International Journal of Enhanced Research in Science, Technology & Engineering
ISSN: 2319-7463, Vol. 11 Issue 12, December-2022, Impact Factor: 7.957

Interacting Two Fluid Viscous Dark Energy Models in Brans-Dicke Theory of Gravitation

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ABSTRACT

In this paper, we study a class of solution of scalar tensor theory proposed by Brans-Dicke with barotropic fluid and bulk viscous fluid for the new class of Bianchi model. We consider two cases of an interacting and non-interacting two fluid and obtained general results. By using i) the special law of variation for Hubble's parameter which is proposed by Berman ii) trace free energy momentum tensor of the two fluid, the solution is obtained. The geometrical and physical aspects of the models are also discussed.

Keywords:- Brans-Dicke theory, Dark energy, Two-fluid scenario.

INTRODUCTION

Now a day, this is well established reality, that expansion of Universe is accelerating. One of the observational foundations for the big bang model of cosmology was observed expansion of the universe. Measurement of the expansion rate is a critical part of the study, and it has been found that the expansion rate is very nearly "flat". That is, the universe is close to the critical density, above which it would slow down and collapse inward toward a future "big crunch". One of the great challenges of astrophysics and astronomy is distance measurement over the vast distances of the universe. Since the 1990s it has become apparent that type Ia supernovae offer a unique opportunity for the consistent measurement of distance out to perhaps 1000 Mpc. Measurement at these great distances provided the first data to suggest that the expansion rate of the universe is actually accelerating. That acceleration implies an energy density that acts in opposition to gravity which would cause the expansion to accelerate. In 1998 and early in1999, High-z Supernova Search teams observed an accelerating expansion of the universe (Riess et al.1998; Permutter et al 1999). The current universe is not only expanding but also accelerating , is confirmed by various results including Permutter et al. (1997,1998);Riess et al.(2000); Garnavch et al.(1998); Schmidt et al.(1998); Tonry et al.(2003); Clocchiatti et al.(2006);fluctuation of cosmic microwave background radiation de Bernardis et al (1998);Large scale structure Spergel et al (2003);Tegmark. et al.(2004); Seljak et al.(2005); Adelman-McCarthy et al (2006);Wilkinson Microwave anisotropy probe (WMAP) Bennett. et al.(2003) .

Brans-Dicke theory of gravitation is a well-known competitor of Einstein's more popular theory of general relativity. It is an example of a scalar-tensor theory, a gravitational theory in which the gravitational interaction is mediated by a scalar field as well as the tensor field of general relativity.

Brans – Dicke (1961) field equations for combined scalar and tensor fields are

$$G_{ij} = -8\pi\phi^{-1}T_{ij} - \omega\phi^{-2}(\phi_{,i}\,\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\,\phi^{'k}) - \phi^{-1}(\phi_{i;j} - g_{ij}\phi_{;k}^{;k}) \tag{1.1}$$

and

$$\phi_{;k}^{'k} = 8\pi\phi^{-1}(3+2\omega)^{-1}T \tag{1.2}$$

Where

$G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ is the Einstein tensor, T_{ij} is the tensor of matter, ω is the dimension less coupling constant , comma and semicolon denotes partial and co-varient differentiation respectively.

The equation of motion

$$T_{;j}^{ij} = 0 \tag{1.3}$$

are consequence of the field equations (1) and (2)

Dark energy is the most accepted theory to explain observations since the 1990s that indicate that the universe is expanding at an accelerating rate. In the standard model of cosmology, dark energy currently accounts for 73% of the total mass-energy of the universe, [P.J.E.Peebles 2003]. The dark energy can be characterized by the equation of state (EoS) parameter ω_D, defined by $\omega_D = \frac{p_D}{\rho_D}$ which is negative for DE, where ρ_D and p_D are energy density and fluid pressure respectively [Carroll and Hoffman 2003]. The simplest candidate for the dark energy is a cosmological constant.

The introduction of viscosity into cosmology has been investigated from different view points (Gren 1990, Padmanabhan & Chitre 1987: Barrow 1986; Zimdahi 1996). Misner (1966,1967) noted that the "measurement of the isotropy of the cosmic background radiation represents the most accurate observational datum in cosmology". An explanation of this isotropy was provided by showing that in large class of homogeneous but anisotropy universe, the anisotropy dies away rapidly. It was found that the most important mechanism in reducing the anisotropy is neutrino viscosity at temperatures just above 10^{10} K (when the universe was about 1 s old: cf. Zel'dovich and Novikov 1971). The astrophysical observations also indicates some evidences that cosmic media is not a perfect fluid (Jaffe et al.2005) and the viscosity effect could be concerned in the evolution of the universe (Brevik & Gorbunova,2005; Brevik et al.2005;Cataldo et al.2005).On the other hand, in the standard cosmological model, if the EoS parameter ω is less than -1 so called phantom, the universe shows the future finite time singularity called the Big Rip (Caldwell et al.2003; Nojiri et al.2005.) or cosmic Dooms day. Several Mechanisms are proposed to prevent the future big rip, like by considering quantum effects terms in the action (Nojiri & Odintsov 2004; Elizalde et al.2004), or by including viscosity effects for the universe evolution (Meng et al. 2007). A well known result of the FRW cosmological solutions, corresponding to Universes filled with perfect fluid and bulk viscous stresses, is the possibility of violating dominant energy condition (Barrow 1987,1988;Folomeev & Gurovich 2008;Ren & Meng 2006; Brevikc & Gorbunovac 2005;Nojiri & Odintsov 2005). Setare (2007a,b,c) and Setare and Saridakis (2008) have studied the interacting models of dark energy in different context. Interacting new age graphic viscous dark energy with varying G has been studied by Sheykhi and Setare (2010).

The cosmological evolution of two fluid dilation model of dark energy was investigated by Amirhashchi et al.(2011a,b) Pradhan et al (2011); Saha et al (2012)) have studied the two fluid scenario for dark energy in FRW universe in different context. Recently Singh & Chaubey (2012) have studied interacting dark energy in Bianchi type I space time and kadam (2019) studied Interacting two fluid viscous dark energy models in scalar-tensor theory of gravitation.

Some experimental data implied that our universe is not a perfectly flat universe and recent papers (Spergel et al 2003; Bennett et al 2003;Ichikawa et al 2006) favoured a universe with spatial curvature. Setare et al (2009) have studied the tachyon cosmology in non-interacting and interacting cases in non-flat FRW universe. Bianchi Type VI_0 model with a two fluid source has been investigated by Coley et al (1990); Pant et al.(2002) examined two fluid cosmological models using Bianchi type II space time. Two fluid Bianchi type I models are studied by Oli(2008).

In this paper , we study the evolution of dark energy parameter within frame work of Bianchi Type – III, V , VI_0 and VI_h cosmological models filled with two fluid Viscous dark Energy. Also, we consider both interacting and non- interacting cases.

METRIC AND FIELD EQUATION

We consider the diagonal form of the metric of general class of Bianchi cosmological models as

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2x}dy^2 + C^2e^{-2mx}dz^2 \tag{2.1}$$

Where A, B, C are functions of t only . The metric (2.1) corresponds to a Bianchi type III model for $m = 0$, Bianchi type V model for $m = 1$ & Bianchi type VI_0 model for $m = -1$.

The energy momentum tensor for a two fluid source is given by

$$T_i^j = (\rho + \bar{p})u_i u^j - \bar{p}\delta_i^j \tag{2.2}$$

Where T_j^i represent two fluid energy momentum tensor of bulk viscous dark energy and is barotropic fluid, together with

$$u^i u_i = -1 \tag{2.3}$$

and

$$\bar{p} = p - \xi u^i_{;i} \tag{2.4}$$

Where ρ is the energy density; p is the pressure, ξ is the bulk-viscous coefficient, and u^i is the four-velocity vector of the distribution. Here after the semicolon denotes covariant differentiation.

The universe field with bulk viscous fluid, from equation (2.2) one find

$$T_1^1 = T_2^2 = T_3^3 = -\bar{p}, T_4^4 = \rho \text{ and } T = \rho - 3\bar{p} \tag{2.5}$$

The average scale factor 'a' of Bianchi type model defined by equation (2.1) is defined as

$$a = (ABC)^{\frac{1}{3}} \tag{2.6}$$

The spatial volume V is given by

$$V = a^3 = (ABC) \tag{2.7}$$

The expansion factor θ is defined by $\theta = u^i_{;i}$.

Hence equation (2.3) leads to

$$\bar{p} = p - 3\xi H \tag{2.8}$$

Where H is Hubble's constant defined by

$$H = \frac{a_4}{a} \tag{2.9}$$

The Bianchi identity for the bulk viscous fluid distribution $G^i_{i;j} = 0$ leads to

$T^i_{j;i} = 0$ which yields to

$$\rho u^i + (\rho + \bar{p})u^i_{;i} = 0 \tag{2.10}$$

Which leads to

$$\rho_4 + 3H(\rho + \bar{p}) = 0 \tag{2.11}$$

The set of field equations for metric (2.1) in Scalar tensor theory of gravitation proposed by Brans-Dicke (1961) are

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4}{B}\frac{C_4}{C} - \frac{m}{A^2} + \frac{\omega}{2}(\frac{\phi_4}{\phi})^2 + \frac{\phi_{44}}{\phi} + \frac{\phi_4}{\phi}(\frac{B_4}{B} + \frac{C_4}{C}) = -8\pi\phi^{-1}\bar{p} \tag{2.12}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4}{A}\frac{C_4}{C} - \frac{m^2}{A^2} + \frac{\omega}{2}(\frac{\phi_4}{\phi})^2 + \frac{\phi_{44}}{\phi} + \frac{\phi_4}{\phi}(\frac{A_4}{A} + \frac{C_4}{C}) = -8\pi\phi^{-1}\bar{p} \tag{2.13}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4}{A}\frac{B_4}{B} - \frac{1}{A^2} + \frac{\omega}{2}(\frac{\phi_4}{\phi})^2 + \frac{\phi_{44}}{\phi} + \frac{\phi_4}{\phi}(\frac{A_4}{A} + \frac{B_4}{B}) = -8\pi\phi^{-1}\bar{p} \tag{2.14}$$

$$\frac{A_4}{A}\frac{B_4}{B} + \frac{A_4}{A}\frac{C_4}{C} + \frac{B_4}{B}\frac{C_4}{C} - \frac{(1+m+m^2)}{A^2} - \frac{\omega}{2}(\frac{\phi_4}{\phi})^2 + \frac{\phi_4}{\phi}(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}) = 8\pi\phi^{-1}\rho \tag{2.15}$$

$$(1+m)\frac{A_4}{A} - \frac{B_4}{B} - \frac{mC_4}{C} = 0 \tag{2.16}$$

$$\phi_{44} + \phi_4\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = \frac{8\pi\phi^{-1}}{(3+2\omega)}(\rho - 3\bar{p}) \tag{2.17}$$

$$\rho_4 + (\rho + \bar{p})\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0 \tag{2.18}$$

Where suffices denotes differentiation with respect to t.

Above field equations in the form of H, σ and q can be written as

$$-8\pi\phi^{-1}\bar{p} = H^2(2q-1) + \sigma^2 - \frac{(1+m+m^2)}{3A^2} + 2H(\frac{\phi_4}{\phi}) + \frac{\omega}{2}(\frac{\phi_4}{\phi})^2 + (\frac{\phi_{44}}{\phi}) \tag{2.19}$$

$$8\pi\phi^{-1}\rho = 3H^2 - \sigma^2 - \frac{(1+m+m^2)}{A^2} - \frac{\omega}{2}(\frac{\phi_4}{\phi})^2 + 3H(\frac{\phi_4}{\phi}) \tag{2.20}$$

Where $\bar{p} = p_m + \bar{p}_D$ and $\rho = \rho_m + \rho_D$. Here p_m and ρ_m are pressure and energy density of barotropic fluid and p_D and ρ_D are pressure and energy density of dark fluid respectively.

The equations of state (EoS) for the barotropic fluid ω_m and dark fluid ω_D are given by

$$\omega_m = \frac{p_m}{\rho_m} \tag{2.21}$$

& $$\omega_D = \frac{\bar{p}_D}{\rho_D} \tag{2.22}$$

respectively.

Let us introduce the dynamical scalars such as expansion parameter θ, shear scalars σ^2 and the mean anisotropy parameter A_m as usual

$$\theta = u^i_{;i} = 3H \tag{2.23}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left\{\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right\} - \frac{\theta^2}{6} \tag{2.24}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \tag{2.25}$$

Where $$H = (\ln a)^* = \frac{a_4}{a} = \frac{1}{3}(H_1 + H_2 + H_3) \tag{2.26}$$

For the general class of Bianchi metric equation (2.1), these dynamical scalars have the forms

$$\theta = \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 3H \tag{2.27}$$

SOLUTIONS

To determine the scale factor $a(t)$ and the Brans-Dicke scalar factor ϕ , we use

i)The special law of variation for Hubble's parameter proposed by Berman (1983) that yields constant deceleration parameter models of the universe and

ii) the condition for trace free energy momentum tensor of the two fluid

$$\rho - 3\bar{p} = 0 \tag{3.1}$$

Which may be, physically, looked upon as the condition for radiation in the two fluid scenario analogous to the condition of disordered radiation for perfect fluid in general relativity. However, there will be structural difference between the two situations.

The constant deceleration parameter q is defined by

$$q = \frac{-a\ddot{a}}{\dot{a}^2} = constant \tag{3.2}$$

which acts as the indicator of the existence of inflation of the model. If $q > 0$ the model decelerates in the standard way while $q < 0$ indicates inflation of the universe. Here we are dealing with accelerated expansion of the universe and hence we take $q < 0$ .Now integration of equation (3.2) yields

$$a = (ct + d)^{\frac{1}{1+q}} \tag{3.3}$$

where $c \neq 0$ and d are constants of integration and $1 + q > 0$ for the expansion of the universe, i.e. $-1 < q < 0$.

Integrating equation (2.17) using Eqn.(3.1) and (3.2), we obtain

$$\phi = \delta\left(\frac{1+q}{q-2}\right)t^{\left(\frac{q-2}{q+1}\right)} + \gamma \tag{3.4}$$

Where γ and δ are constants of integration. We get $\gamma = 0$ and $\delta = 1$ without loss of any generality.

From equation (2.7), (2.16) and (3.2), we get

$$A = t^{\frac{3m^2\alpha + 3 - 3\alpha + 3m}{(1+q)(m+1)(m+2)}}, \tag{3.5}$$

$$B = t^{\frac{3m + 3 - 6m\alpha - 3\alpha}{(1+q)(m+2)}} \tag{3.6}$$

$$C = t^{\frac{3\alpha}{(1+q)}} \tag{3.7}$$

The directional Hubble parameter H_1, H_2 and H_3 having the values

$$H_1 = \left\{ \frac{(3m^2\alpha + 3 - 3\alpha + 3m)}{(1+q)(m+1)(m+2)} \right\} \frac{1}{t}$$

$$H_2 = \left\{ \frac{(3m + 3 - 3\alpha - 6m\alpha)}{(1+q)(m+2)} \right\} \frac{1}{t}$$

$$H_3 = \left\{ \frac{3\alpha)}{(1+q)t} \right\}$$

From equation (2.26) the average generalized parameter H is

$$H = \frac{l}{(1+q)t} \tag{3.8}$$

\\

Also from the equations (2.24) and (3.1) we get

$$\sigma^2 = \frac{X_1}{2(1+q)^2 t^2} \tag{3.9}$$

Where $X_1 = \left\{ \frac{(3m^2\alpha + 3 - 3\alpha + 3m)^2}{(m+1)^2(m+2)^2} + \frac{(3 + 3m - 6m\alpha - 3\alpha)^2}{(m+2)^2} + 9\alpha^2 - 3 \right\}$

From the above equations (3.8) and (3.9) it is observed that the Hubble parameter and Shear scalar are the decreasing function of the time t. also it is diverge at $t = 0$.

For $q > 0$; therefore the model represents a decelerating model of the universe. For $q \le 0$, which implies an accelerating model of the universe. Also recent observations of type Ia supernovae (Perlmutter et al. [1,2,3]; Reiss et al. [4,5]; Tonry et al. [8]; Knop et al. [55] reveal that the present universe is accelerating and the value of decelerating parameter lies somewhere in the range $-1 < q \le 0$. It follows that the solutions obtained in this model are consistent with the observations.

In the following sections we deals with two cases i) non interacting model and ii) interacting model

FOR NON-INTERACTING TWO FLUID MODEL

In this section we assume that two fluids do not interact. Therefore, the general form of conservation equation (2.18) leads us for barotropic and dark fluid separately as

$$(\rho_m)_4 + (\rho_m + p_m)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right) = 0 \tag{4.1}$$

$$(\rho_D)_4 + (\rho_D + p_D)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right) = 0 \tag{4.2}$$

Of course, it may be observed a structural difference between equations (4.1) and (4,2), because equation (4.1) is in the form of ω_m which is constant (Akarsu and Kilinc 2010a, 2010b) and hence equation (4.1) is integrable . But equation (4.2) is a function of ω_D. Accordingly, p_D and ρ_D are also function of ω_D. Therefore, we can not integrate equation (4.2) as it is a function ω_D which is an unknown time dependent parameter. Integrate of equation (4.1) and using equation (2.21), we get density and pressure of barotropic fluid as

$$\rho_m = \rho_0 a^{-3(1+\omega_m)} \tag{4.3}$$

$$p_m = \rho_0 \omega_m a^{-3(1+\omega_m)} \tag{4.4}$$

Where ρ_0 be the constant of integration.

By using the scale factor given by Eqn. (3.3) in (2.19) and (2.20), we have obtained effective pressure and energy density as

$$-8\pi\phi^{-1}\bar{p} = \left\{ \frac{\omega(q-2)^2 + X_1 - 6(q-1)}{2(1+q)^2 t^2} - \frac{(m^2 + m + 1)}{3t^{\frac{2(3m^2\alpha + 3 - 3\alpha + 3m)}{(1+q)(m+1)(m+2)}}} \right\} \tag{4.5}$$

$$8\pi\phi^{-1}\rho = \frac{6(q-1) - X_1 - \omega(q-2)^2}{2(1+q)^2 t^2} - \frac{(1+m+m^2)}{t^{\frac{2(3m^2\alpha+3-3\alpha+3m)}{(1+q)(m+1)(m+2)}}} \tag{4.6}$$

Also we obtained pressure and density for dark fluid as

$$8\pi\phi^{-1}\rho_D = \left\{ \frac{6(q-1) - X_1 - \omega(q-2)^2}{2(1+q)^2 t^2} - \frac{(1+m+m^2)}{t^{\frac{2(3m^2\alpha+3-3\alpha+3m)}{(1+q)(m+1)(m+2)}}} - 8\pi\rho_0 \frac{(q-2)}{(1+q)} t^{-\left(\frac{1+q+3\omega_m}{1+q}\right)} \right\} \tag{4.7}$$

$$-8\pi\phi^{-1}\overline{p}_D = \left\{ \frac{\omega(q-2)^2 + X_1 - 6(q-1)}{2(1+q)^2 t^2} - \frac{(m^2+m+1)}{3t^{\frac{2(3m^2\alpha+3-3\alpha+3m)}{(1+q)(m+1)(m+2)}}} + \frac{8\pi(q-2)\rho_0\omega_m t^{\frac{-(1+q+3\omega_m)}{(1+q)}}}{(1+q)} \right\} \tag{4.8}$$

The equation of state (E0S) parameter for dark fluid is

$$\omega_D = \frac{-\left\{ \frac{\omega(q-2)^2 + X_1 - 6(q-1)}{2(1+q)^2 t^2} - \frac{(m^2+m+1)}{3t^{\frac{2(3m^2\alpha+3-3\alpha+3m)}{(1+q)(m+1)(m+2)}}} + \frac{8\pi(q-2)\rho_0\omega_m t^{\frac{-(1+q+3\omega_m)}{(1+q)}}}{(1+q)} \right\}}{\left\{ \frac{6(q-1) - X_1 - \omega(q-2)^2}{2(1+q)^2 t^2} - \frac{(1+m+m^2)}{t^{\frac{2(3m^2\alpha+3-3\alpha+3m)}{(1+q)(m+1)(m+2)}}} - 8\pi\rho_0 \frac{(q-2)}{(1+q)} t^{-\left(\frac{1+q+3\omega_m}{1+q}\right)} \right\}} \tag{4.9}$$

The effective equation of state (EoS) parameter for viscous dark energy is

$$\omega_D^{eff} = \frac{-\left\{ \frac{\omega(q-2)^2 + X_1 - 6(q-1)}{2(1+q)^2 t^2} - \frac{(m^2+m+1)}{3t^{\frac{2(3m^2\alpha+3-3\alpha+3m)}{(1+q)(m+1)(m+2)}}} + \frac{8\pi(q-2)\rho_0\omega_m t^{\frac{-(1+q+3\omega_m)}{(1+q)}}}{(1+q)} - \frac{8\pi\xi(q-2)}{(1+q)^2} t^{\frac{-(2q-1)}{(1+q)}} \right\}}{\left\{ \frac{6(q-1) - X_1 - \omega(q-2)^2}{2(1+q)^2 t^2} - \frac{(1+m+m^2)}{t^{\frac{2(3m^2\alpha+3-3\alpha+3m)}{(1+q)(m+1)(m+2)}}} - 8\pi\rho_0 \frac{(q-2)}{(1+q)} t^{-\left(\frac{1+q+3\omega_m}{1+q}\right)} \right\}} \tag{4.10}$$

The expressions for the matter energy density Ω_m and dark energy density Ω_D are given by

$$\Omega_m = \frac{1}{3}\rho_0(1+q)^2 t^{2-\frac{3(1+\omega_m)}{(1+q)}} \tag{4.11}$$

$$\Omega_D = \frac{(1+q)^3 t^{\frac{3q}{1+q}}}{24\pi(q-2)} \left\{ \frac{6(q-1) - X_1 - \omega(q-2)^2}{2(1+q)^2 t^2} - \frac{(1+m+m^2)}{t^{\frac{2(3m^2\alpha+3-3\alpha+3m)}{(1+q)(m+1)(m+2)}}} - 8\pi\rho_0 \frac{(q-2)}{(1+q)} t^{-\left(\frac{1+q+3\omega_m}{1+q}\right)} \right\} \tag{4.12}$$

FOR INTERACTING TWO FLUID MODEL

In this section we consider the interaction between dark viscous and barotropic fluids. For this purpose we can write the continuity equations as

$$(\rho_m)_4 + (\rho_m + p_m)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = Q \tag{5.1}$$

$$(\rho_D)_4 + (\rho_D + p_D)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = -Q \tag{5.2}$$

Where the quantity Q expresses the interaction between the fluids . Since we are interested in an energy transfer from the dark energy to dark matter, we consider $Q > 0$ this ensures that the second law of thermodynamics is fulfilling. Here we emphasize that the continuity equations (5.1) and (5.2) imply that the interaction term Q should be proportional to a quantity with units of universe of time i.e. $Q\alpha 1/t$. Therefore, a first and natural candidate can be the Hubble factor H multiplied with the energy density. Following Amendola et al. (2007), Gou et al. (2007) And Hassan Amirhashchi et al.[2011], we consider

$$Q = 3H\sigma\rho_m \tag{5.3}$$

Where σ is coupling constant.
Solving equation (5.1) with the help of equation (5.2), we get

$$\rho_m = \rho_{00} a^{-3(1+\omega_m - \sigma_C)} \tag{5.4}$$

$$p_m = \rho_{00}\omega_m a^{-3(1+\omega_m - \sigma_C)} \tag{5.5}$$

Where ρ_{00} be the constant of integration.
The density for dark fluid as

$$8\pi\phi^{-1}\rho_D = \left\{ \frac{6(q-1) - X_1 - \omega(q-2)^2}{2(1+q)^2 t^2} - \frac{(1+m+m^2)}{t^{\frac{2(3m^2\alpha+3-3\alpha+3m)}{(1+q)(m+1)(m+2)}}} - 8\pi\rho_{00}\frac{(q-2)}{(1+q)} t^{-\left(\frac{1+q+3(\omega_m-\sigma_c)}{1+q}\right)} \right\} \tag{5.6}$$

The effective pressure as

$$8\pi\phi^{-1}\bar{p}_D = \left\{ \frac{6(q-1) - \omega(q-2)^2 - X_1}{2(1+q)^2 t^2} + \frac{(m^2+m+1)}{3t^{\frac{2(3m^2\alpha+3-3\alpha+3m)}{(1+q)(m+1)(m+2)}}} - \frac{8\pi\rho_{00}\omega_m(q-2)t^{-\left(\frac{1+q+3(\omega_m-\sigma_c)}{(1+q)}\right)}}{(1+q)} \right\} \tag{5.7}$$

The equation of state (EoS) parameter for dark fluid is

$$\omega_D = \frac{\left\{ \frac{6(q-1) - \omega(q-2)^2 - X_1}{2(1+q)^2 t^2} + \frac{(m^2+m+1)}{3t^{\frac{2(3m^2\alpha+3-3\alpha+3m)}{(1+q)(m+1)(m+2)}}} - \frac{8\pi\rho_{00}\omega_m(q-2)t^{-\left(\frac{1+q+3(\omega_m-\sigma_c)}{(1+q)}\right)}}{(1+q)} \right\}}{\left\{ \frac{6(q-1) - X_1 - \omega(q-2)^2}{2(1+q)^2 t^2} - \frac{(1+m+m^2)}{t^{\frac{2(3m^2\alpha+3-3\alpha+3m)}{(1+q)(m+1)(m+2)}}} - 8\pi\rho_{00}\frac{(q-2)}{(1+q)} t^{-\left(\frac{1+q+3(\omega_m-\sigma_c)}{1+q}\right)} \right\}} \tag{5.8}$$

The effective equation of state (EoS) parameter for viscous dark energy is

$$\omega_D^{eff} = \frac{\left\{ \frac{6(q-1) - \omega(q-2)^2 - X_1}{2(1+q)^2 t^2} + \frac{(m^2+m+1)}{3t^{\frac{2(3m^2\alpha+3-3\alpha+3m)}{(1+q)(m+1)(m+2)}}} - \frac{8\pi\rho_{00}\omega_m(q-2)t^{-\left(\frac{1+q+3(\omega_m-\sigma_c)}{(1+q)}\right)}}{(1+q)} + 24\pi\xi\frac{(q-2)}{(1+q)^2} t^{\frac{(1-2q)}{1+q}} \right\}}{\left\{ \frac{6(q-1) - X_1 - \omega(q-2)^2}{2(1+q)^2 t^2} - \frac{(1+m+m^2)}{t^{\frac{2(3m^2\alpha+3-3\alpha+3m)}{(1+q)(m+1)(m+2)}}} - 8\pi\rho_{00}\frac{(q-2)}{(1+q)} t^{-\left(\frac{1+q+3(\omega_m-\sigma_c)}{1+q}\right)} \right\}} \tag{5.9}$$

The physical quantities i.e. Average scale factor ad spatial volume, expansion scalar and Anisotropic parameter are

1. Average scale factor $= (a) = (ct+d)^{\frac{1}{(1+q)}}$
2. Spatial volume $(V) = (ct+d)^{\frac{3}{(1+q)}}$
3. Expansion scalar $(\theta) = \frac{3c}{(1+q)(ct+d)}$
4. Anisotropic parameter $(A_m) = constant$

It is observed that the spatial volume is zero at $t = \frac{-d}{c} = t_0$ and expansion scalar is finite, which shows that the universe starts evolving with zero volume at $t = t_0$

With an infinite rate of expansion. The scale factors also vanish at $t = t_0$ and hence the model has a point singularity at the initial epoch. Hubble's parameter and shear scalar diverge at the initial singularity. The anisotropy parameter is constant. As t increases the scale factors and spatial volume increase but the expansion scalar and shear scalar decreases. Thus the rate of expansion slows down with increases in time.

CONCLUSION

In this paper, we have studied a class of solution of scalar tensor theory proposed by Brans-Dicke, with barotropic fluid and bulk viscous fluid for the new class of Bianchi model. Also, it has been observed that the ratio $\frac{\theta}{\sigma}$ becomes constant at $t \to \infty$.Hence The model approaches anisotropically and matter is dynamically negligible near the origin; this result match with the result already given by Collins [1977]. It is also observed that EoS parameter and the effective EoS parameter are function of cosmic time. We have considered both an interacting and non-interacting cases of fluids.

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