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Exploring Plate Behavior: Navier's Method Analysis and Validation Using Abaqus

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ABSTRACT

This study explores the application of Navier's method, a classical plate theory, in the analysis of plate structures within structural engineering. Navier's method provides a theoretical framework for understanding plate behavior by representing the displacement field as a double trigonometric series. The research aims to investigate the efficacy of Navier's method in predicting plate behavior and to validate its results through numerical simulations using Abaqus, a finite element analysis (FEA) software.

Keywords: Navier's method, plate analysis, Abaqus, displacement field

INTRODUCTION

Plates, as structural elements characterized by their thinness relative to other dimensions, play a pivotal role across diverse engineering disciplines, from aerospace to civil engineering. The intricate behavior of plates under varying loading conditions necessitates meticulous analysis to ensure structural integrity and optimize performance.

In industries such as aerospace and automotive manufacturing, plates serve as integral components of aircraft wings, vehicle chassis, and structural panels. Similarly, in civil engineering, plates form the foundation of bridges, buildings, and other vital infrastructure. Consequently, precise analysis of plate structures is imperative for guaranteeing safety, reliability, and efficiency in these critical applications.

This project aims to explore plate analysis using Navier's method, which provides valuable insights into the underlying principles governing plate behavior. By combining classical theory with modern computational methods, we seek to bridge the gap between theoretical understanding and practical application. Through a comprehensive investigation of plate behavior using analytical and numerical approaches, our study aims to contribute to the advancement of structural engineering.

STUDY AREA

The study area focuses on the application of Navier's method, a classical plate theory, in the analysis of plate structures in structural engineering. Navier's method provides a theoretical framework for understanding the behavior of plates under various loading conditions by representing the displacement field as a double trigonometric series. The research aims to investigate the effectiveness of Navier's method in predicting the behavior of plate structures and to validate its results through numerical simulations using Abaqus, a finite element analysis (FEA) software.

BACKGROUND

Overview of Plate Analysis Theories in Structural Engineering:

1. Kirchhoff's Plate Theory:

- Scientist: Gustav Kirchhoff
- Year: 1850

- Primarily applicable to thin plates undergoing bending but neglecting the effects of transverse shear deformation

2. Classical Plate Theory (CPT):

- Scientist: William Thomson (Lord Kelvin)
- Year: 1859
- Commonly used for preliminary analysis of thin plates subjected to uniform loading conditions.

3. First-order Shear Deformation Theory (FSDT):

- Scientist: Ray W. Clough
- Year: 1959
- Applicable to moderately thick plates where transverse shear effects are significant and provides more accurate predictions compared to CPT

4. Higher-order Shear Deformation Theory (HSDT):

- Scientist: R. S. Barsoum
- Year: 1976
- Suitable for plates with significant thickness or subjected to complex loading conditions

5. Mindlin-Reissner Plate Theory (MRPT):

- Scientists: Richard Mindlin, Eric Reissner
- Year: 1940
- Suitable for plates with moderate thickness where both transverse shear deformation and rotational inertia effects are important.

6. Navier's Method:

- Scientist: Claude-Louis Navier
- Year: Early 19th century
- Primarily used for simply supported plates subjected to various loading conditions.

Each of these plate theories has played a significant role in advancing our understanding of plate behavior and has practical applications in structural engineering analysis and design.

METHODOLOGY

Theory of bending in Thin Plate

Theory of bending of thin plate was developed by Love in 1888 using the assumptions proposed by Kirchoff and hence, it is popularly known as Kirchoff Love plate theory or more commonly as Kirchoff's plate theory.

Assumptions in theory of bending of thin plates

- The material is homogenous and isotropic
- The deflection is small
- The normal to the middle surface of the plate before deformation remains normal to the plane after deformation. This shows that strain in z-direction is zero.
- The in plane forces are neglected in bending of thin plates.
- The stresses $\sigma z = \tau xz = \tau zx = \tau yz = 0$

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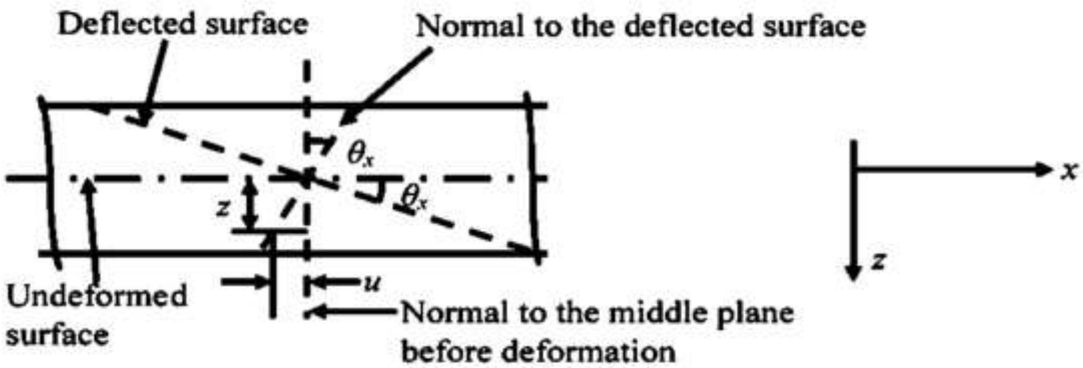

W (x, y) is vertical deflection

$$\emptyset x = slope\ in\ x - direction = \frac{\partial w}{\partial x} \qquad \emptyset y = slope\ in\ y - direction = \frac{\partial w}{\partial y}$$

$$u = -\mathrm{z}\,.\,\emptyset x = -z\ \frac{\partial w}{\partial x}$$

$$\therefore\ \varepsilon x = \frac{\partial u}{\partial x} = -\frac{z\partial^2 w}{\partial x^2}$$

Similarly,

$$\varepsilon y = \frac{\partial v}{\partial y} = -\frac{z\partial^2 v}{\partial y^2} \quad ,$$

$$\gamma xy = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = -2z\ \frac{\partial^2 w}{\partial x\,\partial y}$$

Expression for stress resultant:

Mainly moment is the factor by which plate resist the load i.e., main consideration is moment Therefore, Expression of moments in terms of vertical displacement:

We know, form stress-strain relationship,

$$\sigma x = \frac{E}{1-\gamma^2}\,(\in x + \gamma \in y)$$

$$\sigma y = \frac{E}{1-\gamma^2}\,(\in y + \gamma \in x)$$

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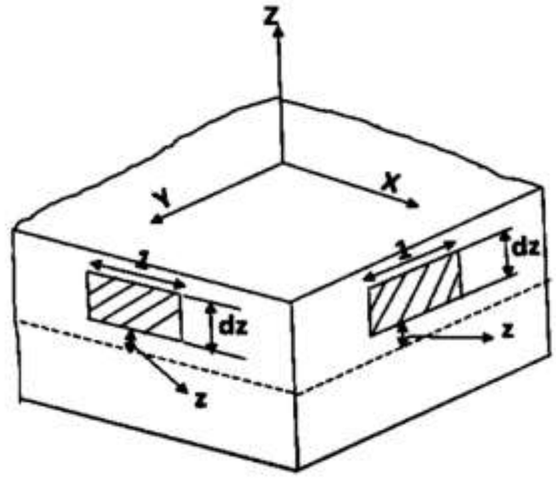

$$Mx= \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma x.\ z\ (dx.1)$$

$$Mx = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{-E}{1-\gamma^2}\ (\in x + \gamma \epsilon y)\ z dx$$

$$= \frac{-E}{1-\gamma^2} \int_{-\frac{h}{2}}^{\frac{h}{2}} z^2 dz\ (\frac{\partial^2 w}{\partial x^2} + \gamma \frac{\partial^2 w}{\partial y^2})$$

$$= \frac{-Eh^3}{12(1-\gamma^2)} \{\frac{\partial^2 w}{\partial x^2} + \gamma \frac{\partial^2 w}{\partial y^2}\}$$

Similarly,

$$My= \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma y.\ z\ (dx.1)$$

$$= \frac{-E}{1-\gamma^2} \int_{-\frac{h}{2}}^{\frac{h}{2}} (\in y + \gamma \epsilon x) z dx$$

$$= \frac{-E}{1-\gamma^2} \int_{-\frac{h}{2}}^{\frac{h}{2}} z^2 dz\ (\frac{\partial^2 w}{\partial y^2} + \gamma \frac{\partial^2 w}{\partial x^2})$$

$$= \frac{-Eh^3}{12(1-\gamma^2)} \{\frac{\partial^2 w}{\partial y^2} + \gamma \frac{\partial^2 w}{\partial x^2}\}$$

So, now the twisting moment,

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$$Mxy= \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau\, xy.\, z\, (dz.1)$$

$$=\frac{-E}{1-\gamma}\int_{-\frac{h}{2}}^{\frac{h}{2}} z^2\ \frac{\partial^2 w}{\partial xy^2}\ dz$$

$$=\frac{-Eh^3}{12(1-\gamma^2)}(1-\gamma)\frac{\partial^2 w}{\partial x.y}$$

Denoting,

$$\frac{-Eh^3}{12(1-\gamma^2)}= \text{D= Flexural rigidity of plate}$$

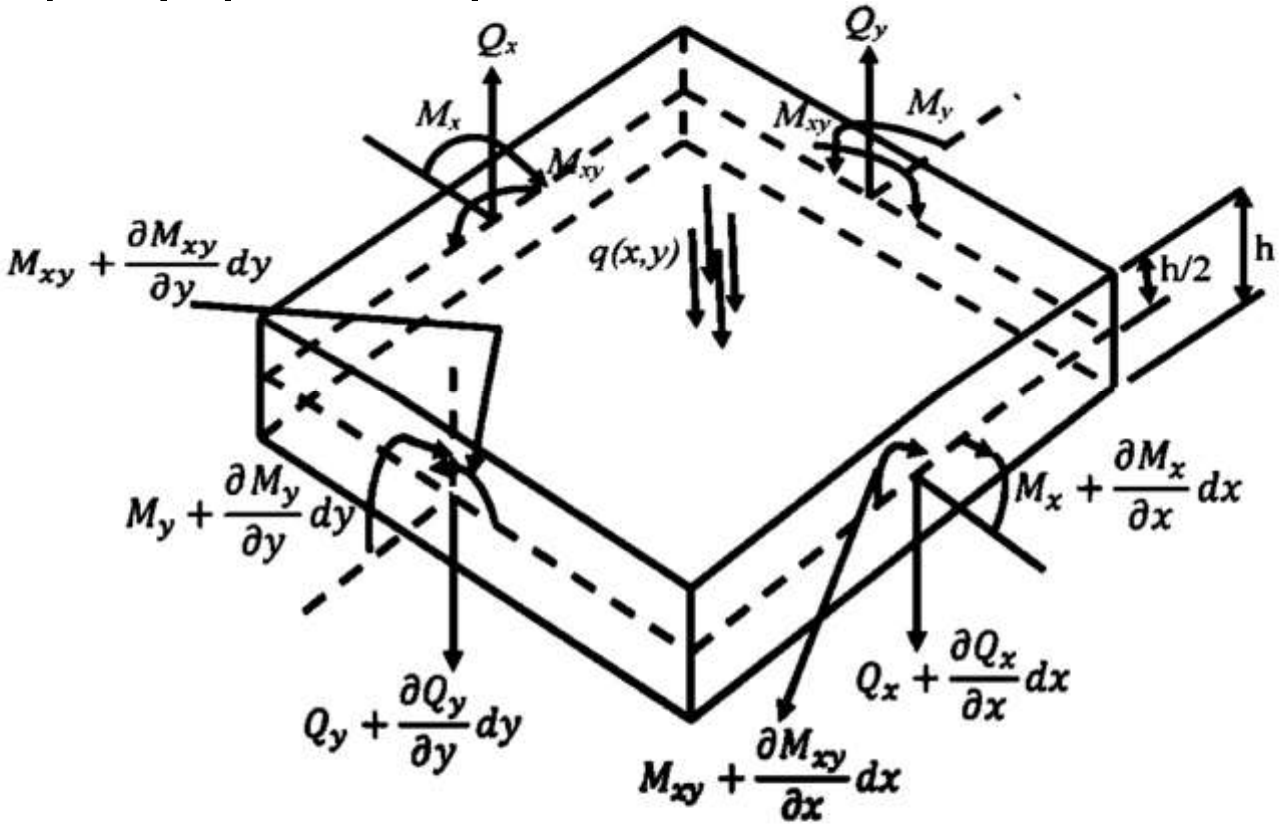
$$Mx= -D\ \{\frac{\partial^2 w}{\partial x^2}+\gamma\frac{\partial^2 w}{\partial y^2}\}$$

$$My= -D\ \{\frac{\partial^2 w}{\partial y^2}+\gamma\frac{\partial^2 w}{\partial x^2}\}$$

$$Mxy= -D\ (1-\gamma)\frac{\partial^2 w}{\partial x.\partial y}$$

Equilibrium equations for thin plate:

- The increment is found by Taylor series expression, taking only first term, neglecting the higher order term
- The q (x, y)' has continuous distribution.
- All quantities plate per unit width of the plate



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$$\sum \boldsymbol{Mx} = \boldsymbol{D}$$

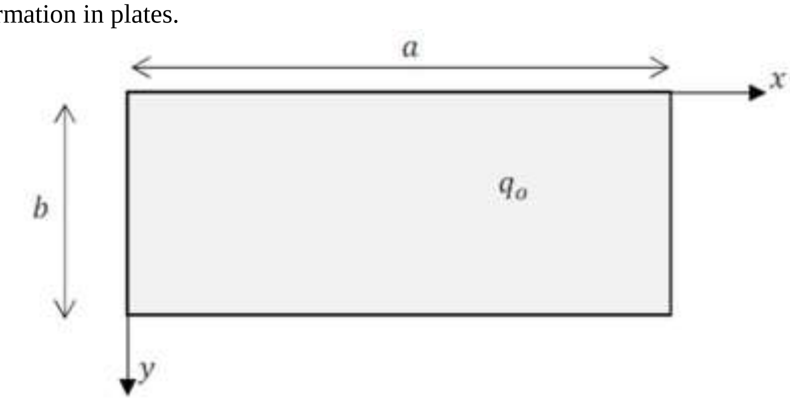
$$\frac{\partial Mx}{\partial x} + \frac{\partial Mxy}{\partial y} = Qx$$

$$\sum \boldsymbol{My} = \boldsymbol{D}$$

$$\frac{\partial My}{\partial y} + \frac{\partial Myz}{\partial x} = Qxy$$

Navier solution for the simply supported rectangular plate:

Navier's method is a classical technique used in structural engineering to analyze the behavior of plates under various loads. It's particularly useful for analyzing the stress distribution in simply supported plates. The method involves representing the displacement field of the plate as a double trigonometric series and then applying the equations of equilibrium to derive the stress resultants. Navier's method extends Kirchhoff's theory by considering the effects of transverse shear deformation in addition to membrane and bending deformations. It achieves this by representing the plate's displacement field using a double trigonometric series, allowing for a more accurate representation of plate deflections. By incorporating transverse shear strains, Navier's method provides a more refined analysis of stress distribution and deformation in plates.



The Navier solution for the rectangular plate simply supported on all sides and under a uniformly distributed load is presented in Chapter 5 of Timoshenko's text. [2]

$$w = \frac{16q_o}{\pi^6 D} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}}{mn\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2} \qquad m = 1, 3, 5, \ldots \text{ and } n = 1, 3, 5, \ldots$$

This solution for the displacement field satisfies the equilibrium conditions,

$$\nabla^4 w = \frac{q(x, y)}{D}$$

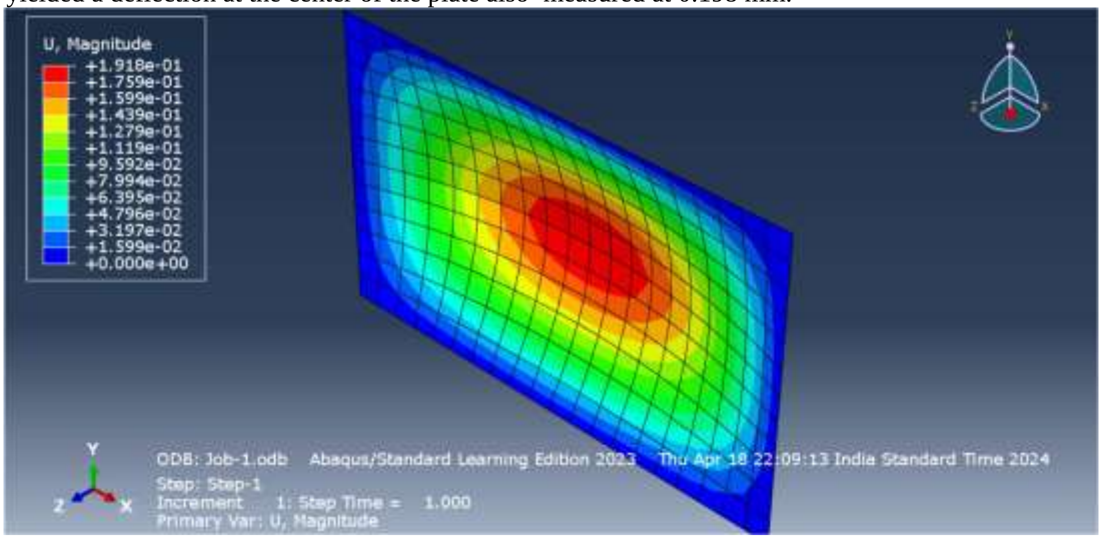
The maximum deflection of the plate is at its center and is found by substituting x = a/2, y = b/2 in formula.

a = 5000mm b = 3000mm
E = 2 x 10^5 N/mm^2 h = 100mm
ν = 0.3 q = 0.005 N/mm^2

By putting the following parameters into the equation, maximum deflection at centre was found to be 0.198mm.

Analysis of plate using FEM software package:

To validate our analytical findings and ensure accuracy, we also performed numerical simulations using Abaqus CAE, a widely-used FEA software known for its robustness in structural analysis. We inputted the same loading conditions and material properties into the software and conducted simulations to predict the plate's behavior. Remarkably, the numerical simulation yielded a deflection at the center of the plate also measured at 0.198 mm.



Deflection at center by classical method	Deflection at center by Abaqus CAE
0.198mm	0.198mm

CONCLUSION

Despite modern computational advancements like finite element analysis (FEA), classical theories such as Navier's method remain relevant in understanding plate behavior. Developed in the 19th century, Navier's method provides fundamental insights into how plates respond to different loads, forming a basis for modern engineering practices.

In conclusion, the exploration of plate analysis using Navier's method offers significant contributions to structural engineering knowledge and practice. By delving into the fundamental principles governing plate behavior, engineers gain valuable insights that enhance problem-solving capabilities and foster innovation in addressing contemporary challenges.

REFERENCES

[1]. Bhavikatti, S.S. Theory of Plates and Shells. New Age International Publishers.

[2]. Timoshenko, S.P., & Woinowsky-Krieger, S. Theory of Plates and Shells (Xth ed.). Tata McGraw Hill.

[3]. Maunder, R. "Navier Soluon for the Simply Supported Rectangular Plate under UDL." Copyright © Ramsay Maunder Associates Limited (2004 – 2018). All Rights Reserved.