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Transportation Problems Solve by Linear Programming and Modified Vogel's Approximation Method

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ABSTRACT

In this paper, we define a VM(Vogel's Modified)approximation method to find out the original possible solution to the transportation problem in order to reduce the cost. The most attractive feature of this method is that it requires very simple arithmetic and logical calculations as compared to the existing method and numerical examples are also given for the same. And this problem can also be solved by linear programming problem. And the solution has been discovered by Lingo Software. And it is found that the results of both the methods are same. This reduces the transportation cost.

Keyword: Transportation, Minimization costs, Sources, supply, Demand, MVAM, LPP, LINGO.

INTRODUCTION

Operation Research is basically a branch of mathematics which deals with quick and effective decisions for any organization especially in the application of mathematics to provide a scientific basis for the management to take timely and effective decisions for their problems Provides a solid base to take on. In India, operations research came into existence in 1949 with the opening of an OR unit at the Regional Research Laboratory at Hyderabad.

The scope of operations research is wide and has been successfully applied in the following sectors such as industry, defense, planning, agriculture and public utilities. Linear programming is an approach in mathematics that is widely used today. The programming concept is used to allocate resources appropriately. 'Programming' means to make decisions systematically. The task requirement of using a linear programming model is to maximize profit while minimizing cost.

Linear programming is a relatively young mathematical discipline, founded in 1947 by G. B. Since the invention of the simplex method by Dantzig. Historically, developments in linear programming have been driven by its applications in economics and management. Dantzig initially signed with the U.S. Served in. The Air Force developed simpler methods for solving planning problems, and planning and scheduling problems still dominate applications of linear programming. One reason is that linear programming is a relatively new field. In the term linear programming, refers to mathematical programming.

Manny Researcher worked in the field of Operations Research to increase profits. According to Anetting et al (2013), the Usmer Water Company reported applying linear programming techniques to determine optimal outputs.
The authors Akpan et al (2016) worked to use the concept of the simplex algorithm; An aspect of linear programming for allocating raw materials to competing variables (large loaf, giant loaf, and small loaf) in a bakery with the aim of maximizing profit.

In this context, it refers to a planning process that allocates resources—labour, materials, machines, capital—in the best possible (optimal) way so that costs are minimized or benefits are maximized. In LP, these resources are known as decision variables. The criteria for selecting the best values of the decision variables are known as the objective function.

The transport problem is a special class of linear programming problem. Where the objective is to reduce the cost of distributing the product from multiple sources to multiple destinations while satisfying both the supply limit and demand requirement.

Definitions

Transportation Model is balanced if Supply $\sum_{i=1}^{m} ai$ = Demand $\sum_{j=1}^{n} bj$

Otherwise unbalanced if Supply $\sum_{i=1}^{m} ai \neq$ Demand $\sum_{j=1}^{n} bj$

A feasible solution to a transportation problem is a set of non-negative allocations xij that satisfies row and column restrictions.

A viable solution to the transport problem is said to be an original viable solution if it does not contain more than m+n-1 non-negative allocations. where m is the number of rows and n is the number of columns of the transport problem. A feasible solution (not necessarily basic) is said to be optimal if it minimizes the total transportation cost

Mathematical Formulation of Transportation Problem

Let xij ≥ 0 be the quantity shipped to the destination j from the origin i

The mathematical formulation of the problem is as follows.

Minimize $Z = \sum_{i-1}^{m} . \sum_{j=1}^{n} c_{ij} x_{ij}$

Subject to $\sum_{j=1}^{n} x_{ij} = a_i$

$\sum_{i=1}^{m} x_{ij} = b_j$

$X_{ij} \geq 0$ for all i and j

Where Z= Total transportation cost to be minimized

C_{ij}= Per unit cost in transporting goods to the j-th destination from i-th origin.

X_{ij}= the quantity transported to the j-th destination from the i-th origin.

a_i = the amount available at the i-th origin

b_j= the demand of the j-th destination

Tabular form of Transportation Problem

Destinations / Sources	D_1	D_2		D_n	Supply
S_1	C_{11} X_{11}	C_{12} X_{12}	……	C_{1n} X_{1n}	a_1
S_2	C_{21} X_{21}	C_{22} X_{22}	……	C_{2n} X_{2n}	a_2
: :	…..	…..	……	……	……
S_m	C_{m1} X_{m1}	C_{m2} X_{m2}	……	C_{mn} X_{mn}	a_m
Demand	b_1	b_2	…….	b_n	

METHODOLOGY

The following method are always used to find the solution for the transportation problem and we available in every text book of Operations Research

Initial Basic Feasible Solution Methods

1. North West Corner Method
2. Column Minimum Method
3. Row Minimum Method
4. Least Cost Method
5. Vogel's Approximation Method

Optimal Method

1. Modified Distribution (MODI) Method
2. Stepping Stone Method

In this observe, primary idea is to get higher preliminary solution for the transportation hassle. Consequently, study focused on Vogel's method. Many researchers offer many methods to clear up transportation hassle. "Modified Vogel's Approximation Method for Solving Transportation Problem" [2] via Abdul Sattar Soomro et.al., "An Improved Vogel's Approximation Method for the transportation Problem" [5] via Serder Korukogu and Serkan Balliy., "A Modified Vogel's Approximation Method for Obtaining a Good Primal Solution of Transportation Problem" [4] via M.Wali Ullah et.al., "An Improved Algorithm to Obtain Initial Basic Feasible Solution For the Transportation Problem" [1] via A. Seethalakshmy, and N. Srinivasan. ,"Modified Extremum Difference Method (MEDM) For Solving Cost Minimizing Transportation Problem",[3] via Md. Amirul Islam et.al. Developed the method is very helpful as having less computation and is very closer to the VAM solution.

Here we solve this transportation Problem by Linear Programming Method. And also Introduce new Method which is called a Modified Vogel's Approximation Method. And we find the same solution by both the methods.

Algorithm of Modified Vogel's Approximation Method (MVAM):-
Step 1:- Observe whether the transportation problem is balanced or not. If it is balanced then go to next step.
Step 2:- Find the smallest cost from each row and subtract the smallest cost from each element of the row.
Step 3 Find the smallest cost from each column and subtract the smallest cost from each element of the column.
Step 4:- The new matrix will have at least 1 zero. Allocate least supply/demand to the zero element of the row or column. If there may be more zero then we can take least supply or demand which ever less. Delete corresponding columns or rows where supply or demand is met then visit next step.
Step 5 examine the least of deliver or demand whichever is least then allocate the least (supply or call for) at the place of least cost of associated row or column.
Step 6:- Repeating the step 4 to step 5 until satisfaction of all the supply and demand is met.
Step 7:-Now total minimum cost is calculated as sum of the product of cost and corresponding allocate value of supply/demand.

Numerical Example
The XYZ Paper mill Company's CEO asks to see next month's log hauling schedule to his three Paper mills. He wants to make sure he keeps a steady, adequate flow of logs to his Paper mills to capitalize on the good lumber market. Secondary, but still important to him, is to minimize the cost of transportation. The harvesting group plans to move to three new logging sites. The distance from each site to each Paper mills is in Table 1. The average haul cost is 10 rupees per mile for both loaded and empty trucks. The logging supervisor estimated the number of truckloads of logs coming off each harvest site daily. The number of truckloads varies because terrain and cutting patterns are unique for each site. Finally, the Paper mills managers have estimated the Mini truckloads of logs their mills need each day. All these estimates are in

Table 1.

Logging Site	Distance to mill (KM)			Maximum Mini truckloads/day per logging site
	Mill P	Mill Q	Mill R	
1	8	15	50	20
2	10	17	20	30
3	30	26	15	45
Mill demand (mini truckloads/day)	30	35	30	

The next step is to determine costs to haul from each site to each mill.

Table 2.

Logging Site			
	Mil P	Mil Q	Mil R

1	160	300	1000
2	200	340	400
3	600	520	300

The cost of roundtrip is rupees 10 per km.
There(8 km × 10) + (8 km × 10) = Rs160
(15 km × 10) + (15 km × 10) = Rs300
(50 km × 10) + (50 km × 10) = Rs1000
(10 km × 10) + (10 km × 10) = Rs200
(17 km × 10) + (17 km × 10) = Rs340
(20 km × 10) + (20 km × 10) = Rs400
(30 km × 10) + (30km × 10) = Rs600
(26km × 10) + (26 km × 10) = Rs520
(15 km × 10) + (15 km × 10) = Rs300

We can set the LP problem up as a cost minimization; that is, we want to minimize hauling costs and meet each of the Paper mills daily demand while not exceeding the maximum number of Mini truck loads from each site. We can formulate the problem as:
Let Xij = costs from Site i to Mill j
i = 1, 2, 3 (logging sites) j = 1, 2, 3 (Paper mills)
Objective function:
MIN 32X11 + 40X21 + 120X31 + 60X12 + 68X22 +104X32 + 200X13 + 80X23 + 60X33
Subject to:
Truckloads to Mill A X11 + X21 + X31 ≥ 30
Truckloads to Mill B X12 + X22 + X32 ≥ 35
Truckloads to Mill C X13 + X23 + X33 ≥ 30
Truckloads from Site 1 X11 + X12 + X13 ≤ 20
Truckloads from Site 2 X21 + X22 + X23 ≤30
Truckloads from Site 3 X31 + X32 + X33 ≤ 45
X11, X21, X31, X12, X22, X32, X13, X23, X33 > 0

For the computer solution: in the edit box of LINGO, type.
The solution of this transportation problem by Linear Programming to use LINGO Software

Variable	Value	Reduced Cost
X_{11}	0.000000	0.000000
X_{12}	20.00000	0.000000
X_{13}	0.000000	920.0000
X_{21}	30.00000	0.000000
X_{22}	0.000000	0.000000
X_{23}	0.000000	280.0000
X_{31}	0.000000	220.0000
X_{32}	15.00000	0.000000
X_{33}	30.00000	0.000000

Row	Slack or Surplus	Dual Price
1	28800.00	-1.000000
2	0.000000	220.0000
3	0.000000	180.0000
4	0.000000	0.000000
5	0.000000	-380.0000
6	0.000000	-520.0000
7	0.000000	-300.0000
8	0.000000	0.000000
9	20.00000	0.000000
10	0.000000	0.000000
11	30.00000	0.000000
12	0.000000	0.000000
13	0.000000	0.000000

14	0.000000	0.000000
15	15.00000	0.000000
16	30.00000	0.000000

Table 3.

Iteration of the LINGO output

Logging Site	Mill	Mini truck loads Per day	Cost per load	Total cost
1	P	0	160	0
1	Q	20	300	6000
1	R	0	1000	0
2	P	30	200	6000
2	Q	0	340	0
2	R	0	400	0
3	P	0	600	0
3	Q	15	520	7800
3	R	30	300	9000
Total				Rs. 28800

The Rs. 28800 represents the minimum daily cost of Peppermill Company from the three logging sites to the three Peppermills.

Transportation Problem Solution by Modified Vogel's Approximation method

Table 4.

LOGGING SITE				**Supply**
	Mill P	**Mill Q**	**Mill R**	
1	160	300	1000	20
2	200	340	400	30
3	600	520	300	45
Demand	30	35	30	

Solution : $\sum a_i = \sum b_j = 95$

Table 5.

Logging Site				**Supply**
	Mill P	Mill Q	Mill R	
1	20 160	300	1000	20
2	10 200	20 340	400	30
3	600	15 520	30 300	45

Demand	30	35	30	

The transportation cost is
Z = 20*160+10*200+20*340+15*520+30*300 = 28,800/-

RESULT & CONCLUSION

In this paper we use Lingo software to reduce costs by constructing a transport problem and a linear programming problem. And this problem is solved by the modified Vogel's approximation method. We see that both the methods give comparable results. The result is that the least amount is Rs. 28800. We can use these methods for other types of transportation problem. Both the methods are equally effective and are very simple, easy to understand and give exactly the same results.

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