

Study of Heat and Mass transfer through a channel filled with porous medium with moving plates & viscous dissipation in the presence of magnetic field

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ABSTRACT

The study of fluid motions through porous medium has endured much attention due to its significance not only to the field of academics but also to the industry. Particularly heat and transfer through channel/ cylinder which are filled with porous medium. Many authors have studied the flows in presence of Magnetic field but with clear fluid i.e. without porous medium while others have taken the plates at rest. Considering all these things, in this Paper, a non-Darcy viscous dissipating MHD flow and heat transfer through a channel filled with porous medium with moving plates is considered. A simulation method called Differential Transform Method or Taylor Transform method is employed to solve the governing equation. The effects of the thermo physical properties of highly porous medium and of the fluid are considered to be constant. Analytical solutions for velocity and temperature are obtained to predict the flow and temperature field. A parametric study is conducted to discuss the influence of different factors on the flow and heat transfer performance. Skin friction coefficient and Nusselt Number is calculated whose tables are also incorporated with graphs.

Keywords: MHD, Porous Media, Heat Transfer, Differential Transform Method (DTM)

INTRODUCTION

The study of fluid motions through porous medium has endured much attention due to its significance not only to the field of academics but also to the industry. Fluid motions have many applications in many industrial and biological processes like food industry, irrigation problems, oil exploitation, motion of blood in the cardiovascular system, chemistry and bio-engineering, soap and cellulose solutions, and in biophysical sciences where the human lungs are considered as a porous layer, etc. Engineering problems like magneto hydrodynamics (MHD) generators, plasma studies, nuclear reactors, geothermal energy extraction, and the boundary layer control in the field of aerodynamics are many applications of the study of flow of an electrically conducting fluid. Vafai [1] studied the effects of variable porosity and inertial forces on convective flow and heat transfer in porous media and forced convection in packed beds in the vicinity of impermeable boundary. Thiyagaraja et al. [2] analyzed fluid flow and heat transfer at the interface region in deepness for three general and fundamental classes of problems in porous media. There are the interface regions between two different porous media, the interface region between a fluid region and a porous medium, and the interface region between an impermeable medium and a porous medium. Ettfagh et al. [3] discussed the significance and relevance of non Darcian effects associated with the buoyancy driven convection in open ended cavities filled with fluid-saturated porous medium. Karimi et al.[4] explained numerically double-diffusive natural convection in a square cavity filled with a porous medium and the effects of non-Darcian fluid are analyzed by investigating the average heat and mass transfer rates.. Marafie et al.[5] analyzed forced convection flow through a channel filled with porous medium and to represent the fluid transport within the porous medium the Darcy- Forchheimer-Brinkman model is used.

A non- thermal equilibrium, two- equation model is used to represent the fluid and solid energy transport. Chakra borty [6] studied MHD flow and heat transfer of a dusty viscoelastic stratified fluid down an inclined channel in porous medium under variable viscosity. Chamkha [7] studied unsteady laminar hyderomagnetic flow and heat transfer in porous channels with temperature dependent properties. Nield et al.[8] studied the thermal development of forced

convection parallel plate channel filled by saturated porous medium with uniform temperature at wall and the effects of axial conduction and viscous dissipation. On foundation of Darcy model, Hooman et al.[9] discussed the effects of viscous dissipation on thermal entranced heat transfer in a parallel plate channel filled with a saturated porous medium. Mankinde et al.[10] discussed the collaborate effects of a transverse magnetic field and radiative heat transfer to unsteady flow of a conducting optically thin fluid through a channel filled with saturated porous medium and non-uniform walls temperature.

Figen Kangalgi et.al.[11] studied to obtain exact and approximate solutions for the nonlinear dispersive KdV and mKdV equations with initial profile. Rawat et al.[12] employed the Nakamura-Sawada rheological model to analyze the pulsatile hydro magnetic flow and heat transfer of a non-Newtonian biofluid through a saturated non-Darcian porous medium channel with viscous heating. Taklifi et al.[13] studied the unsteady magneto hydrodynamic (MHD) periodic flow of a non-Newtonian fluid through a porous channel. The effects of the rheological behavior of fluid on velocity and shear stress profiles along channel width at different time periods have been also depicted. Sharma M. et al.[14] studied the steady magneto hydrodynamic (MHD) flow and heat transfer of electrically conducting viscous incompressible fluid through non-Darcian porous medium bounded between two horizontal infinite impermeable parallel plates with viscous and Joule dissipation effects.. Sepasgozar et al. [15] explained the solution of momentum and heat transfer equations of non-Newtonian fluid flow in an axisymmetric channel with porous wall by using differential transform method. Han Wei et al.[16] discussed about the effective thermal conductivities of composite material and porous media by using machine learning methods which are very useful tools to fast predict the effective thermal conductivities of composite materials and porous media. H.J. Xu[17] discussed about the fully developed forced convection heat transfer in a micro channel partially filled with a porous medium core which is accomplished by assuming local thermal non equilibrium effect between solid and liquid phases. Ganesan [18] studied the magnetic effect on an impulsively moving semi-infinite vertical cylinder in the presence of constant heat flux and magnetic field applied normal to the surface. Aristov [19] discussed two-dimensional time-dependent viscous fluid flow between transversely and longitudinally moving rigid planes. Bhattacharya [20] investigated the effective thermal conductivity, permeability and inertial coefficient of metal foam samples with different porosity.

In the present study, we will apply Differential Transform Method (DTM) which is numerical method based on idea of Taylor series. The differential transform method was first proposed by Zhou JK.[21].Chen et al.[22] studied two-point boundary-value problems using the differential transformation method. Biazar J. [23] applied DTM to solve quadratic Riccati differential equations, Ali j [24] solved the fifth and sixth order boundary value problems along with two conditions in a finite domain and Mirzaee F [25] solved many linear and non- linear ordinary differential equations using differential transform method (DTM). This method is numerical technique to find approximate solutions of linear and non-linear intial value problems and Eigen value problems. Most of the methods are computationally intensive because they are trial-and-error in nature, or need complicated symbolic computations but it gives the exact, approximate, and purely numerical solution for the systems of differential equations and it reduces the size of computational domain and is applicable to many problems easily.

As we have considered the research work of many authors, some have considered the flow through parallel plates but in rest, others have considered parallel moving plates. Some authors considered Non-Darcy fluid flow and some have considered Darcy fluid flow. In the present study, we investigate heat and mass transfer through a channel filled with porous medium with moving plates & a non-Darcy viscous dissipation MHD flow in the presence of magnetic field. The equations which express MHD flow are a combination of continuity equation and Navier-Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism. After non-dimensional zing the governing equations, Differential method is used to find velocity profile and temperature profile and the effect of variation of various physical parameters is examined at velocity profile and temperature profile through graphs.

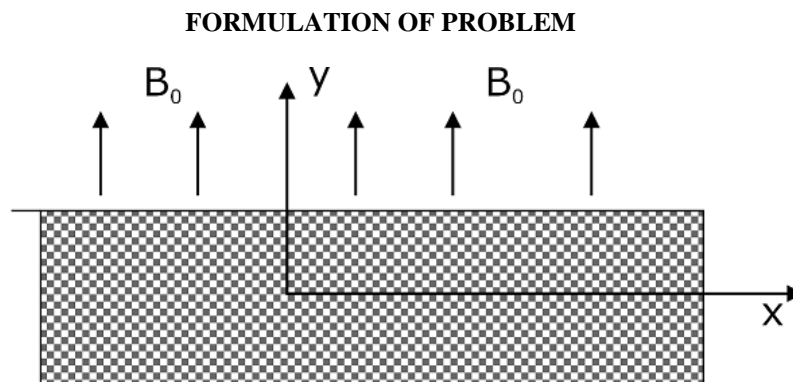


Figure 1.Physical model of the problem

Consider two semi-infinite parallel plates which are parallel to the x-axis and kept at a distance 2d apart, the y-axis is normal to the channel filled with porous medium with moving plates. Temperature of lower plate and upper plate are T_1 and optimum at the middle of channel. At transverse to the flow direction, a static magnetic field of strength B_0 is applied. The effects of the thermo physical properties of highly porous medium and of the fluid are supposed to be constant. In certain porous medium the porosity may not be uniform due to channeling close to the wall. But in the present study the porosity and permeability considered constant near the walls of the channel.

The Maxwell equations for MHD flow are, Shercliff [26]

$$\text{Div} \vec{B} = 0 \quad (1)$$

$$\text{Curl} \vec{B} = \mu_m \vec{J}, \text{ (Amper's law)} \quad (2)$$

$$\text{curl} \vec{E} = \partial \vec{B} / \partial t \text{ (Faraday's law)} \quad (3)$$

Where E represents electric field, B denotes magnetic field, μ_m is the electric permeability and J is the current density. The generalized Ohm's law defines the current density as

$$\vec{J} = \sigma (\vec{E} + \vec{V} \times \vec{B}) \text{ (Ohm's law, without Hall Effect)} \quad (4)$$

Where σ is the electrical conductivity of the fluid

The induced electromagnetic force $F^{(em)}$ is defined as

$$\vec{F}^{(em)} = \vec{J} \times \vec{B} = \sigma (\vec{E} + \vec{V} \times \vec{B}) \times \vec{B} \quad (5)$$

Under the above considerations the governing equations may be defined as

The equation of continuity for incompressible fluid is

$$\nabla \cdot \vec{q} = 0 \quad (6)$$

The momentum equation for the flow through a non-Darcy porous medium by Darcy –Brinkman –Forchheimer model, Nield and Bejan(1992) [27] in the presence of magnetic field takes the following form

$$\rho D\vec{q}/Dt = -\nabla p + \mu_{eff} \nabla^2 \vec{q} - \left\{ \frac{\mu}{K} \vec{q} + \frac{\rho C_d}{\sqrt{K}} |\vec{q}| \vec{q} \right\} + \vec{J} \times \vec{B} \quad (7)$$

Where q represents the velocity field for the flow of incompressible viscous electrically conducting fluid, ρ denotes the density of fluid, μ_{eff} denotes the effective viscosity of fluid in the porous medium, μ represents the viscosity of fluid, C_d denotes the drag coefficient, K represents the permeability, $B(0, B_0, 0)$ represents the magnetic field.

Following Cowling (1957) and Gupta (1960) [28], there is no applied or polarization voltage so that $\vec{E} = 0$. The resultant electromagnetic force $\vec{F}^{(em)} = \vec{J} \times \vec{B} = -\sigma B_0^2 u \hat{j}$.

The energy equation with the consideration of viscous dissipation and joule's dissipation can be written as

$$\rho C_p \frac{DT}{Dt} = k \nabla^2 T + \Phi + \frac{J^2}{\sigma} \quad (8)$$

Where C_p represents the specific heat, T denotes the fluid temperature, Φ represents viscous dissipation term. As the plate are of semi-infinite length therefore the variation along the x-direction are negligible in compare to variation along y-direction. Hence velocity and temperature are taken independent of x. Also, there is no flow along z-direction and plates are extended to semi-infinite length in z-direction, resulting $w=0$ and $\frac{\partial}{\partial z}(\cdot) = 0$. on incorporating these assumption and in view of the physical configuration of the problem, the equations (7) and (8) reduce to

$$\mu_{eff} \frac{\partial^2 u}{\partial y^2} = \frac{\partial p}{\partial x} + \left\{ \frac{\mu}{K} u + \rho \frac{C_d}{\sqrt{K}} u^2 \right\} + \sigma B_0^2 u \quad (9)$$

$$k \frac{\partial^2 T}{\partial y^2} + \sigma B_0^2 u^2 + \mu \left(\frac{\partial u}{\partial y} \right)^2 = 0 \quad (10)$$

The boundary conditions are:

$$\begin{aligned} Y=0; & \quad \frac{\partial u}{\partial y} = 0 & \quad \frac{\partial T}{\partial y} = 0 \\ Y=d; & \quad u = v_0 & \quad T = T_1 \end{aligned} \quad (11)$$

METHOD OF SOLUTION

Introduce following dimensionless quantities which are used in equation of motion and equation of energy and boundary conditions

$$x^* = \frac{x}{d} \quad Y^* = \frac{y}{d} \quad u^* = \frac{u}{v_0} \quad \theta = \frac{T - T_1}{T - T_0}$$

$$D = \frac{K}{d^2} \quad F = \frac{\rho C_d G d^4}{\mu^2 \sqrt{K}} \quad P = \frac{\mu_{eff}}{\mu} \quad H = \sqrt{\frac{\sigma d^2 B_0^2}{\mu}} \quad Bm = \frac{G^2 d^4}{K \mu (T_1 - T_0)} \quad R = \frac{G d^2}{\mu v_0}$$

Where $G = -\frac{\partial p}{\partial x}$ is the pressure gradient, D represents the Darcy number, F denotes Forchheimer number, P is ratio of effective viscosity to the viscosity of the fluid, H denotes the Hartman number, Bm represents the Brinkman number. The equation of motion and energy in dimensionless form are given by

$$\frac{d^2 u^*}{dy^2} = -\frac{R}{P} + \frac{1}{P} \left(\frac{1}{D} + H^2 \right) u^* - F/Pu^{*2} \quad (12)$$

$$\frac{d^2 \theta}{dy^2} + Bm \left(\frac{\partial u^*}{\partial y} \right)^2 + H^2 Bm u^{*2} = 0 \quad (13)$$

The corresponding boundary conditions are:

$$\begin{aligned} y^*=0 & \quad \frac{\partial u^*}{\partial y} = 0 & \quad \theta = 0 \\ y^*=1 & \quad u^*=1 & \quad \theta=1 \end{aligned} \quad (14)$$

There is no loss of generality taking variable without asterisk from dimensionless form of the governing equations. Therefore, the equation of motion and energy in the dimensionless form are given by

$$\frac{d^2 u}{dy^2} = -\frac{R}{P} + \frac{1}{P} \left(\frac{1}{D} + H^2 \right) u - \frac{F}{P} u^2 \quad (15)$$

$$\frac{d^2 \theta}{dy^2} + Bm \left(\frac{\partial u}{\partial y} \right)^2 + H^2 Bm u^2 = 0 \quad (16)$$

The corresponding boundary conditions are:

$$\begin{aligned} y=0 & \quad \frac{\partial u}{\partial y} = 0 & \quad \theta = 0 \\ y=1 & \quad u=1 & \quad \theta=1 \end{aligned} \quad (17)$$

Solution for Velocity Profile

The Differential Transform Method (DTM) is applied for solving (non-linear ordinary differential equation) momentum equation (15).

The differential transform(s) of the derivative $\frac{d^s u(x)}{dx^s}$ is defined as

$$U(s) = \frac{1}{s!} \left[\frac{d^s u(x)}{dx^s} \right]_{x=x_0}$$

Table 1. The fundamental mathematical operation in DTM

Function	Differential Transform
$u(y) = f(y) \pm g(y)$	$U(s) = F(s) \pm G(s)$
$u(y) = \lambda g(y)$	$U(s) = \lambda G(s)$
$u(y) = \frac{\partial g(y)}{\partial y}$	$U(s) = (s+1)G(s+1)$
$u(y) = \frac{\partial^m g(y)}{\partial y^m}$	$U(s) = (s+1) \dots (s+m)G(s+m)$
$u(y) = y^m$	$U(s) = \delta(s - m) = \begin{cases} 1, & s = m \\ 0, & \text{otherwise} \end{cases}$
$u(y) = f(y)g(y)$	$U(s) = \sum_{r=0}^s F(r)G(s-r)$
$u(y) = f_1(y) f_2(y) \dots f_m(y)$	$U(s) = \sum_{s_1=1}^s \dots \sum_{s_{m-1}=0}^{s_2} F_1(s_1)F_2(s_2 - s_1) \dots F_m(s - s_{m-1})$

(18)

And inverse differential transform of U(s) is defined as

$$U(x) = \sum_{s=0}^{\infty} U(s)(x - x_0)^s \quad (19)$$

On applying differential transform method, the momentum equation (15) with transform parameter 'r' gives following recurrence relation

$$\frac{(r+2)!}{r!} U(r+2) = MU(r) + N \sum_{t=0}^r U(t)U(r-t) + Z\delta(r) \quad (20)$$

$$\text{Where } M = \frac{1}{P} \left(\frac{1}{D} + H^2 \right) N = -\frac{F}{P} Z = -\frac{R}{P}$$

$$\delta(r) = \begin{cases} 1 & r = 0 \\ 0 & r \neq 0 \end{cases} \quad (21)$$

DTM of the boundary condition for velocity at lower plate i.e. $\left[\frac{\partial u}{\partial y} \right]_{y=0} = 0$ gives

$$U(1) = 0 \quad (22)$$

The initial value of u(0) is unknown, therefore its DTM U(0) is also not known, so taking $u(0) = \alpha$ (constant)

(23)

Where α is an unknown constant to be determined under prescribed boundary conditions over the flow field.

Applying DTM on (23) gives

$$U(0) = \alpha \tag{24}$$

Put $r=0, 1, 2,3,4,5$ the recurrence relation (20) gives

$$U(2) = \frac{1}{2}(M\alpha + N\alpha^2 + Z)$$

$$U(3) = 0$$

$$U(4) = \frac{N^2\alpha^3}{12} + \frac{MN\alpha^2}{8} + \left(\frac{M^2}{24} + \frac{NZ}{12}\right)\alpha + \frac{MZ}{24}$$

$$U(5) = 0$$

From equation (19), inverse differential transform of $U(s)$ can be defined as

$$U(y) = \sum_{s=0}^5 U(s)y^s \tag{25}$$

Put value of $U(0)$ to $U(5)$ the solution of the problem up to 4th order is given by

$$U(y) = \alpha + \frac{1}{2}(M\alpha + N\alpha^2 + Z)y^2 + \left(\frac{N^2\alpha^3}{12} + \frac{MN\alpha^2}{8} + \left(\frac{M^2}{24} + \frac{NZ}{12}\right)\alpha + \frac{MZ}{24}\right)y^4 \tag{26}$$

At the boundary condition $u(1) = 1$, the equation (26) provides a third degree polynomial in α as given by

$$\frac{N^2}{12}\alpha^3 + \left(\frac{N}{2} + \frac{MN}{8}\right)\alpha^2 + \left(1 + \frac{M}{2} + \frac{M^2}{24} + \frac{NZ}{12}\right)\alpha + \left(\frac{Z}{2} + \frac{MZ}{24} - 1\right) = 0 \tag{27}$$

The fluid flow profile obtained from equation (27) by calculating value of α . The value of physical parameters P, D, F, H and R are obtained by computing the value of α through MATLAB and the result for velocity profile are analyzed by graphs.

Solution of Energy Equation

The solution of the heat equation (16) is obtained by using differential transform method.

The differential transform $\varphi(s)$ of the derivative $\frac{d^s u(x)}{dx^s}$ is defined as

$$\varphi(s) = \frac{1}{s!} \left[\frac{d^s u(x)}{dx^s} \right]_{x=x_0} \tag{28}$$

And inverse differential transform of $\theta(s)$ is defined as

$$\theta(x) = \sum_{s=0}^{\infty} \varphi(s)(x - x_0)^s \tag{29}$$

On applying differential transform method, the Energy equation (16) with transform parameter 'r' gives following recurrence relation

$$\frac{(r+2)!}{r!} \theta(r+2) = E \sum_{t=0}^r u(t)u(r-t) + F \sum_{t=0}^r (t+1)(r-t+1)u(t+1)u(r-t+1) \tag{30}$$

Where $E = -H^2 Bm$, $F = -Bm$

DTM of the boundary condition for temperature at lower plate i.e. $\left[\frac{\partial \theta}{\partial y} \right]_{y=0} = 0$ gives

$$\varphi(1) = 0 \tag{31}$$

The initial value of $\theta(0)$ is unknown, therefore its DTM $\varphi(0)$ is also not known, so taking

$$\varphi(0) = \beta (\text{constant}) \tag{32}$$

Where β is an unknown constant to be determined under prescribed boundary conditions over the flow field

Putting $r=0, 1, 2,3,4,5$ the recurrence relation (30) gives

$$\varphi(2) = \frac{E\alpha^2}{2}$$

$$\varphi(3) = 0$$

$$\varphi(4) = \frac{E\alpha(M\alpha + N\alpha^2 + Z)}{12} + \frac{F(M\alpha + N\alpha^2 + Z)^2}{12}$$

$$\varphi(5) = 0$$

$$\varphi(6) = \frac{E\alpha}{15} \left(\frac{N^2\alpha^3}{12} + \frac{MN\alpha^2}{8} + \left(\frac{M^2}{24} + \frac{NZ}{12} \right) \alpha + \frac{MZ}{24} \right) + \frac{E}{120} (M\alpha + N\alpha^2 + Z)^2 + \frac{4E}{15} (M\alpha + N\alpha^2 + Z) \left(\frac{N^2\alpha^3}{12} + \frac{MN\alpha^2}{8} + \left(\frac{M^2}{24} + \frac{NZ}{12} \right) \alpha + \frac{MZ}{24} \right) \tag{33}$$

From equation (29), inverse differential transform of $\varphi(s)$ can be defined as

$$\theta(y) = \sum_{s=0}^5 \varphi(s)y^s \tag{34}$$

Put value of $\varphi(0)$ to $\varphi(6)$ the solution of the problem upto 6th order is given by

$$\theta(y) = \beta + \frac{E\alpha^2}{2} y^2 + \left(\frac{E\alpha(M\alpha + N\alpha^2 + Z)}{12} + \frac{F(M\alpha + N\alpha^2 + Z)^2}{12} \right) y^4 + \left(\frac{E\alpha}{15} \left(\frac{N^2\alpha^3}{12} + \frac{MN\alpha^2}{8} + \left(\frac{M^2}{24} + \frac{NZ}{12} \right) \alpha + \frac{MZ}{24} \right) + \frac{E}{120} (M\alpha + N\alpha^2 + Z)^2 + \frac{4E}{15} (M\alpha + N\alpha^2 + Z) \left(\frac{N^2\alpha^3}{12} + \frac{MN\alpha^2}{8} + \left(\frac{M^2}{24} + \frac{NZ}{12} \right) \alpha + \frac{MZ}{24} \right) \right) y^6 \tag{35}$$

At the boundary condition $\theta(1) = 1$, the value of β is obtained from the equation (35)

$$\beta = 1 - \left(\frac{E\alpha^2}{2} + \frac{E\alpha(M\alpha + N\alpha^2 + Z)}{12} + \frac{F(M\alpha + N\alpha^2 + Z)^2}{12} + \frac{E\alpha}{15} \left(\frac{N^2\alpha^3}{12} + \frac{MN\alpha^2}{8} + \left(\frac{M^2}{24} + \frac{NZ}{12} \right) \alpha + \frac{MZ}{24} \right) + \frac{E}{120} (M\alpha + N\alpha^2 + Z)^2 + \frac{4E}{15} (M\alpha + N\alpha^2 + Z) \left(\frac{N^2\alpha^3}{12} + \frac{MN\alpha^2}{8} + \left(\frac{M^2}{24} + \frac{NZ}{12} \right) \alpha + \frac{MZ}{24} \right) \right) \tag{36}$$

The Temperature profile obtained from equation (34) by calculating value of β . The value of physical parameters P, D, Fr, H, Bm and R are obtained by computing the value of β through MATLAB.

SKIN FRICTION COEFFICIENT

The non-dimensional shearing stress on the upper plate in terms of local skin-friction coefficient is obtained and computed values are given in the Table 1.

$$S_f = [\partial u / \partial y]_{y=1} = Z + \frac{MZ}{6} + \left(M + \frac{M^2}{6} + \frac{NZ}{3} \right) \alpha + \left(N + \frac{MN}{2} \right) \alpha^2 + \frac{N^2}{3} \alpha^3 \quad (37)$$

NUSSELT NUMBER

The non-dimensional coefficient of heat transfer at the upper plate is obtained and computed values are given in the Table 2.

$$N_s = -(\partial \theta / \partial y)_{y=1} = (E\alpha^2 + 4 \left(\frac{E\alpha(M\alpha + N\alpha^2 + Z)}{12} + \frac{F(M\alpha + N\alpha^2 + Z)^2}{12} \right) + 6 \left(\frac{E\alpha(N^2\alpha^3}{15} + \frac{MN\alpha^2}{8} + \left(\frac{M^2}{24} + \frac{NZ}{12} \right) \alpha + \frac{MZ}{24} \right) + \frac{E}{120} (M\alpha + N\alpha^2 + Z)^2 + \frac{4E}{15} (M\alpha + N\alpha^2 + Z) \left(\frac{N^2\alpha^3}{12} + \frac{MN\alpha^2}{8} + \left(\frac{M^2}{24} + \frac{NZ}{12} \right) \alpha + \frac{MZ}{24} \right)) \quad (38)$$

RESULT AND DISCUSSION

Velocity profile and Temperature profile are plotted to find the effect of various parameters. In figure 2 and 3, the effect of Hartman number on velocity profile is shown with relative viscosity 0.5 and 2 respectively as the Hartman number increases in case of relative viscosity 0.5, the fluid flows accelerates while in case of relative viscosity equal to 2, the flow gets accelerated. The velocity of fluid decreases with the increase in Darcy number at P=0.5 while it accelerates in case of P=2 as shown in figure 4 and 5. In case P=0.5 the fluid velocity increases sharply as Forchheimer number increases from 5 to 10 but in case of P=2, the fluid flow is not much affected by the increase in Forchheimer number as in figure 6 and 7. With the increase in Reynolds number the flow retards in case of P=0.5, while it is accelerated in case of P=2 as shown in figure 8 and 9. Also the impact of relative viscosity is shown in figure 10 in fluid flow, if the effective viscosity is more than the fluid viscosity than fluid flow shown as in figure. The temperature of the fluid increases with the increase in Hartman number as in figure 11 in case of P=0.5, where P=2 first the fluid temperature increases and after that decreases with increase in Hartman number as shown in figure 12. With the increase in Darcy number the temperature shows decreasing trends with increase in Darcy number for P=0.5 while the temperature increases with increase in Darcy number in case of P=2 as in figure 13 and 14. As the trends for fluid flow shows with increase in Forchheimer number as in figure 15 and 16. The Temperature decreases with increase in relative viscosity as in figure 17. The increase in Reynolds number and Brinkman number increase the temperature of the fluid in case of P=0.5 while the temperature decreases in case of P=2 as in figures 18, 19, 20 and 21 respectively. Table 1 shows that when ratio of effective viscosity to viscosity of fluid and Darcy number is increased than rate of Heat Transfer decreased and when Hartmann and Forchheimer number is increased than rate of heat transfer increased. There is slight increment in rate of heat transfer when Reynolds and Brinkman number is increased. The magnitude of skin-friction coefficient is computed at the Upper -plate and tabulated in Table 2. The skin friction coefficient is maximum when the viscosity of fluid is greater than the effective viscosity of fluid. Skin-friction coefficient is increased when Darcy number is increased. When the Value of Forchheimer number is between 5 and 20, than much decrement in skin friction coefficient, but when Reynolds number is increased than skin friction coefficient is slightly decreased.

CONCLUSION

There is slight increment in rate of heat transfer when Reynolds and Brinkman number is increased. When ratio of effective viscosity to viscosity of fluid and Darcy number is increased than rate of Heat Transfer decreased and when Hartmann and Forchheimer number is increased than rate of heat transfer increased. The skin friction coefficient is maximum when the viscosity of fluid is greater than the effective viscosity of fluid. The temperature of the fluid increases with the increase in Hartman number.

Table 1 Values of Nusselt Number at the Upper Plate

P	R	D	H	F	Bm	Ns
0.5	0.01	0.1	5	10	0.01	60.5971
1	0.01	0.1	5	10	0.01	16.3457
2	0.01	0.1	5	10	0.01	0.0133
0.5	0.01	0.1	5	10	0.01	60.5971
0.5	0.01	0.5	5	10	0.01	22.7306
0.5	0.01	1	5	10	0.01	19.6926
0.5	0.01	0.1	1	10	0.01	0.0129
0.5	0.01	0.1	3	10	0.01	2.6734
0.5	0.01	0.1	5	10	0.01	60.5971
0.5	0.01	0.1	5	1	0.01	0.0014
0.5	0.01	0.1	5	5	0.01	0.0015
0.5	0.01	0.1	5	10	0.01	60.5971
0.5	0.1	0.1	5	10	0.01	0.0015
0.5	0.5	0.1	5	10	0.01	0.0018
0.5	1	0.1	5	10	0.01	0.0024
0.5	0.01	0.1	5	10	0.5	0.0732
0.5	0.01	0.1	5	10	1	0.1438
0.5	0.01	0.1	5	10	2	0.2773

Table 2 Values of Skin -Friction Coefficient at the Upper Plate

P	R	D	H	F	Sf
0.5	0.01	0.1	5	10	-63.5293
1	0.01	0.1	5	10	-31.8536
2	0.01	0.1	5	10	3.0250
0.5	0.01	0.1	5	10	-63.5293
0.5	0.01	0.5	5	10	-37.1335
0.5	0.01	1	5	10	-34.2804
0.5	0.01	0.1	1	10	-0.5803
0.5	0.01	0.1	3	10	-17.1536
0.5	0.01	0.1	5	10	-63.5293
0.5	0.01	0.1	5	1	3.6467
0.5	0.01	0.1	5	5	3.7227
0.5	0.01	0.1	5	10	-63.5293
0.5	0.1	0.1	5	10	3.6474
0.5	0.5	0.1	5	10	3.6144
0.5	1	0.1	5	10	3.5874

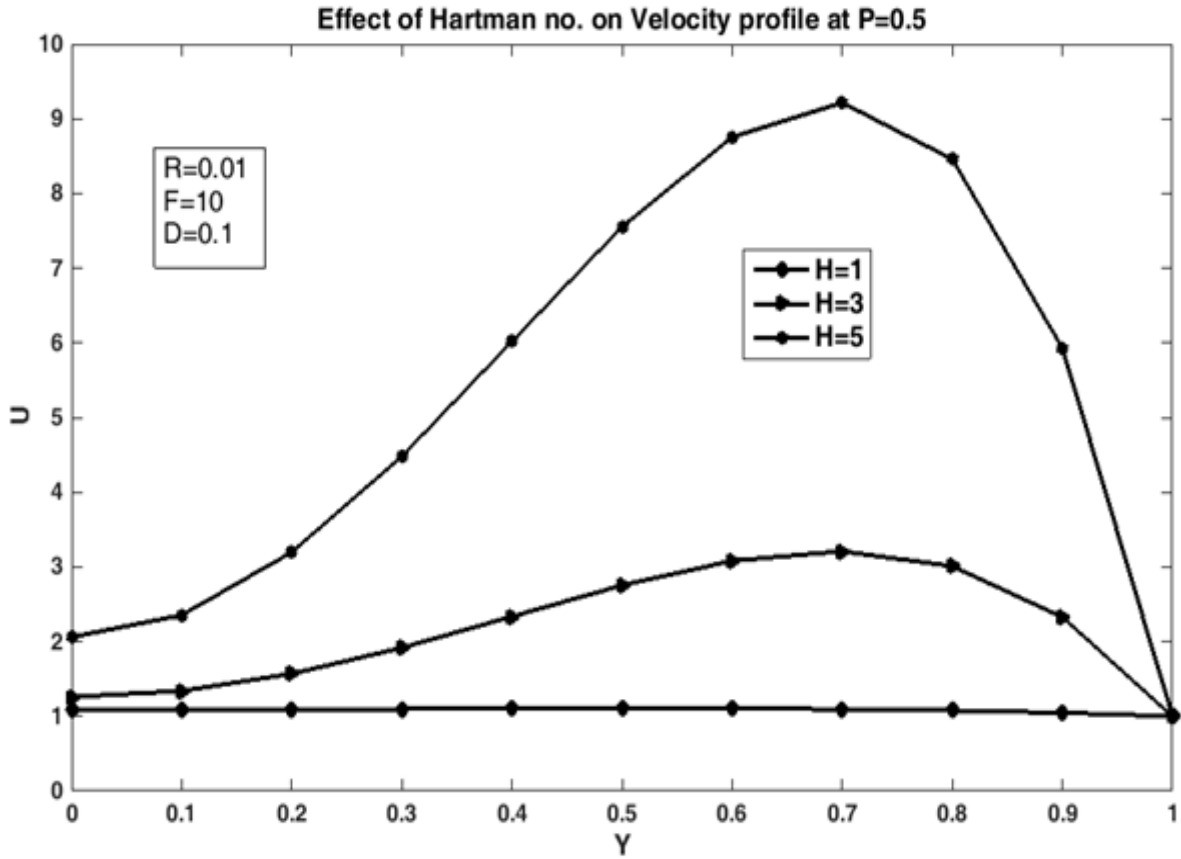


figure 2

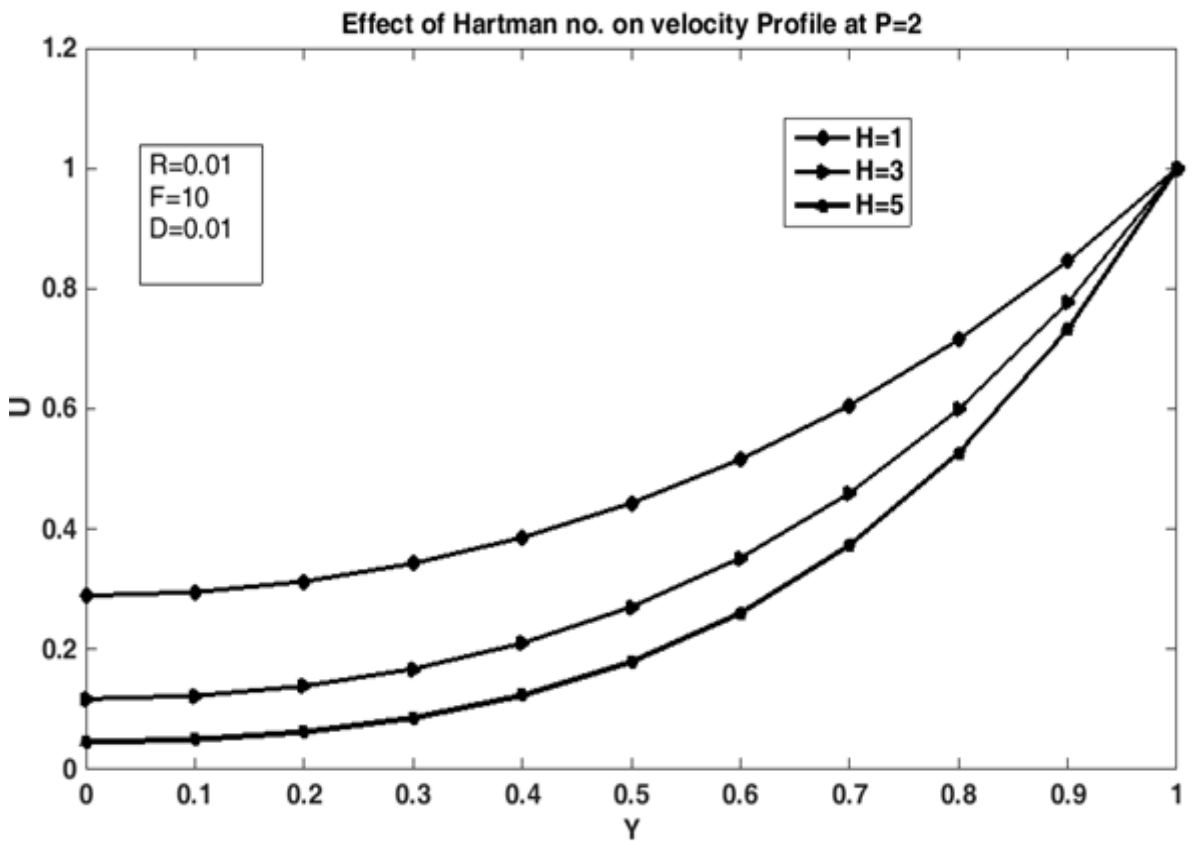
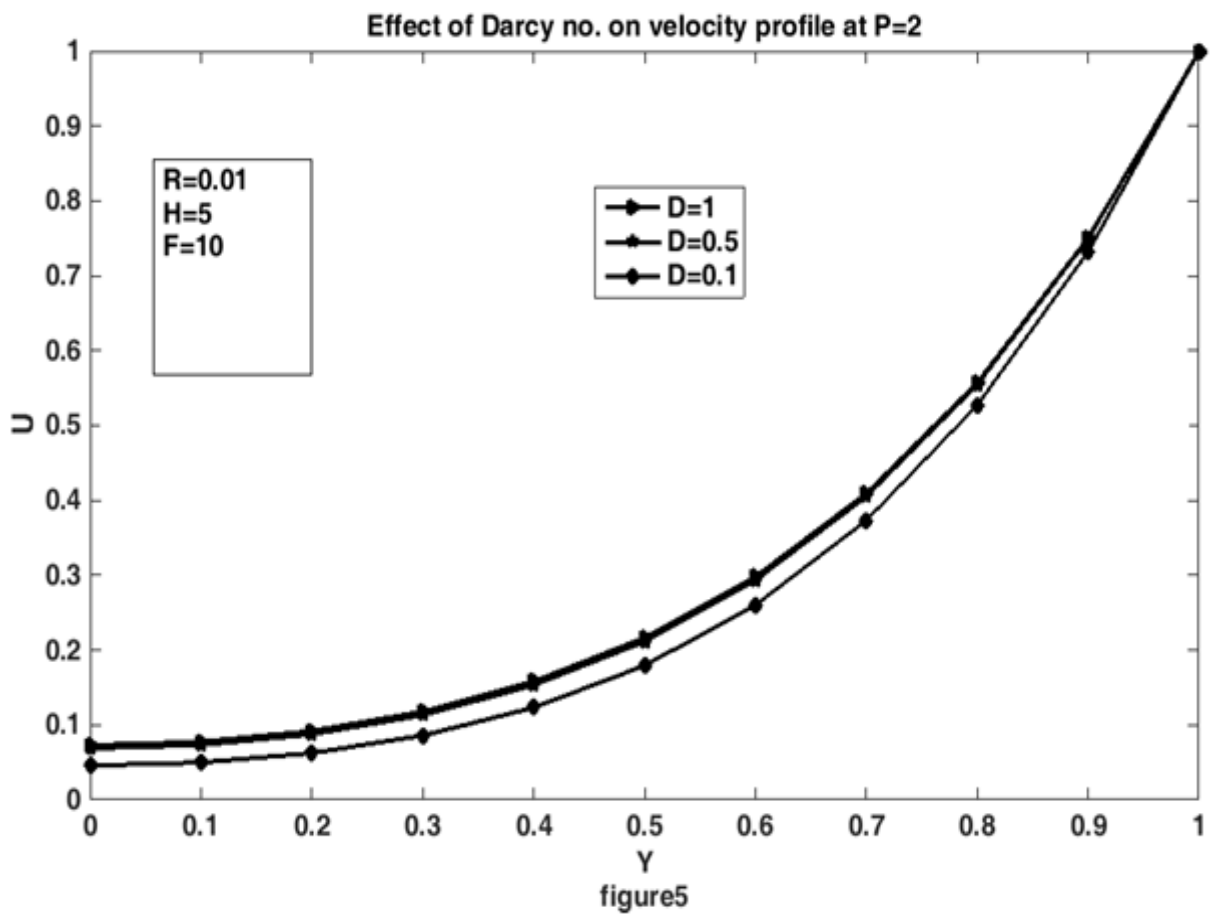
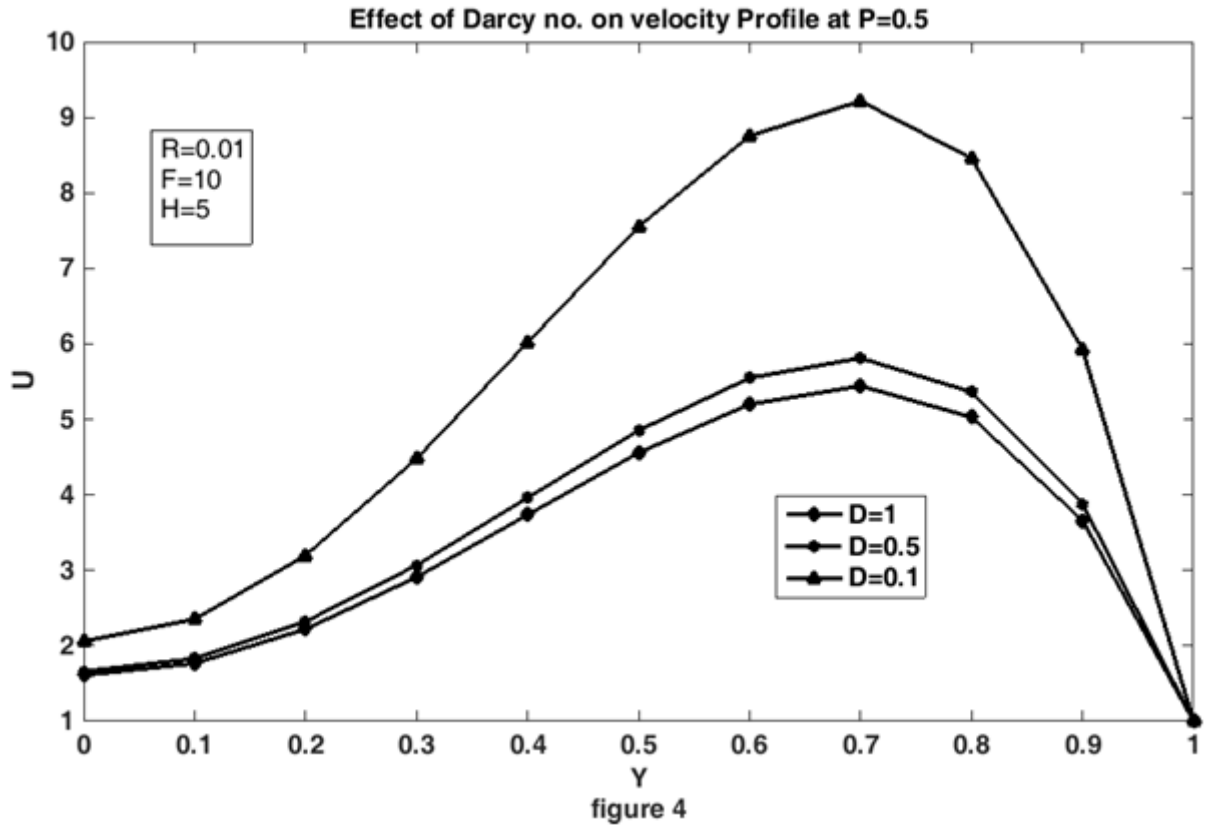
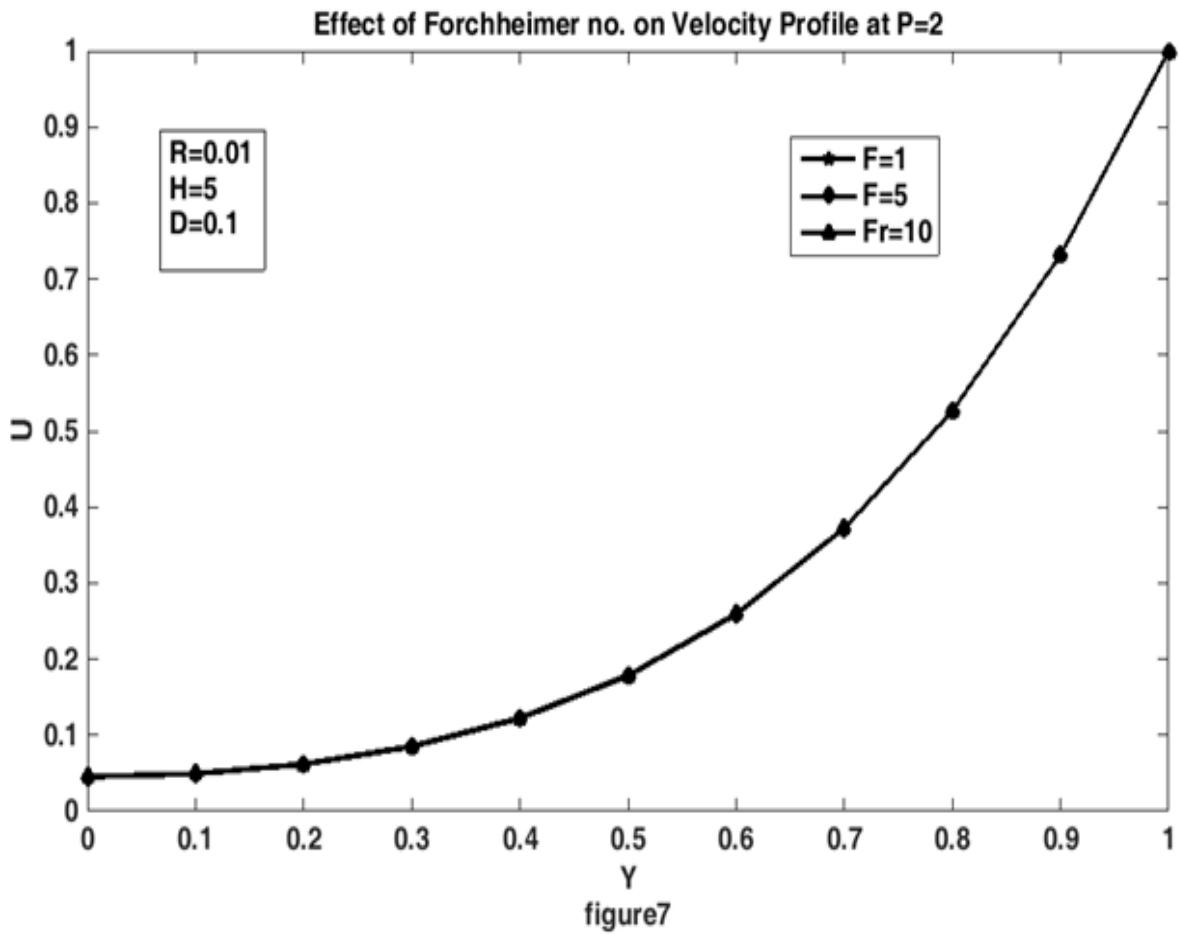
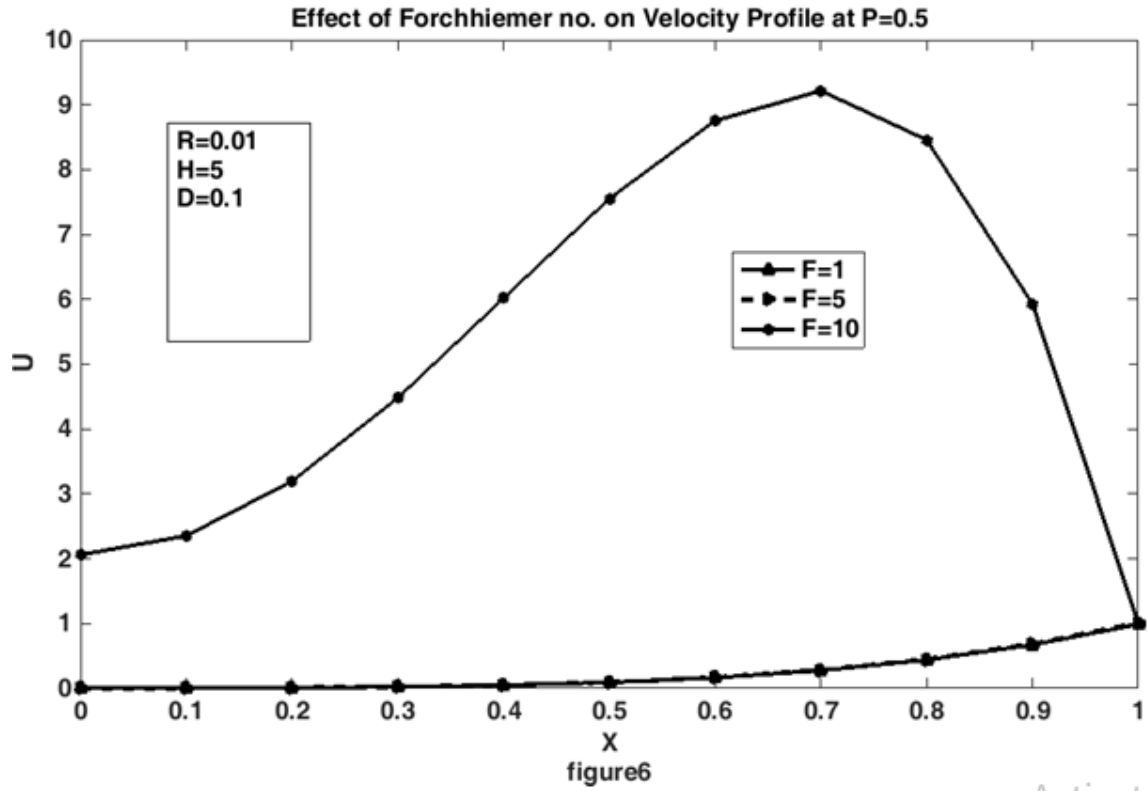
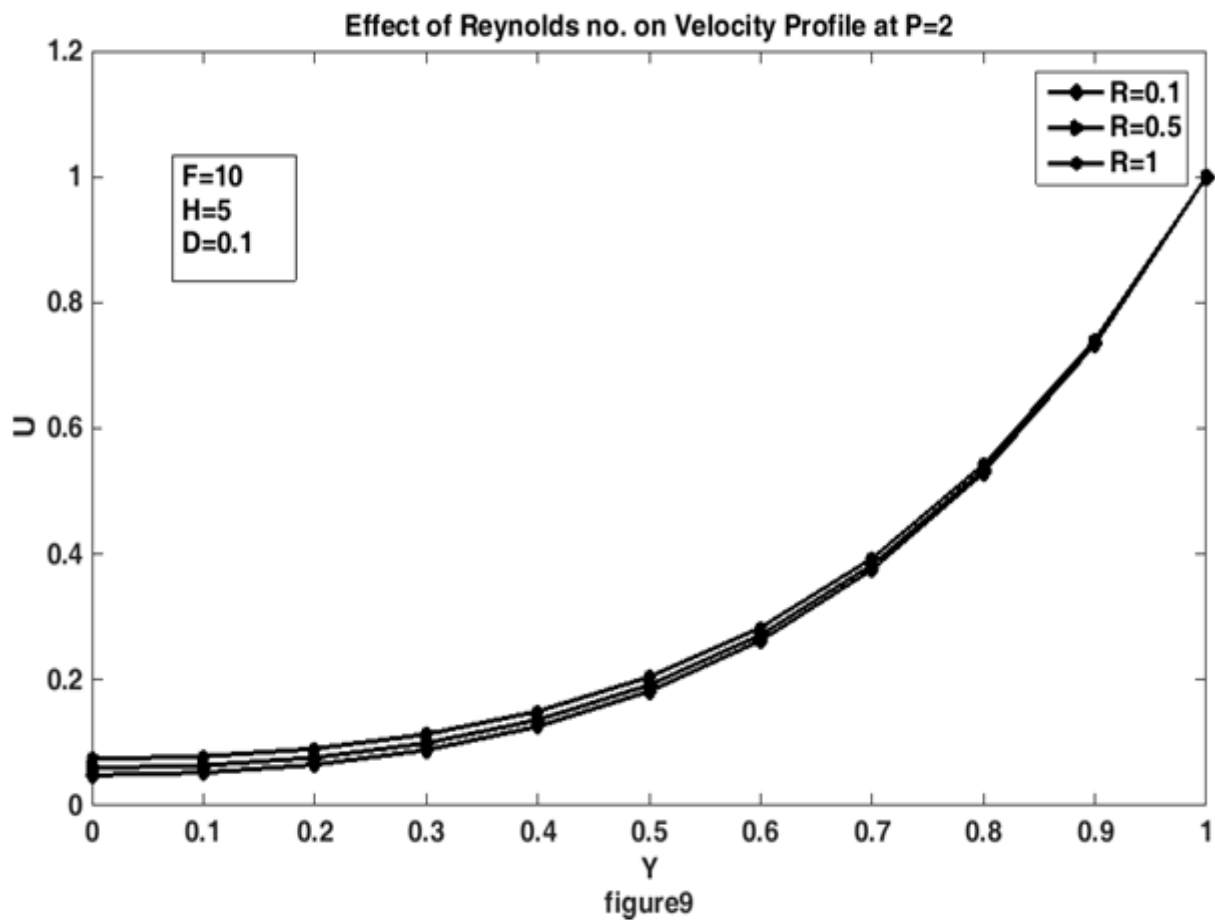
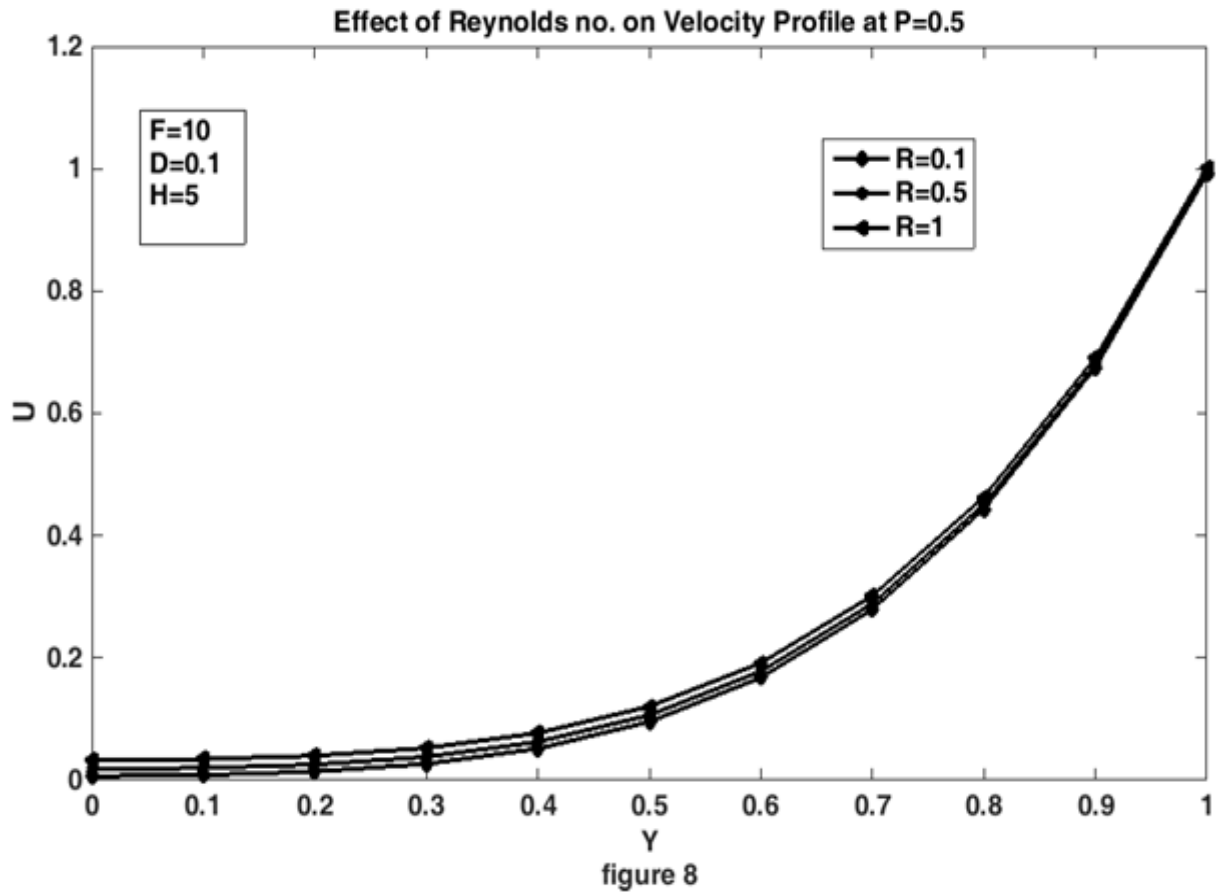


figure 3







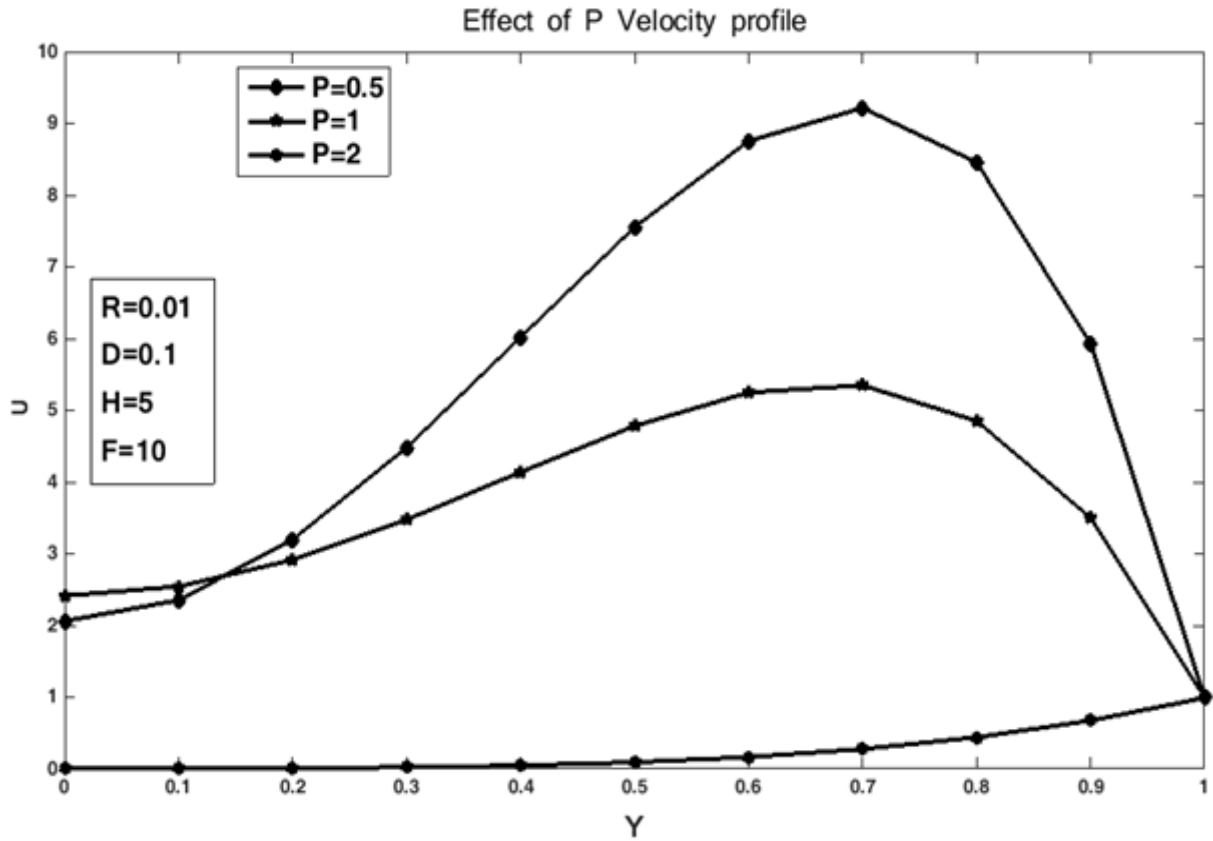


figure 10

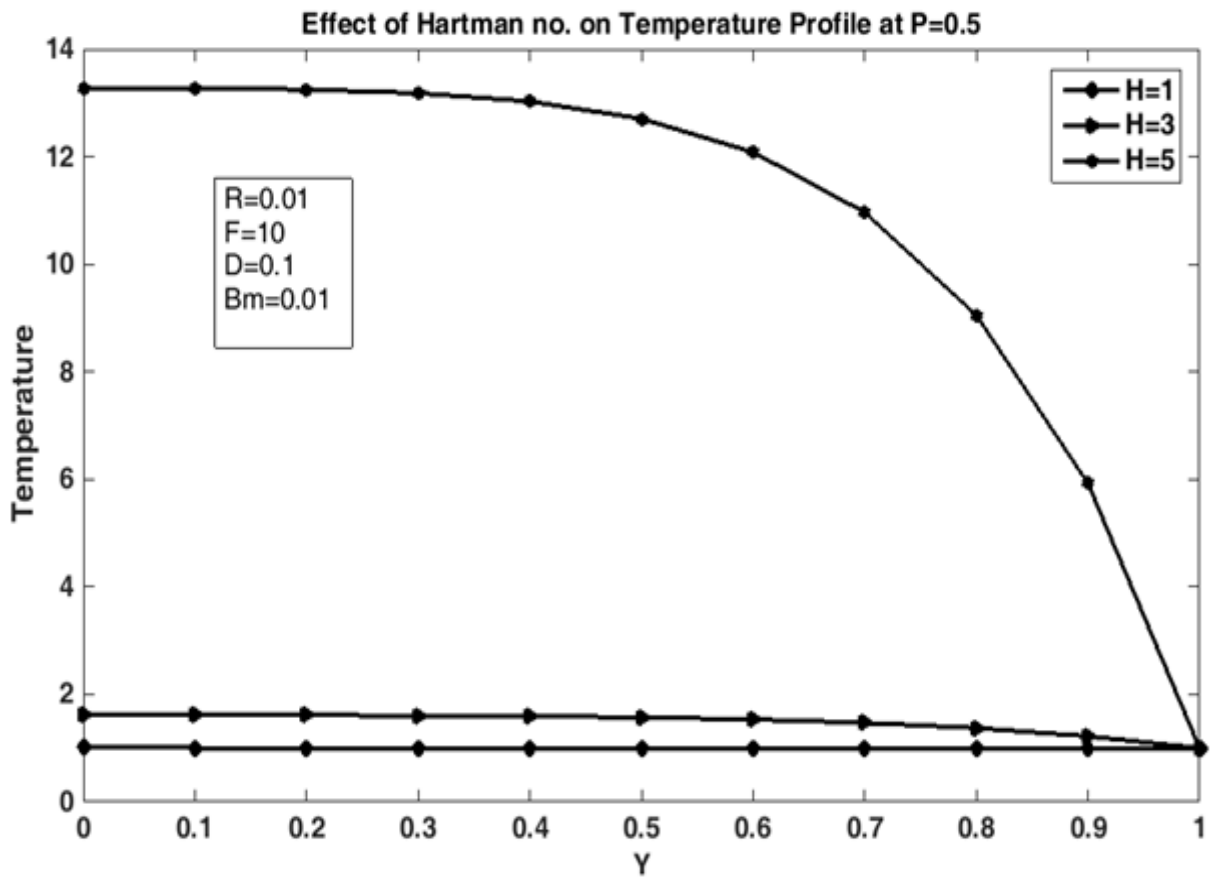
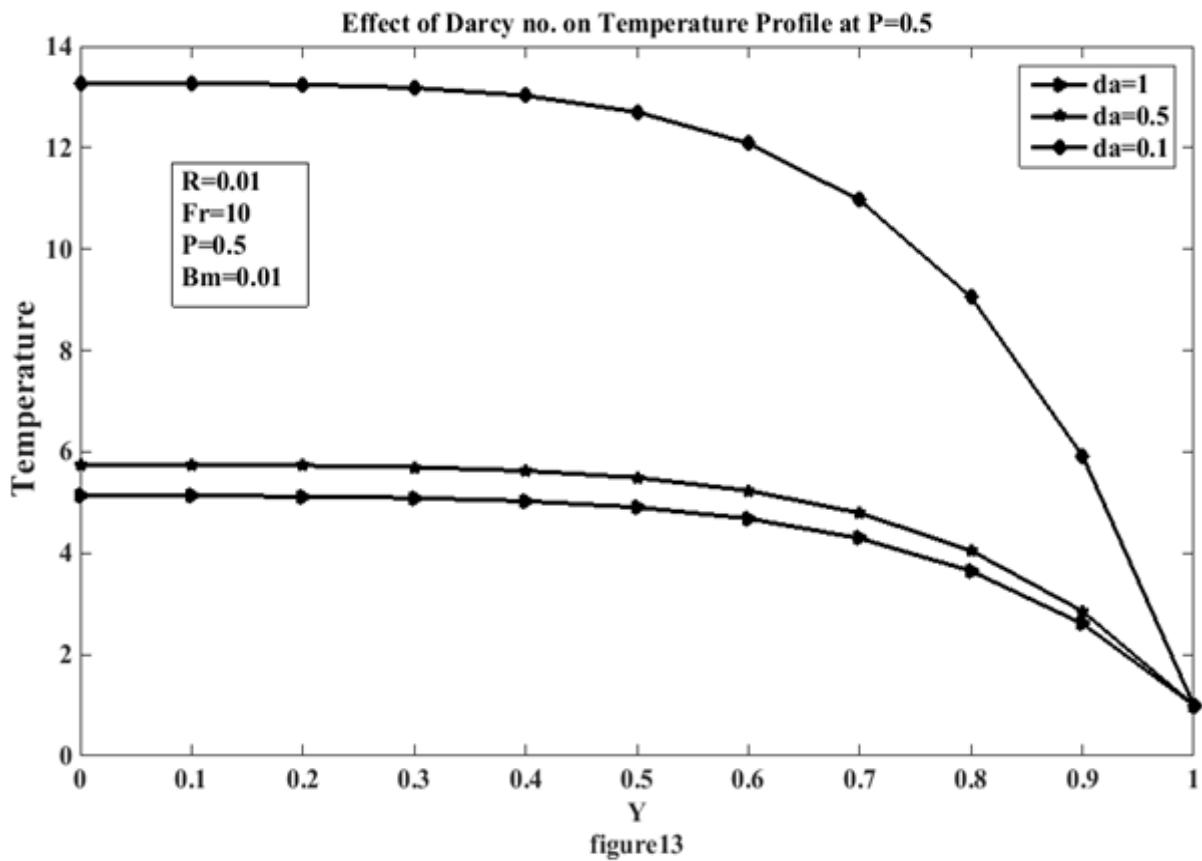
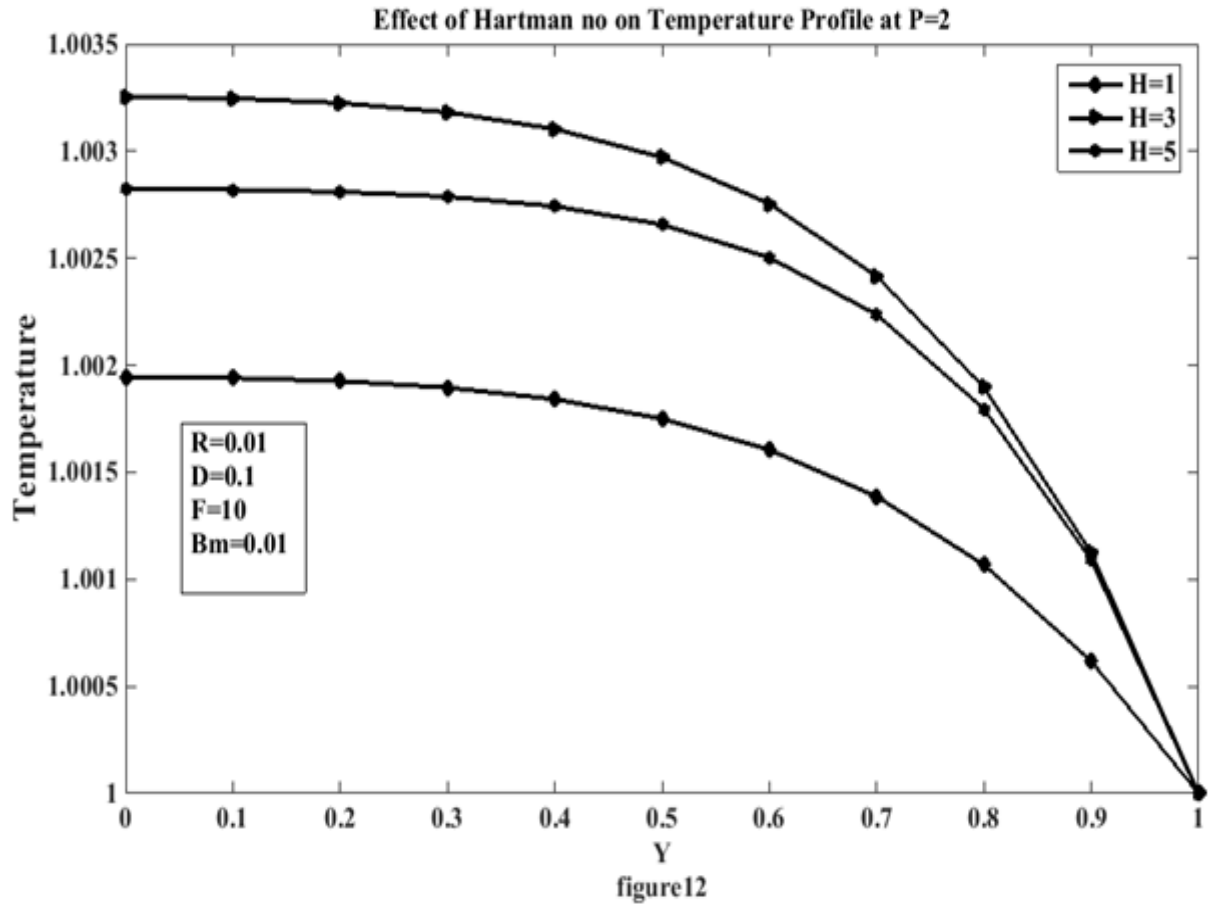
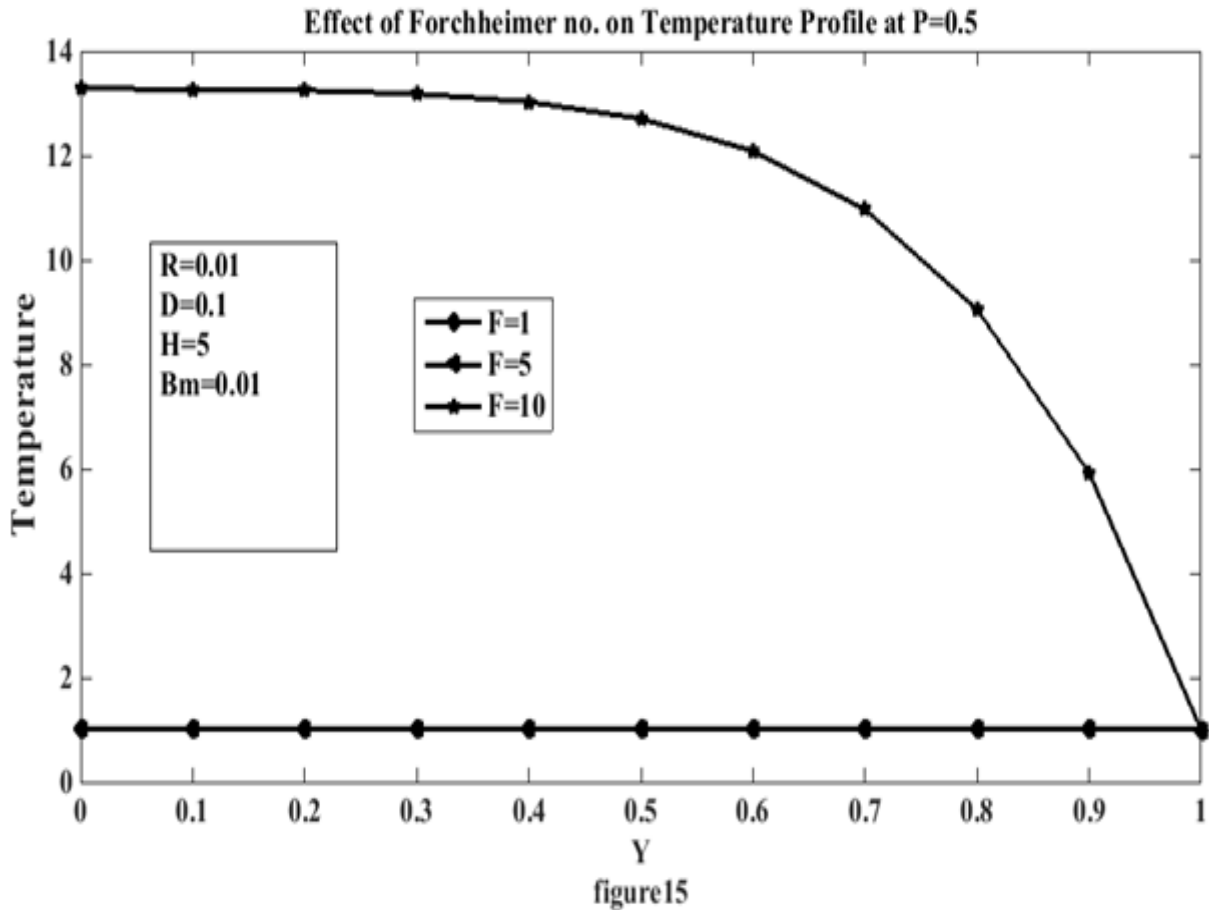
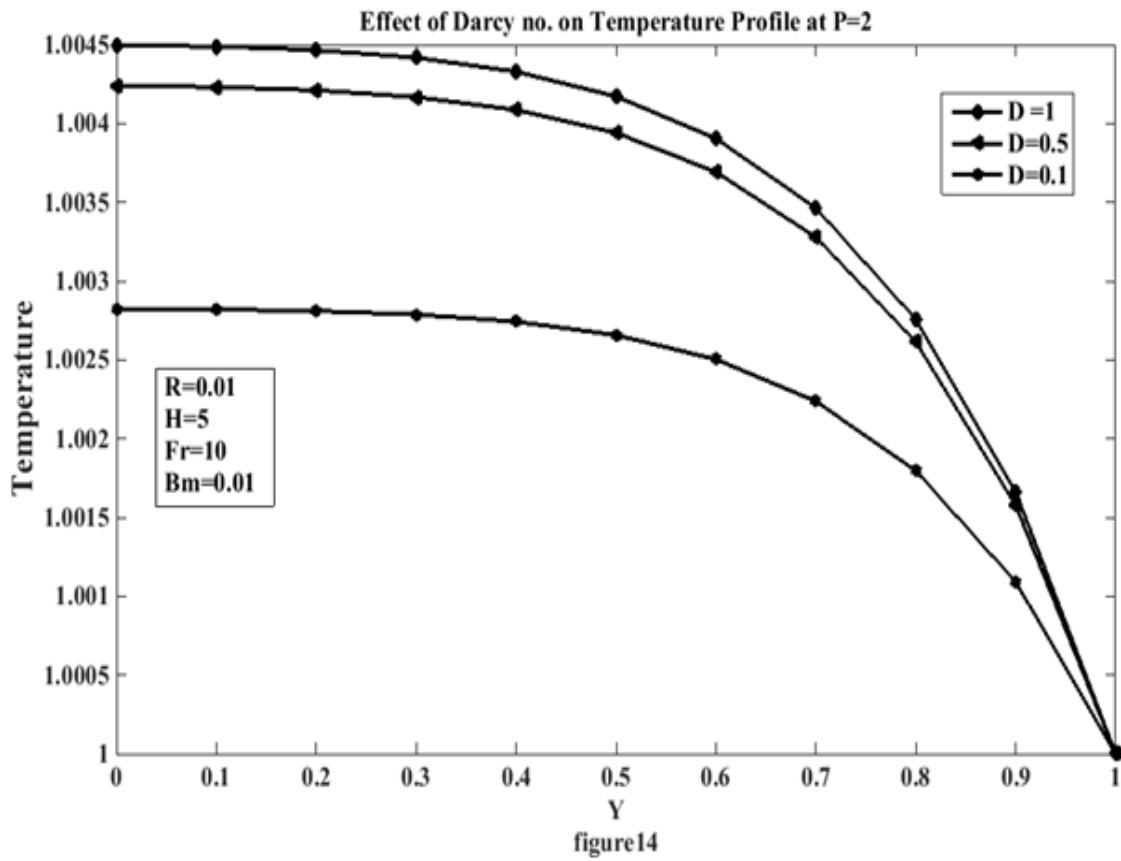


figure11





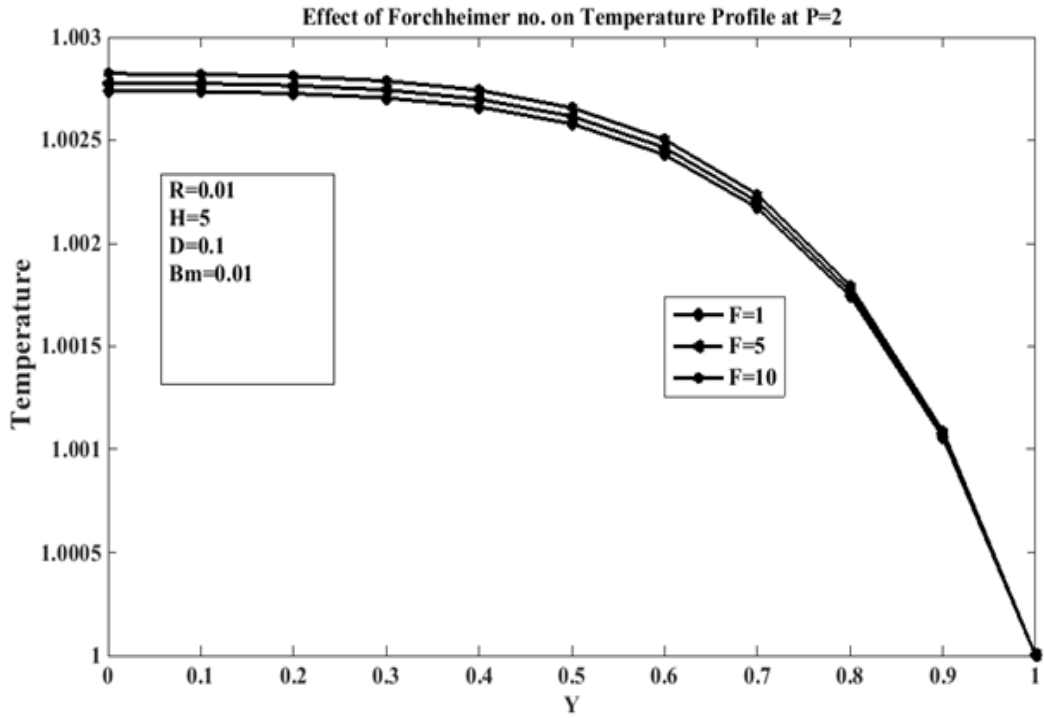


Figure 16

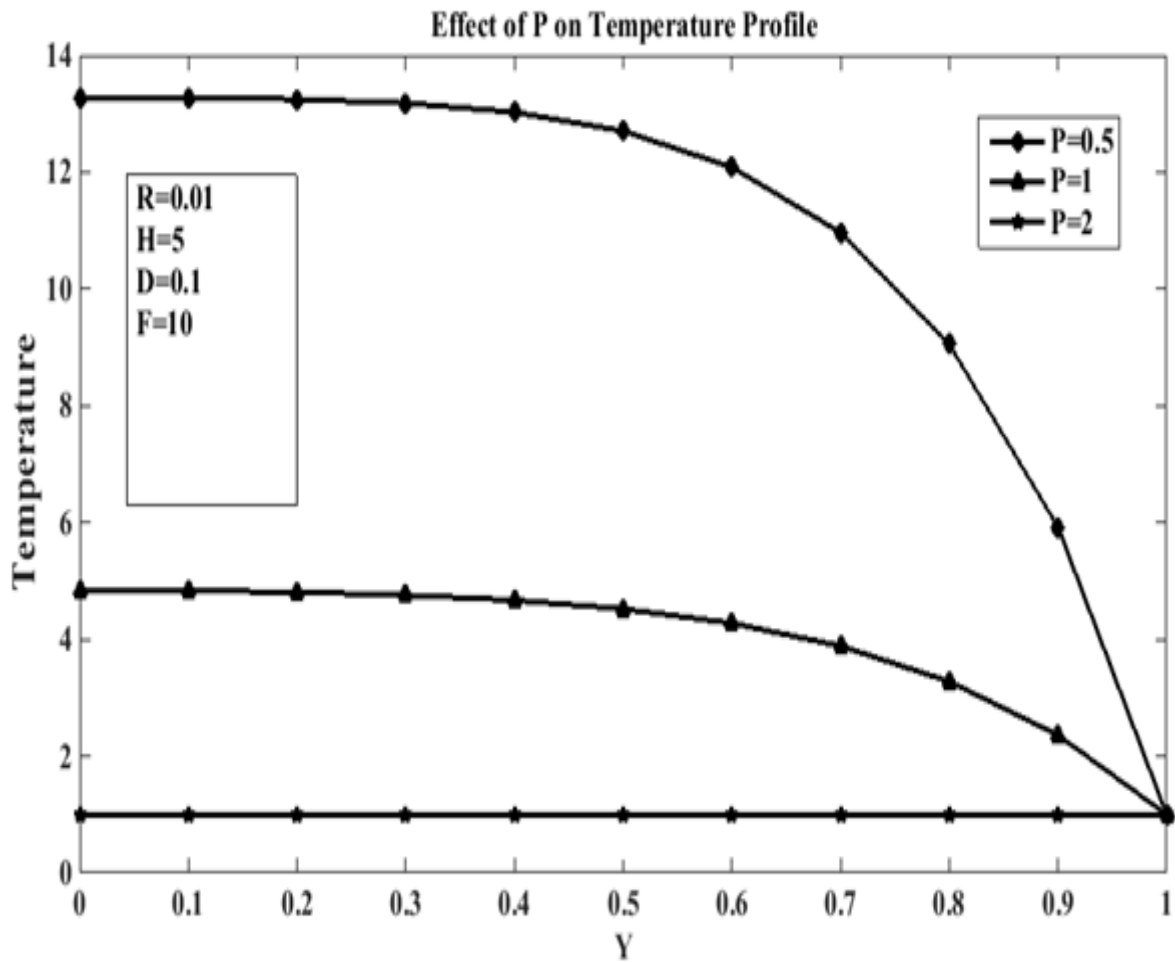


figure17

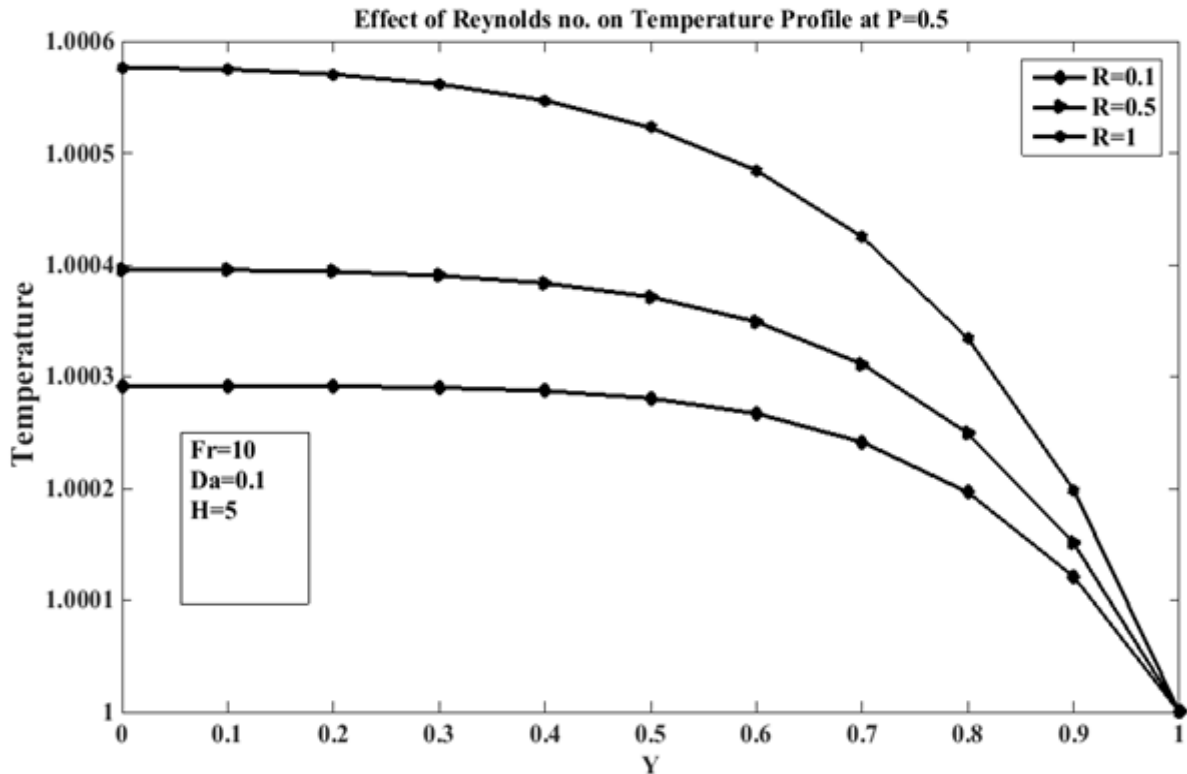


figure18

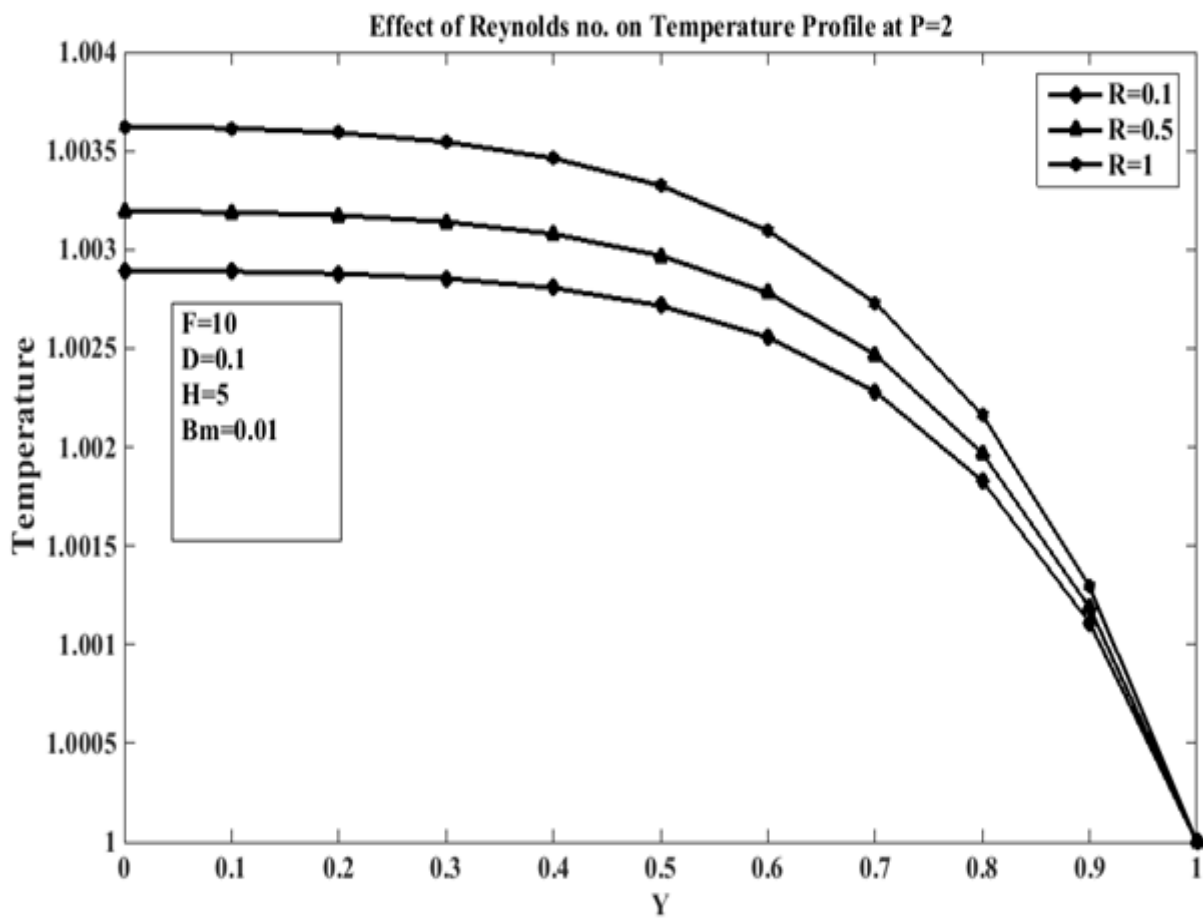
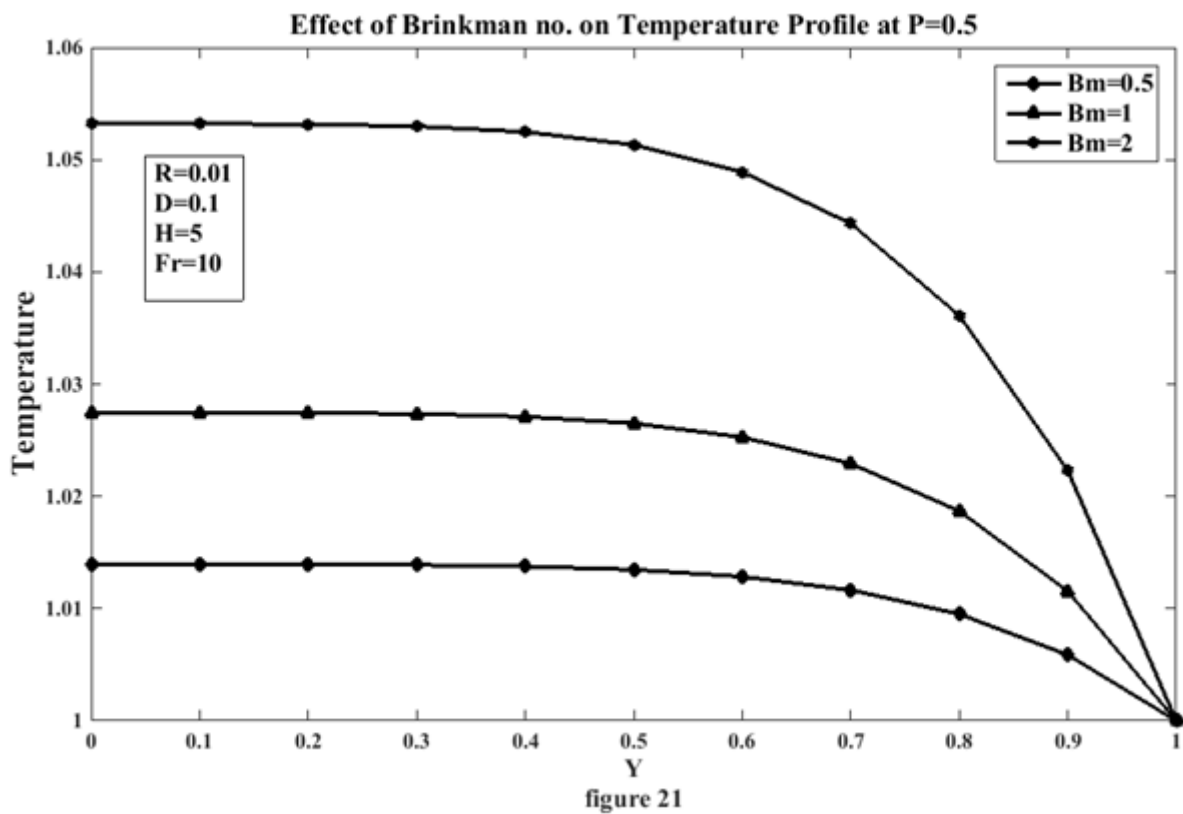
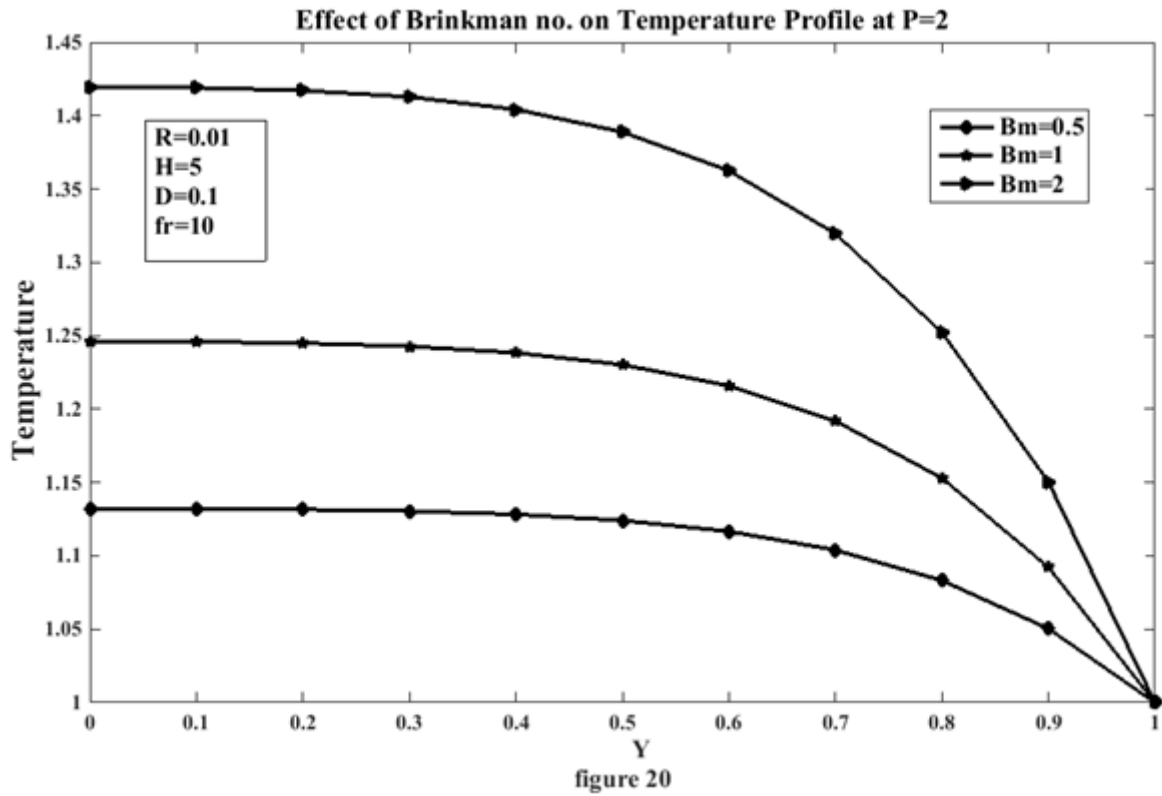


figure19



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