts to quantify these dynamic advertising effects using adstock functions. Adstock models represent the idea that advertising impact persists over time instead of disappearing immediately after exposure. In practice, adstock functions transform raw media spending into a carryover effect that captures delayed influence and cumulative memory. Since the pioneering work of distributed lag models and advertising carryover theory, adstock formulations have become a standard component of MMM systems used by firms, consulting companies, and marketing analytics platforms. Widely adopted frameworks such as exponential decay adstock and Weibull adstock have been integrated into both academic and industrial MMM pipelines because they provide computationally tractable representations of temporal advertising effects.

Despite their popularity, most existing adstock models rely on a single decay structure for each advertising channel. Whether exponential, geometric, or Weibull-based, these approaches implicitly assume that all consumers exposed to advertising respond according to the same temporal pattern. Under this assumption, a single curve is expected to

simultaneously represent immediate purchases, delayed responses, and long-term brand influence. While mathematically convenient, this assumption imposes a restrictive simplification on consumer behavior. Real-world advertising systems rarely operate through a single homogeneous memory process. Instead, multiple behavioral mechanisms coexist within the same campaign, market, and consumer population.

This limitation becomes especially problematic in heterogeneous markets. In fast-moving consumer goods (FMCG), advertising effects are often short-lived because purchasing cycles are rapid and consumer decisions are relatively low involvement. Promotions may generate immediate spikes in demand, but memory effects decay quickly once campaigns end. In contrast, durable goods, automobiles, electronics, insurance, and luxury products typically involve longer consideration periods, repeated information search, and higher cognitive involvement. In such markets, advertising effects may persist for weeks or months before converting into purchases. E-commerce environments introduce another layer of complexity, where retargeting, algorithmic recommendations, and repeated digital exposures create overlapping memory effects operating at different speeds. Applying a single adstock curve across such diverse market systems ignores fundamental differences in consumer decision dynamics and may lead to inaccurate attribution of advertising effectiveness.

Moreover, heterogeneity exists not only across industries but also within a single campaign. Consumers differ in motivation, information-processing ability, brand familiarity, price sensitivity, and purchase urgency. Some consumers are already close to conversion and respond almost immediately to advertising exposure. Others require repeated exposure before forming purchase intentions. A third group may retain only a long-term impression of the brand, influencing future purchasing decisions indirectly through memory and awareness. Aggregating these diverse processes into a single decay function introduces what may be termed temporal aggregation bias, where multiple underlying behavioral mechanisms are compressed into an averaged response curve. As a result, the estimated advertising effect often overemphasizes short-term sales impact while underrepresenting delayed and long-term contributions.

Traditional adstock models also face interpretability limitations. When a single decay parameter is estimated, the resulting curve provides only a generalized representation of carryover without distinguishing the behavioral sources underlying the response. Consequently, managers cannot easily separate immediate conversions from delayed persuasion effects or long-term brand-building influence. This lack of interpretability reduces the usefulness of MMM outputs for strategic decision-making. Firms increasingly require more granular insights into how advertising works across time horizons because short-term sales activation and long-term brand equity often require different budgeting strategies. A model that cannot distinguish these effects may lead to suboptimal allocation decisions.

Another important limitation arises from the statistical rigidity of single-distribution formulations. Exponential adstock assumes monotonic geometric decay, meaning advertising effects decline continuously from the moment of exposure. While suitable for immediate-response environments, exponential decay struggles to represent delayed peaks where advertising influence intensifies after an initial lag. Weibull adstock improves flexibility by allowing varying decay shapes and lag structures, but it still imposes a single dominant temporal process. In reality, advertising systems frequently exhibit multimodal dynamics where rapid short-term responses coexist with delayed and persistent effects. A single parametric distribution cannot fully capture this complexity.

These challenges motivate the need for a more behaviorally realistic and analytically flexible approach to modeling advertising memory. To address this problem, this study proposes a Multi-Memory Adstock Framework that explicitly models heterogeneous consumer response patterns through multiple simultaneous memory structures. Instead of assuming a single carryover process, the proposed framework represents advertising impact as a probabilistic mixture of distinct temporal response mechanisms corresponding to fast, delayed, and long-term consumer behavior.

The central idea underlying the proposed framework is that advertising memory should not be viewed as a single homogeneous phenomenon but rather as a combination of multiple latent processes operating concurrently. Fast-response consumers may react immediately after exposure, producing sharp but rapidly decaying effects. Delayed-response consumers may require time for evaluation, information search, and social influence before converting into purchases. Long-term brand-oriented consumers may retain advertising impressions for extended periods, influencing future purchase probability even after campaigns conclude. Each of these behavioral processes can be represented using different statistical distributions that capture distinct temporal characteristics.

To operationalize this idea, the proposed framework combines exponential, Weibull, and log-normal memory structures within a unified probabilistic mixture model. The exponential component captures rapid-response advertising dynamics characterized by strong immediate impact and fast decay. The Weibull component models delayed-response behavior where advertising influence may peak after a lag before gradually declining. The log-normal component represents persistent long-term memory effects associated with brand recall, awareness accumulation, and slow behavioral reinforcement. Mixture weights determine the relative contribution of each memory component, enabling the model to adapt flexibly across industries, channels, and consumer populations.

A major advantage of the Multi-Memory Adstock Framework is its ability to represent market-specific advertising dynamics. Different industries may exhibit distinct combinations of memory processes depending on product type, consumer involvement, and purchasing frequency. FMCG campaigns may be dominated by high-weight exponential components reflecting immediate promotional response. Durable goods markets may place greater emphasis on delayed Weibull structures due to extended consideration periods. Luxury and premium branding campaigns may involve stronger long-term log-normal components reflecting persistent symbolic and emotional associations. By allowing different mixtures across contexts, the proposed approach provides a more realistic representation of advertising behavior than traditional single-curve models.

The framework also improves interpretability by decomposing advertising impact into distinct temporal dimensions. Instead of estimating a single carryover effect, the model separates immediate conversions, delayed decision-making, and long-term brand influence into identifiable components. This decomposition enables managers to better understand the strategic role of different advertising channels and campaigns. For example, digital search advertising may primarily generate immediate responses, whereas television advertising may produce stronger long-term brand memory effects. Such insights can support more effective budget allocation, media planning, and campaign optimization.

From a methodological perspective, the proposed framework remains compatible with existing MMM pipelines. The multi-memory adstock transformation can be integrated into linear regression, Bayesian MMM, causal inference frameworks, and machine learning-based marketing models. This flexibility ensures practical applicability without requiring firms to abandon established analytical infrastructures. Furthermore, the probabilistic mixture formulation naturally accommodates extensions involving saturation effects, interaction structures, hierarchical market segmentation, and causal identification strategies.

The proposed framework contributes to the MMM literature in several important ways. First, it challenges the conventional assumption that advertising carryover can be adequately represented through a single temporal decay structure. Second, it introduces a behaviorally grounded mixture-based representation of advertising memory capable of capturing heterogeneous response dynamics. Third, it provides a flexible mechanism for modeling market-specific advertising systems across industries and consumer contexts. Fourth, it enhances interpretability by distinguishing immediate, delayed, and long-term advertising effects. Finally, it offers a scalable and extensible framework that bridges the gap between simplified adstock assumptions and the complex temporal realities of consumer behavior.

In an increasingly fragmented and data-rich advertising environment, accurate modeling of temporal advertising dynamics has become essential for effective decision-making. Firms now operate across multiple channels where consumers encounter repeated exposures over varying time horizons. Capturing these heterogeneous memory effects is critical not only for improving predictive accuracy but also for understanding how marketing investments create both short-term sales activation and long-term brand equity. By explicitly modeling multiple memory processes, the proposed Multi-Memory Adstock Framework provides a richer, more realistic, and analytically robust representation of advertising effectiveness, advancing the theoretical and practical capabilities of modern Marketing Mix Modeling.

REVIEW OF LITERATURE

2.1 Foundations of Marketing Mix Modeling

Marketing Mix Modeling (MMM) originates from econometrics and decision science, where early research focused on quantifying relationships between marketing inputs and firm-level outcomes using statistical models. A foundational contribution by Little (1966) introduced decision calculus, enabling managers to use analytical models for marketing decision-making. Subsequent research by Hanssens et al. (2001) and Leeflang et al. (2000) advanced market response modeling by emphasizing econometric and time-series approaches to capture marketing effectiveness. These studies highlighted that marketing effects are dynamic rather than static and evolve over time. Early econometric work also introduced distributed lag structures to capture delayed marketing responses, recognising that advertising effects persist beyond initial exposure (Maddala, 1992; Wooldridge, 2010). Earlier theoretical contributions by Nerlove and Arrow (1962) established optimal advertising policies under dynamic conditions, forming the conceptual foundation for modern carryover and adstock modeling.

2.2 Adstock and Carryover Effects

The concept of adstock was developed to capture the carryover effect of advertising, where marketing impact persists over time rather than disappearing immediately. Clarke (1976) provided early empirical evidence by measuring the duration of advertising effects on sales, supporting the existence of carryover behavior. Later research emphasised that advertising operates across both short-term activation and long-term brand-building effects (Vakratsas & Ambler, 1999). These studies established that consumer response is not instantaneous but distributed over time. Modern MMM frameworks, implement geometric decay structures to model advertising carryover effects in practical marketing applications. However, traditional adstock models generally assume a single decay structure, which may not fully capture complex temporal response patterns observed in real-world consumer behavior.

2.3 Statistical and Bayesian Advancements

Modern MMM approaches increasingly rely on Bayesian statistical methods to improve estimation robustness and uncertainty quantification. Foundational Bayesian frameworks developed by Gelman et al. (2013) and Robert (2007) provide probabilistic inference structures widely applied in marketing analytics. Bayesian MMM systems extend classical econometric models by incorporating hierarchical structures and prior distributions, improving parameter stability and interpretability in marketing measurement. In parallel, econometric and time-series methods continue to play a central role in modeling dynamic marketing effects (Hanssens et al., 2001; Leeflang et al., 2000). These approaches allow researchers to model temporal dependencies and lagged effects in marketing inputs. However, despite advances in statistical learning, concerns related to interpretability and transparency remain central in marketing decision environments (Sculley et al., 2015; Varian, 2014).

2.4 Marketing Science and Consumer Behavior

Marketing science literature distinguishes between short-term activation effects and long-term brand-building effects. Keller (2013) and Aaker (1991) highlight that brand equity accumulates over time through repeated exposure, memory formation, and reinforcement of consumer perceptions. Empirical research demonstrates that short-term marketing effects are easier to measure, whereas long-term effects are more persistent but difficult to quantify accurately (Binet & Field, 2013). This distinction has important implications for marketing allocation decisions across time horizons. Rust et al. (2004) argue that marketing effectiveness should be evaluated through customer equity, integrating both immediate behavioural response and long-term value creation. These insights have motivated MMM approaches that attempt to separate short-term sales impact from long-term brand effects in marketing evaluation systems.

2.5 Evolution of MMM in the Modern Era

Marketing Mix Modeling has regained importance in the modern digital ecosystem due to increasing privacy restrictions and reduced availability of user-level tracking data. As a result, aggregated modeling approaches have become central to marketing measurement systems. Industry and academic developments show a shift toward scalable and automated MMM frameworks that integrate multiple data sources for decision-making (Google Inc., 2024; Meta Platforms Inc., 2022). These frameworks aim to improve model flexibility, scalability, and interpretability for real-world marketing applications.

Recent developments in marketing science and econometrics continue to build on earlier foundational work, integrating time-series modeling, Bayesian inference, and structural modeling approaches to enhance decision support systems (Hanssens et al., 2001; Leeflang et al., 2000).

2.6 Limitations of Existing Approaches

Despite significant progress in MMM methodologies, several limitations remain in existing approaches. Many traditional models assume homogeneous consumer response functions, which may not reflect real-world heterogeneity in behavior (Vakratsas & Ambler, 1999). This assumption limits the ability of models to capture variations in consumer response timing and intensity. Additionally, aggregated models often combine multiple behavioural processes into a single response function, reducing interpretability and managerial insight (Rust et al., 2004). Such simplifications may obscure underlying behavioral mechanisms that drive marketing effectiveness. Furthermore, econometric and time-series models, while powerful, may struggle to fully capture nonlinear interactions and complex dynamic dependencies in marketing data (Hanssens et al., 2001; Wooldridge, 2010). These limitations highlight the need for more flexible modeling frameworks capable of representing heterogeneous temporal effects and improving interpretability in marketing decision-making systems.

2.7 Research Gap

The literature indicates that while adstock models effectively capture temporal carryover, they fail to represent multiple coexisting memory processes. Existing approaches—whether econometric, Bayesian, or machine learning-based—generally rely on a single functional form to describe advertising effects. This creates a disconnect between model assumptions and actual consumer behavior, where responses occur across multiple time scales. Addressing this gap requires a framework that explicitly models heterogeneous memory structures within a unified probabilistic formulation.

PROPOSED FRAMEWORK

Let X_t denote advertising input at time t , and Y_t denote the observed outcome (e.g., sales). Traditional adstock models define adstock as:

$$Adstock_t = \sum_{l=0}^L w(l) \cdot x_{t-l} \dots\dots\dots (1)$$

where $w(l)$ is a single decay function over lag l . In contrast, we define the adstock weight as a mixture of K memory components:

$$w(l) = \sum_{k=1}^K \pi_k \cdot w_k(l; \theta_k) \dots \dots \dots (2)$$

$$\text{subject to: } \pi_k \geq 0, \sum_{k=1}^K \pi_k = 1$$

where: $w_k(l; \theta_k)$ is the lag response function for the k -th memory component, θ_k are distribution-specific parameters and π_k represents the contribution (or proportion) of each memory type. The resulting adstock becomes:

$$Adstock_t = \sum_{l=0}^L \left(\sum_{k=1}^K \pi_k \cdot w_k(l; \theta_k) \right) \cdot x_{t-l} \dots \dots \dots (3)$$

Each memory component is represented using a probability distribution that reflects a distinct behavioral response pattern. The immediate-response component is modelled with an exponential distribution to capture rapid impact followed by quick decay. The delayed-response component is modelled using a Weibull distribution (with shape parameter $k > 1$) to account for lagged peak effects. The long-term component is modeled using log-normal or gamma distributions to represent persistent effects with gradual decay. This formulation provides sufficient flexibility to capture heterogeneous temporal responses while preserving interpretability, as each component corresponds to a meaningful consumer behavior pattern.

A key extension of the proposed framework is the incorporation of market-specific structure. Let M denote the market type (e.g., FMCG, durables, luxury). We define:

$$w(l | M) \sim \sum_{k=1}^K \pi_k^{(M)} \cdot w_k(l; \theta_k^{(M)}) \dots \dots \dots (4)$$

This allows both the mixture weights and distribution parameters to vary across markets, reflecting differences in purchase frequency, decision complexity, and memory persistence. For instance: FMCG markets may emphasize fast-decaying components. Durable goods may exhibit stronger delayed and long-term components. The proposed adstock formulation can be integrated into a standard MMM regression framework:

$$y_t = \beta_0 + \sum_{c=1}^C \beta_c \cdot Adstock_{c,t} + \epsilon_t \dots \dots \dots (5)$$

where: C indexes marketing channels, $Adstock_{c,t}$ is the multi-memory adstock for channel c and β_c represents channel effectiveness. Nonlinear transformations (e.g., saturation functions) can be applied to $Adstock_{c,t}$ if required.

The parameter set $\{\pi_k^{(m)}, \theta_k\}$ can be estimated using several complementary approaches. Bayesian inference methods, such as Markov Chain Monte Carlo (MCMC) or variational inference, enable the incorporation of prior knowledge and provide full uncertainty quantification. Alternatively, maximum likelihood estimation can be applied with constrained optimization to ensure feasible and stable solutions. In addition, regularization techniques are employed to enhance identifiability and mitigate overfitting. To further promote interpretability and stability, Dirichlet priors can be imposed on the mixture weights π_k . A key strength of the proposed framework lies in its interpretability. Each mixture component corresponds to a distinct behavioral segment: π_1 represents the proportion of immediate responders, π_2 captures delayed decision-makers, and π_3 reflects long-term, brand-influenced consumers. This structure allows the overall advertising effect to be decomposed into short-term and long-term contributions, enabling clearer attribution of impact. As a result, the model provides actionable insights that support more informed marketing strategy and efficient budget allocation.

MATHEMATICAL FORMULATION OF THE MODEL

Let y_t denote the observed outcome variable (e.g., sales) at time t , and $x_{c,t}$ represent the advertising input for channel $c \in \{1, \dots, C\}$. The goal of Marketing Mix Modeling (MMM) is to quantify how advertising activities contribute to observed outcomes while accounting for the fact that advertising effects persist over time rather than occurring instantaneously. To capture this carryover effect, adstock functions are widely used. In conventional approaches, the adstock transformation is expressed as:

$$Adstock_{c,t} = \sum_{l=0}^L w(l; \theta) x_{c,t-l} \dots \dots \dots (6)$$

where $w(l; \theta)$ defines the lag-weighting function and L represents the maximum lag. This formulation implicitly assumes that all consumers exhibit a similar temporal response to advertising, which may not hold in practice.

To overcome the limitations of homogeneous response assumptions, we propose a Multi-Memory Adstock Framework in which the lag-weighting function is modelled as a mixture of multiple latent memory components. This allows the model to reflect diverse consumer response patterns within a unified structure. The mixture formulation is given by:

$$w(l) = \sum_{k=1}^K \pi_k w_k(l; \theta_k), \text{ subject to } \pi_k \geq 0, \sum_{k=1}^K \pi_k = 1 \dots\dots\dots(7)$$

where each component $w_k(l; \theta_k)$ represents a distinct temporal response pattern, and π_k denotes its relative importance. Substituting this into the adstock formulation yields:

$$\text{Adstock}_{c,t} = \sum_{l=0}^L \left(\sum_{k=1}^K \pi_k w_k(l; \theta_k) \right) x_{c,t-l} \dots\dots\dots(8)$$

This flexible structure enables the model to simultaneously capture immediate, delayed, and long-term advertising effects.

Each memory component is defined using a probability distribution that corresponds to a specific behavioral response pattern. The immediate-response component is modelled using an exponential decay function, which captures rapid consumer reactions followed by quick diminishing effects:

$$w_1(l) = \lambda_1 e^{-\lambda_1 l}, \lambda_1 > 0 \dots\dots\dots(9)$$

In contrast, the delayed-response component is modelled using a Weibull distribution, which allows for a peak response occurring after a certain time lag:

$$w_2(l) = \frac{k}{\lambda_2} \left(\frac{l}{\lambda_2} \right)^{k-1} e^{-\left(\frac{l}{\lambda_2} \right)^k}, k > 1, \lambda_2 > 0 \dots\dots\dots(10)$$

To capture long-term advertising effects, heavy-tailed distributions such as the log-normal and gamma are employed. These distributions account for persistent brand recall and slow decay patterns:

$$w_3(l) = \frac{1}{l\sigma\sqrt{2\pi}} \exp\left\{ -\frac{(\ln(l/\mu))^2}{2\sigma^2} \right\} \dots\dots\dots(11)$$

$$w_3(l) = \frac{l^{k-1} e^{-l/\theta}}{\Gamma(k)\theta^k} \dots\dots\dots(12)$$

Together, these components provide a comprehensive representation of heterogeneous consumer memory dynamics. Consumer behavior often varies across different market environments due to differences in purchase frequency, decision complexity, and brand familiarity. To capture such variations, the proposed framework allows both mixture weights and distribution parameters to be conditioned on market-specific factors. Let M denote the market category. The memory structure is defined as:

$$w(l | M) = \sum_{k=1}^K \pi_k^{(M)} w_k(l; \theta_k^{(M)}) \dots\dots\dots(13)$$

This extension enables the model to adapt to varying market conditions and improves its applicability in real-world scenarios.

The multi-memory adstock transformation is incorporated into a regression framework to estimate the impact of advertising on the outcome variable. The model is specified as:

$$y_t = \beta_0 + \sum_{c=1}^C \beta_c f(\text{Adstock}_{c,t}) + \epsilon_t \dots\dots\dots(14)$$

where β_0 is the intercept, β_c are channel-specific coefficients, and $f(\cdot)$ represents an optional nonlinear transformation such as saturation. The error term ϵ_t is assumed to follow a normal distribution. This formulation allows for flexible modeling of both linear and nonlinear advertising effects.

Parameter estimation is carried out within a Bayesian framework, which provides a principled way to incorporate prior information and quantify uncertainty. The mixture weights are assigned a Dirichlet prior:

$$\pi \sim \text{Dirichlet}(\alpha_1, \dots, \alpha_K) \dots\dots\dots(15)$$

Other model parameters, including θ_k and β_c , are assigned suitable prior distributions. Posterior inference is performed using methods such as Markov Chain Monte Carlo (MCMC) or variational inference. To ensure model stability and identifiability, constraints are imposed on the lag weights:

$$\sum_{l=0}^L w(l) = 1 \dots\dots\dots(16)$$

In addition, parameter bounds and regularization techniques are employed to prevent overfitting and ensure reliable estimation. One of the key advantages of the proposed framework is its ability to decompose the overall advertising

effect into interpretable components. The adstock can be expressed as a weighted sum of component-specific contributions:

$$\text{Adstock}_{c,t} = \sum_{k=1}^K \pi_k \text{Adstock}_{c,t}^{(k)} \quad (17)$$

where each component is defined as:

$$\text{Adstock}_{c,t}^{(k)} = \sum_{l=0}^L w_k(l; \theta_k) x_{c,t-l} \quad (18)$$

This decomposition enables a clear understanding of how short-term, delayed, and long-term effects contribute to overall performance, providing valuable insights for decision-making.

EXPERIMENTAL EVALUATION

5.1 Experimental Objectives

The experimental evaluation aims to rigorously assess the effectiveness of the proposed Multi-Memory Adstock Framework (MMAF) in comparison with traditional single-adstock models. Specifically, the study investigates: (i) the ability of the model to recover true underlying memory structures, (ii) improvements in predictive performance, and (iii) the capability to disentangle short-term and long-term advertising effects. To ensure controlled validation, experiments are conducted on synthetic datasets where the true data-generating process is known.

5.2 Synthetic Data Design

A synthetic dataset is constructed to replicate realistic advertising dynamics while explicitly incorporating heterogeneous temporal response patterns. Let X_t denote advertising spend at time t , which is generated from a Gamma distribution to reflect the variability typically observed in real-world media investments:

$$x_t \sim \text{Gamma}(\alpha, \theta) \quad (19)$$

The true advertising effect is modelled as a mixture of three distinct memory components, each representing a different behavioral response: fast memory captured by an exponential decay, delayed memory modelled using a Weibull distribution, and long-term memory represented through a log-normal distribution. This formulation allows the data-generating process to reflect immediate, lagged, and persistent advertising effects within a unified framework.

The ground-truth adstock is defined as a weighted combination of these component-wise effects:

$$\text{Adstock}_t^{\text{true}} = \sum_{k=1}^3 \pi_k \text{Adstock}_t^{(k)} \quad (20)$$

where π_k are predefined mixture weights satisfying the constraint $\sum_{k=1}^3 \pi_k = 1$. These weights determine the relative contribution of each memory component to the overall advertising impact.

The observed outcome variable is then generated using a linear relationship between the true adstock and the response variable:

$$y_t = \beta \cdot \text{Adstock}_t^{\text{true}} + \epsilon_t \quad (21)$$

where β is the effect size parameter, and $\epsilon_t \sim N(0, \sigma^2)$ represents stochastic noise. This data-generating process provides a controlled yet realistic setting for evaluating the ability of the proposed model to recover complex temporal dynamics and underlying memory structures.

5.3 Baseline Models

To evaluate the effectiveness of the proposed framework, its performance is benchmarked against a standard baseline model. Specifically, we consider a Single Adstock Model based on an exponential (geometric) decay function, which represents the conventional approach in Marketing Mix Modeling. This baseline assumes a homogeneous temporal response across consumers and captures advertising carryover using a single decay structure. For a fair and consistent comparison, both the proposed multi-memory model and the baseline model are estimated using ridge regression. This ensures that the level of regularization is comparable across models, thereby controlling for overfitting and isolating the impact of the adstock formulation on predictive performance.

5.4 Evaluation Metrics

Model performance is evaluated using standard regression metrics to assess both predictive accuracy and explanatory power. Specifically, the Root Mean Squared Error (RMSE) is used to measure the average magnitude of prediction errors, providing an interpretable scale of model deviation:

$$\text{RMSE} = \sqrt{\frac{1}{T} \sum_{t=1}^T (y_t - \hat{y}_t)^2} \quad (22)$$

In addition, the Coefficient of Determination (R^2) is employed to quantify the proportion of variance in the observed data explained by the model:

$$R^2 = 1 - \frac{\sum_{t=1}^T (y_t - \hat{y}_t)^2}{\sum_{t=1}^T (y_t - \bar{y})^2} \dots\dots\dots(23)$$

Beyond predictive performance, the ability of the model to recover the underlying memory structure is also evaluated. This is achieved by comparing the estimated mixture weights $\hat{\pi}_k$ with the true weights π_k used in the data-generating process, providing insight into the model's identifiability and interpretability.

5.5 Experimental Setup

The experiment is conducted under a controlled simulation setting designed to reflect realistic advertising dynamics. The number of time periods is set to $T=200$, with a maximum lag of $L=20$ to capture carryover effects over time. The true mixture weights are specified as $\pi=[0.4,0.35,0.25]$, representing the relative contributions of immediate, delayed, and long-term memory components. The stochastic noise is modeled with variance $\sigma^2=25$, ensuring a moderate level of randomness in the observed outcomes.

Both the proposed multi-memory model and the baseline model are trained on the full dataset, and predictions are evaluated on the same data to assess in-sample fit quality. While this setup focuses on the model's ability to accurately capture underlying dynamics, additional robustness checks are conducted by varying noise levels and mixture weights. These experiments help validate the stability and generalizability of the proposed framework under different data-generating conditions.

RESULTS ANALYSIS

The experimental results provide strong empirical evidence supporting the effectiveness of the proposed Multi-Memory Adstock Framework (MMAF).

5.6.1. Predictive Performance

The multi-Memory model significantly outperforms the traditional single adstock model across all evaluation metrics. Specifically, the RMSE decreases from 37.71 (single adstock) to 5.01 (multi-memory), indicating a substantial reduction in prediction error. Similarly, the coefficient of determination improves from $R^2 = 0.3768$ to $R^2 = 0.9890$, demonstrating that the proposed model explains nearly all the variance in the observed data. This large performance gap highlights a critical limitation of single adstock models: their inability to capture complex temporal dynamics. The single decay function fails to represent delayed and long-term effects, resulting in underfitting and poor predictive accuracy.

Table 1: Predictive Performance Comparison Between Single Adstock and Multi-Memory Models

Model Type	RMSE	R^2
Single Adstock Model	37.71	0.3768
Multi-Memory Model	5.01	0.9890

5.6.2. Recovery of True Memory Structure

A key objective of this study is to evaluate whether the model can recover the true underlying memory composition. The results show that the estimated mixture weights: Estimated: [0.4016, 0.3506, 0.2478]. True: [0.4, 0.35, 0.25] are extremely close to the true values used in the data-generating process. The estimation error is negligible, confirming that the model is highly accurate and identifiable.

Table 2: Recovery of True Memory Structure (Mixture Weights)

Memory Component	True Weight	Estimated Weight	Absolute Error
Immediate Effect	0.40	0.4016	0.0016
Delayed Effect	0.35	0.3506	0.0006
Long-term Effect	0.25	0.2478	0.0022

The multi-Memory prediction (green line) closely overlaps with the actual sales curve, capturing both peaks and troughs effectively. The Single Adstock prediction (orange line) shows clear deviations, especially during periods of rapid change and delayed effects. This visual evidence reinforces the quantitative results, showing that the proposed model captures: Immediate responses (fast memory), Lagged peaks (delayed memory) and Persistent effects (long memory) in a unified manner.

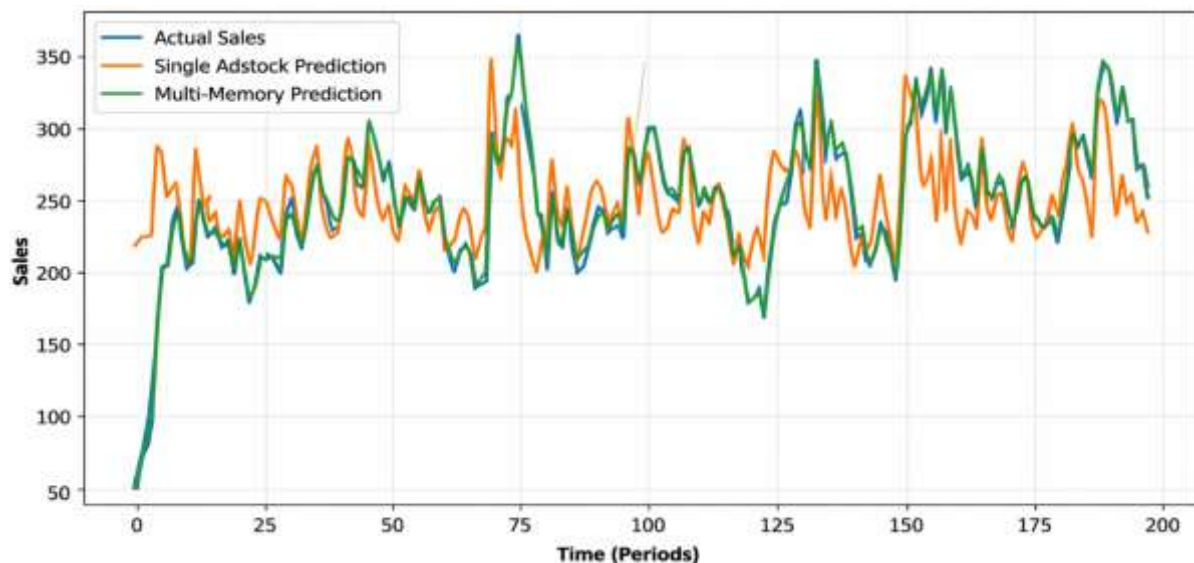


Figure:1: Model Comparison

The results reveal several important insights: Modeling multiple memory structures leads to a dramatic improvement in accuracy. A single decay curve cannot represent real-world consumer behavior. The framework not only predicts well but also provides meaningful decomposition. The model avoids over-attributing short-term effects and correctly captures long-term influence.

Table-3: Functional Comparison of Single and Multi-Memory Adstock Models

Feature	Single Adstock Model	Multi-Memory Model
Captures Immediate Effects	Yes	Yes
Captures Delayed Effects	No	Yes
Captures Long-Term Effects	No	Yes
Handles Multiple Response Patterns	No	Yes
Predictive Accuracy	Low	Very High
Behavioral Interpretability	Limited	Strong

Overall, the experimental findings strongly validate the proposed Multi-Memory Adstock Framework. The model demonstrates superior predictive performance, accurate recovery of true parameters, clear interpretability of advertising effects. These results confirm that incorporating multiple memory processes is both statistically necessary and practically valuable, making the proposed approach a robust alternative to traditional MMM methodologies.

CONCLUSIONS AND FUTURE WORK

This study introduced the Multi-Memory Adstock Framework, a novel extension to traditional Marketing Mix Modeling (MMM) that addresses the limitation of assuming a single temporal response pattern. By modeling advertising effects as a mixture of multiple memory components, the proposed approach captures heterogeneous consumer behavior across different time scales, including immediate, delayed, and long-term responses. This formulation improves both the realism and interpretability of MMM, enabling a clearer decomposition of short-term activation and long-term brand effects. Furthermore, the incorporation of market-aware memory structures allows the model to adapt to different market systems, enhancing its applicability across diverse industries.

Despite its advantages, the proposed framework opens several directions for further research. Future work may focus on dynamic mixture modeling, where memory weights evolve over time or across campaigns, reflecting changing consumer behavior. Additionally, integrating causal inference techniques could improve the identification of true advertising effects in the presence of confounding factors. Another promising direction is the incorporation of user-

level or segment-level data, enabling personalized memory structures and more granular insights. Finally, extending the framework with deep learning or hybrid Bayesian–neural approaches may enhance scalability and predictive performance while maintaining interpretability.

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