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Trans-Sasakian Manifold with a Semi – Symmetric Metric Connection

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ABSTRACT

Oubina, J.A.[1] defined and initiated the study of Trans-Sasakian manifolds. Blair [2], Prasad and Ojha [3], Hasan Shahid [4] and some other authors have studied different properties of C-R-Sub –manifolds of Trans-Sasakian manifolds. Golab, S. [5] studied the properties of semi-symmetric and Quarter symmetric connections in Riemannian manifold. Yano, K. [6] has defined contact conformal connection and studied some of its properties in a Sasakian manifold. Mishra and Pandey [7] have studied the properties in Quarter symmetric metric F-connections in an almost Grayan manifold.
In this paper we have studied the properties of a Trans- Sasakian manifold equipped with a semi-symmetric metric connection.

Key words: Riemannian curvature tensor, Trans-Sasakian manifold, C-R-Sub –manifolds of Trans-Sasakian manifolds, Semi-symmetric and Quarter symmetric connections in Riemannian manifold, Almost Grayan manifold.

INTRODUCTION

Let M_n (n = 2m + 1) be an almost contact metric manifold endowed with a (1,1)-type structure tensor F, a contravariant vector field T, a -1 form A associated with T and a metric tensor ‘g’ satisfying :---

(1.1)(a) $F^2X = -X + A(X)T$
(1.1)(b) $FT = 0$
(1.1)(c) $A(FX) = 0$
(1.1)(d) $A(T) = 1$
and
(1.2)(a) $g(\bar{X},\bar{Y}) = g(X, Y) – A(X)A(Y)$
Where
(1.2)(b) $\bar{X} \overset{\text{def}}{=} FX$
And
(1.2)(c) $g(T, X) \overset{\text{def}}{=} A(X)$
For all C^∞- vector fields X,Y in $M_n$also, a fundamental 2-form ‘F in M_n is defined as
(1.3) $'F(X,Y) = g(\bar{X},Y) = -g(X,\bar{Y}) = -'F(Y,X)$
Then, we call the structure bundle {F,T,A,g}an almost contact-metric structure [1]
An almost contact metric structure is called normal [1], if
(1.4)(a) $(dA)(X,Y)T + N(X,Y) = 0$
Where
(1.4)(b) $(dA)(X,Y) = (D_XA)(Y) - (D_YA)(X)$, D is the Riemannian connection in M_n .
And
(1.5) $N(X, Y) = (D_{\bar{X}}\ F)(Y) - (D_{\bar{Y}}\ F)(X) - \overline{(D_X F)(Y)} + \overline{(D_Y F)(X)}$
Is Nijenhenus tensor in M_n .
An almost contact metric manifold M_n with structure bundle {F,T,A,g} is called a Trans-Sasakian manifold [3]&[1],if
(1.6) $(D_XF)(Y)=\alpha\{g(X,Y)T – A(Y)X\}+\beta\{'F(X,Y)T – A(Y)\bar{X}\}$
Where, β are non -zero constants.
Itcan be easily seen that a Trans-Sasakian manifold is normal. In view of (1.6) one can easily obtain in M_n, the relations
(1.7) $N(X, Y) = 2\alpha\, 'F(X,Y)T$
(1.8)$(dA)(X,Y) = -2\alpha\, 'F(X,Y)$
(1.9)$(D_XA)(Y) + (D_YA)(X) = 2\beta\{g(X,Y) – A(Y)A(X)\}$
(1.10) $(D_X\, 'F)(Y ,Z) + (D_Y\, 'F)(Z ,X) + (D_Z\, 'F)(X ,Y)$

= 2β[A(Z)‘F(X,Y) +A(X) ‘F(Y,Z) +A(Y) ‘F(Z,X)]
(1.11)(a)$(D_X A)(Y) = -\alpha$‘F(X,Y) + β{g(X,Y) –A(X)A(Y)}
(1.11)(b)$(D_X T) = -\alpha\bar{X}$ + β{X –A(X)T}

REMARK (1.1): In the above and in what follows, the letters X,Y,Z …..etc. an C^∞- vector fields in M_n .

ON A SEMI –SYMMETRIC METRIC CONNECTION IN TRANS-SASAKIAN MANIFOLD

We consider a semi-symmetric metric connection B given by [8]
(2.1) $B_X Y = D_X Y + A(X)Y – g(X,Y)T$
Whose torsion tensor is given by
(2.2) S(X,Y) = A(Y)X –A(X)Y
The curvature tensor with respect to B, say R(X,Y,Z) is given by
(2.3) $R(X,Y,Z)= B_X B_Y Z – B_Y B_X Z – B_{[X,Y]}Z$
Using (2.1) in it,we get
(2.4) $R(X,Y,Z)=K(X,Y,Z) + (D_X A)(Z)Y - (D_Y A)(Z)X$
$- g(Y,Z)D_X T + g(X,Z)D_Y T +A(Z)A(Y)X - A(Z)A(X)Y$
+g(X,Z)Y - g(Y,Z)X - A(X)g(Y,Z)T -A(Y)g(X,Z)T
Again, using (1.11)(b) in (2.4), we obtained
(2.5) R(X,Y,Z)=K(X,Y,Z) +α{‘F(Y,Z)X – ‘F(X,Z)Y +g(Y,Z)$\bar{X}$- g(X,Z)$\bar{Y}$}
+(2β+1){g(X,Z)Y -g(Y,Z)X}
-(β+1){A(Y)g(X,Z)T -A(X)g(Y,Z)T +A(X)A(Z)T-A(Y)A(Z)X}
Contracting (2.5) with respect to X, we get
(2.6)(a) R(Y,Z) = Ric(Y,Z) +α(n-2) ‘F(Y,Z) –{(2n-3)β
+(n-2)}g(Y,Z) +(β+1)(n-2)A(Y)A(Z)
Or
(2.6)(b) R(Y) = K(Y) +α(n-2)$\bar{Y}$ + {(2n-3)β + (n-2)}Y
+ (β+1)(n-2)A(Y)T
Contracting which with respect to Y, we get
(2.6)(c) $r = k -2\beta(n-1)^2 –(n-1)(n-2)$

Where R(Y,Z), r are Ricci tensor and scalar curvature with respect to B and Ricci and k are respectively the same with respect to Riemannian connection D.

Now, suppose the curvature tensor with respect to B vanishes, i.e. R(X,Y,Z) = 0 then from (2.6)(c), we see that the manifold M_n is of constant scalar curvature k and is given by

(2.7) $\beta = \frac{k}{2(n-1)^2} + \frac{(n-2)}{2(n-1)}$
Also the equation (2.6)(a), in view of the above fact and (2.7) becomes
(2.8) Ric(Y,Z) = -α(n-2) ‘F(Y,Z) + $\frac{k}{2(n-1)^2}$ [(2n-3)g(Y,Z)
-(n-2)A(Y)A(Z)] + $\frac{(n-2)}{2(n-1)}$ [(4n-5)g(Y,Z) –(3n-4)A(Y)A(Z)]
Barring Y in (2.8), we have
(2.9)(a)Ric($\bar{Y}$,Z) = α(n-2)g($\bar{Y}$,$\bar{Z}$) +$\frac{(2n-3)}{2(n-1)^2}$ k ‘F(Y,Z)+ $\frac{(n-2)(4n-5)}{2(n-1)}$ ‘F(Y,Z)
Further,barring Z in (2.8), we obtained
(2.9)(b)Ric(Y,$\bar{Z}$) = - α(n-2)g($\bar{Y}$,$\bar{Z}$) +$\frac{(2n-3)}{2(n-1)^2}$ k ‘F(Z,Y)+ $\frac{(n-2)(4n-5)}{2(n-1)}$ ‘F(Z,Y)
Adding (2.9)(a) and (2.9)(b), we get
(2.10) Ric($\bar{Y}$,Z) + Ric(Y,$\bar{Z}$) = 0
Thus, we have

THEOREM (2.1): Let $M_n$be a Trans-Sasakian manifold admitting a semi –symmetric metric connection B by (2.1) Let the curvature tensor with respect to B vanish, then M_n is of constant scalar curvature and

Ric($\bar{Y}$,Z) + Ric(Y,$\bar{Z}$) = 0
Holds good in M_n.
Now, from (2.9)(a) and (2.9)(b), we have
(2.11) K($\bar{Y}$) = $\overline{K(Y)}$ = α(n-2){Y-A(Y)T}+$\frac{(2n-3)}{2(n-1)^2}$ k $\bar{Y}$+ $\frac{(n-2)(4n-5)}{2(n-1)}\bar{Y}$
Contracting which with respect to Y, we have
α(n-1)(n-2) = 0

which gives $\alpha = 0$,for $n > 2$
then, we have

THEOREM (2.2): A Trans-Sasakian manifold M_n, $n \geq 3$, equipped with a semi-symmetric metric connection B given by (2.1) becomes a $(0,\beta)$ type Trans Sasakian manifold if the curvature tensor with respect to B vanishes.

CONCLUSION

If in a Trans-Sasakian manifold admitting a semi –symmetric metric connection B, and if curvature tensor with respect to B vanish, then M_n is of constant scalar curvature and $Ric(\bar{Y},Z) + Ric(Y,\bar{Z}) = 0$,Holds good in M_n. Again A Trans-Sasakian manifold M_n , $n \geq 3$, equipped with a semi-symmetric metric connection B becomes a $(0,\beta)$ type Trans Sasakian manifold if the curvature tensor with respect to B vanishes.

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