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Effects of Body Acceleration on Couple Stress Blood Flow in Stenosed Porous artery in the Presence of Magnetic Field

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ABSTRACT

The present problem deals with effects of body acceleration on the blood flow through porous medium. The blood is assumed to be incompressible, electrically conducting couple stress fluid in the presence of magnetic field. The equation of motion for the fluid is exactly solved by using Laplace and finite Hankel transform. The analytical expressions for axial velocity, flow rate and shear stress are obtained and discussed graphically to provide physical interpretation.

Key words and phrases: stenosed tube, porous media, incompressible, couple stress fluid, body acceleration, magnetic field.

INTRODUCTION

In the present scenario, the study of blood flow in stenosed human arteries is of great interest for scientific research. The term stenosis denotes the narrowing of artery due to arteriosclerotic plaque or other types of abnormal tissue development. This results into circulatory disorders by reducing or occluding the blood supply which may results in cardiovascular diseases especially heart attack, stroke etc.Lee and Fung (1970) studied the blood flow in locally narrowing tube at low Reynolds number in the range 0-25 by using conformal mapping method. Oka et al. (1970) discussed hydro dynamical theory of the steady slow motion of blood through a capillary with permeable wall. They were much interested to consider the exchange of fluid across the permeable wall upon the motion of the fluid within rigid circular tube. Popel et al. (1974) have investigated a continuum approach for blood flow with couple stresses. Steady flow of blood through modelled vascular stenosis has been investigated by Mcdonald (1979). Shukla et al. (1980) discussed the effect of stenosis on non-Newtonian flow of blood in an artery but later on they took into account the effect of radial distribution of cells and the existence of the peripheral plasma layer near the wall and studied the blood flow through an artery with mild stenosis by considering the blood as a power law fluid. Srivastava and Srivastava (1983) have developed a two- phase model of pulsatile blood flow with entrance effects.

The efffects of couple stresses on the blood flow through an artery with mild stenosis have been investaged by Sinha and Singh (1984). Srivastava (1985) studied the flow of couple stress fluid through stenotic blood vessels. Numerical studies of fluid flow through tubes with double constrictions have been developed by Lee (1990). The flow fields in the neighbourhood of double constrictions in a circular cylindrical tube are studied numerically. The effects on the streamline, velocity and vorticity distributions as the flow passes through the constrictions in the tube are studied in the Reynolds number range 05–200. Haldar and Ghosh (1994) have studied Newtonian blood flow through indented artery under the effect of magnetic field in the presence of erythrocytes.

They have taken variable blood viscosity according to Einstein relation and obtained expressions for blood velocity, pressure and flow rate. Theoretical analysis of flow properties of aggregating red cell suspension in narrow horizontal tubes has been presented by Murata (1998). He has proposed a sedimentation model in which he considered constant values of hematocrit and Newtonian viscosity in the circular core region, containing red cell aggregates. Srivastava (2003) has investigated the flow of couple stress fluid through an artery in the presence of mild stenosis. The blood flow in central layer (core region) is assumed to be couple stress fluid as it is the suspension of erythrocytes (RBC) and a peripheral layer of plasma (Newtonian fluid). Pralhad and Schultz (2004) have studied the modeling of arterial stenosis and its application to blood diseases. Blood flow has been assumed to be represented by a couple stress fluids. Bali and Awasthi (2007) have discussed the effect of magnetic field on the resistance to blood flow through stenotic artery and the problem of pulsatile blood flow in the artery having single mild stenosis. According to physiological importance of body acceleration, many investigations have been developed for blood flow under the influence of body acceleration with and without stenosis. Rathod and Shakera (2009) have discussed the pulsatile flow of blood through a porous medium under the influence of periodic body acceleration by considering blood as a couple stresses, incompressible, electrically conducting fluid in the presence of magnetic field. When blood flow to a tissue becomes blocked or reduced, necrosis will eventually occur. Local exposure of a magnetic field could potentially result in blood vessel relaxation and increased blood flow.

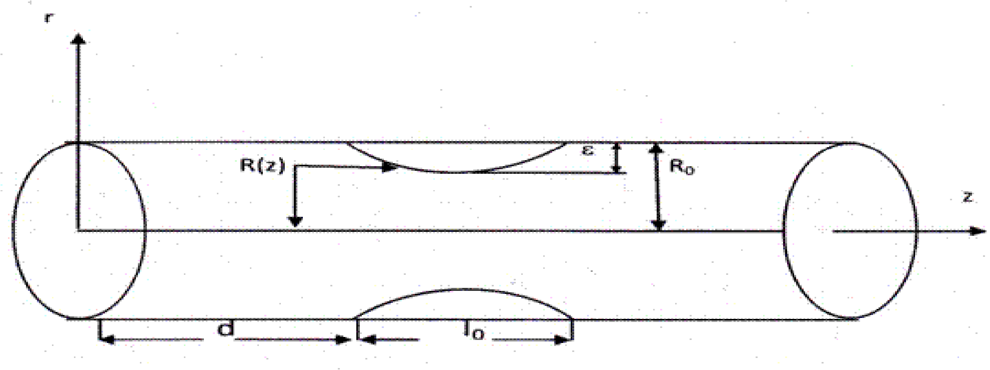
The effects of magnetic field on blood vessels and the cardiovascular system are very interesting. Ali et al. (2009) presented a mathematical model for blood flow through stenosed artery. Blood has been assumed as Newtonian fluid and it has been realized that various hydrodynamic effects play important role in the development of several diseases related to blood flow. Varshney et al. (2010) discussed the effect of magnetic field on the blood flow in artery having multiple stenosis and they found that all the flow characteristics are affected by the presence of multiple stenosis and magnetic field of different intensity. Singh and Rathee (2011) studied the effect of externally applied magnetic field on the blood flow through stenosed artery in porous medium.

The advantage of this study is that we can measure strength of magnetic field up to which we can control the blood flow in hypertensive patients and those who have blockage in their arteries. Bali and Awasthi (2012) represented a Casson fluid model for multiple stenosed arteries in the presence of magnetic field. The effect of magnetic field, height of stenosis, volumetric flow rate in stenotic region and the wall shear stress at the surface of stenosis are obtained. Rathee and Singh (2013) presented a theoretical mathematical model to represent a two-layered model of blood flow through stenotic tube in porous medium under the effect of magnetic field. The blood has been assumed to be a Newtonian fluid of variable viscosity in core region and plasma fluid as a Newtonian fluid of constant viscosity in peripheral region of the stenotic tube. Mathur and Jain (2013) gave mathematical modeling of non-Newtonian blood flow through artery in the presence of stenosis and results were compared with published literature.

They considered blood as Power law fluid and provided strong evidence that hydrodynamic factor such as resistance to flow, wall shear stress and apparent viscosity may play a vital role in the development and the progression of arterial stenosis. Kumar et al. (2014) have designed a model to investigate a non-Newtonian blood flow through multiple stenoses. They observed that the wall shear stress decrease with the increase of blood viscosity. The unsteady slip flow of blood through constricted artery has been studied by Gaur and Kumar (2015).

The study shows that the axial velocity, wall shear stress and volumetric flow rate decrease along the axial distance with passage of time. Liu et al. (2015) used non-Newtonian models to simulate blood flow in a stenotic right artery and studied the influence of different boundary conditions on the phasic variations and spatial distribution patterns of blood flow. Siddiqui et al. (2015) discussed a biomedical approach to study the effect of body acceleration and slip velocity through stenotic artery and it is observed that body acceleration enhances the axial velocity and flow rate.Siddiqui et al. (2016)analysed the unsteady blood flow through stenosed artery with slip effects.Sankad and Nagathan(2019) explained the transport of MHD couple stress fluid through peristalsis in a porous medium under the influence of heat transfer and slip effects. Ismael et al.(2019) presented a study of gravitational and magnetic effects on coupled stress bi-phase liquid suspended with Crystal and Hafnium particles down in steep channel. Abubakar and Adeoye (2019) discussed the effects of radiative heat and magnetic field on blood flow in an inclined tapered stenosed porous artery. Liu and Liu(2020) modelled the blood flow analysis in tapered stenosed arteries with the influence of heat and mass transfer. Nasrin et al. (2020) discussed the blood flow analysis inside a stenotic artery using Power-law fluid model.

Formulation of the Problem: Let us consider a one dimentional pulsatile flow of blood through a porous medium in a straight and rigid circular tube by considering blood as couple stress, non-Newtonian, incompressible and electrically conducting fluid in the presence of magnetic field with periodic body acceleration. The flow is considered as axially symmetric and fully developed. The geometry of stenosed artery as described by Haldar and Ghosh (1994)



$$\frac{R(z)}{R_0} = 1 - A\left[l_0^{s-1}\left(z-d\right) - (z-d)^s\right] , \quad d \le z \le d + l_0 \tag{1}$$

where, $s(\geq 2)$ is a parameter determining the shape of stenosis, d is the position of stenosis, l_0 is the length of stenosis, R(z) is a radius of stenosed vessel, z denotes the axial position and A is a parameter, given by

$$A = \frac{\varepsilon}{R_0 l_0^{\,s}} \frac{s^{s/s-1}}{(s-1)} , \tag{2}$$

ε being the maximum height of stenosis at $z = d + \frac{l_0}{s^{1/s-1}}$ such that $\varepsilon / R_0 << 1$.

The pressure gradient and body acceleration G are given by

$$-\frac{\delta p}{\delta z} = A_0 + A_1 \cos(\omega t) \qquad t \geq 0 \tag{3}$$

$$G = a_0 \cos(\omega_1 t + \phi), \qquad t \geq 0 \tag{4}$$

where A_0 is the steady-state part of the pressure gradient, A_1 is the amplitude of the oscillatory part, $\omega = 2\pi f$ and f is the heart pulse frequency, a_0 is the amplitude of the body acceleration, $\omega_1 = 2\pi f_1$ and f_1 is the body acceleration frequency, ϕ is the phase difference, z is the axial distance and t is time. Now the equation of motion for flow as discussed by Rathod and Shakera (2009) under the above assumptions in cylindrical polar co-ordinates is

$$\rho \frac{\delta u}{\delta t} = -\frac{\delta p}{\delta z} + \rho G + \mu \nabla^2 u - \eta \nabla^2(\nabla^2 u) - \sigma B_0^{\,2} u - \frac{\mu}{K} u \tag{5}$$

where

$$\nabla^2 \equiv \frac{1}{r}\left(\frac{\partial}{\partial r}\left(r \frac{\partial}{\partial r}\right)\right) \tag{6}$$

where $u(r,t)$ is velocity in the axial direction, ρ and μ are the density and viscosity of blood, η is the couple stress parameter, σ is the electrical conductivity, B_0 is the external magnetic field, K is the permeability of the isotropic porous medium and r is the radial co-ordinate.

Let us introduce the following dimensionless quantities:

$$u^* = \frac{u}{\omega R_0},\ r^* = \frac{r}{R_0},\ t^* = t\,\omega,\ A_0^* = \frac{R_0}{\mu\omega}A_0,\ A_1^* = \frac{R_0}{\mu\omega}A_1,\ a_0^* = \frac{\rho R_0}{\mu\omega}a_0,\ z^* = \frac{z}{R_0},\ K^* = \frac{K}{R_0^2}$$

(7)

After dropping stars, above equation in new variables becomes

$$\alpha^2 \frac{\delta u}{\delta t} = A_0 + A_1\cos t + a_0\cos(bt+\phi) + \frac{1}{r}\frac{\delta}{\delta r}\left(r\frac{\delta u}{\delta r}\right) - \frac{1}{\bar{\alpha}^2}\left(\frac{1}{r}\frac{\delta}{\delta r}\left(r\frac{\delta}{\delta r}\right)\right)\left(\frac{1}{r}\frac{\delta}{\delta r}\left(r\frac{\delta u}{\delta r}\right)\right) - \left(H^2+M^2\right)u$$

(8) where $\bar{\alpha}^2 = \frac{R_0^{\,2}\mu}{\eta}$ couple stress parameter, $\alpha = R_0\left(\frac{\omega\rho}{\mu}\right)^{\frac{1}{2}}$ is Womersley parameter, $H = B_0 R_0\left(\frac{\sigma}{\mu}\right)^{\frac{1}{2}}$ is the Hartmann number, $M^2 = \frac{1}{K}$ is permeability parameter, $b = \frac{\omega_1}{\omega}$ and R_0 isthe radius of the tube.

We assume that at $t < 0$, only the pumping action of the heart is present and at $t = 0$, the flow in the artery corresponds to the instantaneous pressure gradient, i.e. $-\frac{\delta p}{\delta z} = A_0 + A_1$ (10)

The initial and boundary conditions for this problem are

(i) $$u(r,0) = \frac{2}{a}\sum_{n=1}^{\infty}\frac{\bar{\alpha}^2 J_0(r\lambda_n)}{\lambda_n J_1(a\lambda_n)}\frac{\left[A_0+A_1+a_0\cos\phi\right]}{\left[\lambda_n^4+\bar{\alpha}^2\left(\lambda_n^2+H^2+M^2\right)\right]}$$ (11)

(ii) $u(r,t) = 0\,;\qquad r = a$ (12)

(iii) $u(r,\,t)$ *is finite at* $r = 0$ (13)

where $a = \frac{R(z)}{R_0}$

Required Integral transforms: If $f(t)$ is continuous function and is of exponential order for $t \geq 0$ then its Laplace transform is defined as

$$\bar{f}(s) = \int_0^\infty e^{-st} f(t)\,dt,\qquad s \succ 0$$

(14)

If $f(r)$ satisfies Dirichlet conditions in finite interval $[0, a]$ and its finite Hankel transform defined as

$$f(\lambda_n) = \int_0^a r\, f(r)\, J_0(r\lambda_n)\,dr$$

(15)

where λ_n are the roots of the equation $J_0(ar) = 0$, J_0 and J_1 are Bessel's functions of first kind. Then at each point of the interval at which $f(r)$, continuous is given by

$$f(r)=\frac{2}{a^2}\sum_{n=1}^{\infty} f(\lambda_n)\frac{J_0(r\lambda_n)}{J_1^2(a\lambda_n)} \qquad (16)$$

Analysis: Transposing the terms of equation (8)

$$\frac{1}{r}\frac{\delta}{\delta r}\left(r\frac{\delta}{\delta r}u(r,t)\right)-\frac{1}{\bar{\alpha}^2}\left[\frac{1}{r}\frac{\delta}{\delta r}\left(r\frac{\delta}{\delta r}\right)\left(\frac{1}{r}\frac{\delta}{\delta r}\left(r\frac{\delta}{\delta r}u(r,t)\right)\right)\right]=\alpha^2\frac{\delta}{\delta t}u(r,t)+\left(H^2+M^2\right)u(r,t) -\left[A_0+A_1\cos t+a_0\cos(bt+\phi)\right]$$

(17)

Employing the Laplace transform to equation (16), we obtain

$$\frac{1}{r}\frac{\delta}{\delta r}\left(r\frac{\delta}{\delta r}\bar{u}(r,s)\right)-\frac{1}{\bar{\alpha}^2}\left[\frac{1}{r}\frac{\delta}{\delta r}\left(r\frac{\delta}{\delta r}\right)\left(\frac{1}{r}\frac{\delta}{\delta r}\right)\left(r\frac{\delta}{\delta r}\bar{u}(r,s)\right)\right]=\alpha^2\left[s\bar{u}(r,s)-u(r,0)\right]+\left(H^2+M^2\right)\bar{u}(r,s) -\left[\frac{1}{s}A_0+\frac{s}{s^2+1}A_1+a_0\left(\frac{s\cos\phi-b\sin\phi}{s^2+b^2}\right)\right]$$

(18)

where

$$\bar{u}(r,s)=\int_0^{\infty} e^{-st}u(r,t)\,dt$$

Now applying finite Hankel transformation to equation (17), we obtain

$$\bar{u}(\lambda_n,s)=\frac{a}{\alpha^2\lambda_n}J_1(a\lambda_n)\left[\frac{1}{s(s+h)}A_0+\frac{s}{(s^2+1)(s+h)}A_1+a_0\frac{s\cos\phi-b\sin\phi}{(s^2+b^2)(s+h)}+\frac{A_0+A_1+a_0\cos\phi}{h(s+h)}\right]$$

(19)

where

$$h=\frac{1}{\alpha^2\bar{\alpha}^2}\left[\lambda_n^4+\bar{\alpha}^2\left(H^2+M^2+\lambda_n^2\right)\right] \qquad (20)$$

Using partial fractions, we have

$$\bar{u}(\lambda_n,s)=\frac{a}{\alpha^2\lambda_n}J_1(a\lambda_n)\left[\frac{1}{h}\left(\frac{1}{s}-\frac{1}{s+h}\right)A_0-\frac{h}{1+h^2}\left(\frac{1}{s}-\frac{s}{s^2+1}-\frac{1}{h(s+h)}\right)A_1\right] +\frac{a}{\alpha^2\lambda_n}J_1(a\lambda_n)a_0\left[\begin{array}{l}\frac{h}{h^2+b^2}\left(-\frac{1}{s+h}+\frac{s}{s^2+b^2}+\frac{b^2}{h(s^2+b^2)}\right)\cos\phi\\ -\frac{b}{h^2+b^2}\left(\frac{1}{s+h}-\frac{s}{s^2+b^2}+\frac{h}{s^2+b^2}\right)\sin\phi\end{array}\right] +\frac{a}{\alpha^2\lambda_n}J_1(a\lambda_n)\left[(A_0+A_1+a_0\cos\phi)\frac{1}{h(s+h)}\right] \qquad (21)$$

Applying Inverse Laplace transformation, we obtain

$$u(\lambda_n, t) = \frac{a}{\alpha^2 \lambda_n} J_1(a\lambda_n)\left[\frac{1}{h}A_0 + e^{-ht}\left(-\frac{h}{1+h^2}A_1 - \frac{a_0}{h^2+b^2}(h\cos\phi + b\sin\phi) + \frac{1}{h}(A_1 + a_0\cos\phi)\right)\right] + \frac{a}{\alpha^2 \lambda_n} J_1(a\lambda_n)\left[\frac{1}{1+h^2}A_1(h\cos t + \sin t) + \frac{a_0}{h^2+b^2}\left[h\cos(bt+\phi) + b\sin(bt+\phi)\right]\right]$$

(22)

Taking inverse Hankel transformation, we obtain

$$u(r,t) = \frac{2}{a^2}\sum_{n=1}^{\infty} u(\lambda_n, t)\frac{J_0(r\lambda_n)}{J_1^2(a\lambda_n)}$$ (23)

$$u(r, t) = \frac{2}{a}\sum_{n=1}^{\infty}\frac{1}{\alpha^2\lambda_n}\left[\begin{array}{l}\frac{1}{h}A_0 + e^{-ht}\left(-\frac{h}{1+h^2}A_1 - \frac{a_0}{h^2+b^2}(h\cos\phi + b\sin\phi) + \frac{1}{h}(A_1 + a_0\cos\phi)\right)\\ + \frac{1}{1+h^2}A_1(h\cos t + \sin t) + \frac{a_0}{h^2+b^2}\left[h\cos(bt+\phi) + b\sin(bt+\phi)\right]\end{array}\right]\frac{J_0(r\lambda_n)}{J_1(a\lambda_n)}$$

Expression for fluid acceleration is

(24)

$$f(r,t) = \frac{\delta u}{\delta t}$$

$$f(r, t) = \frac{2}{a}\sum_{n=1}^{\infty}\frac{1}{\alpha^2\lambda_n}\left[\begin{array}{l}-he^{-ht}\left(-\frac{h}{1+h^2}A_1 - \frac{a_0}{h^2+b^2}(h\cos\phi + b\sin\phi) + \frac{1}{h}(A_1 + a_0\cos\phi)\right)\\ + \frac{1}{1+h^2}A_1(-h\sin t + \cos t) + \frac{a_0}{h^2+b^2}\left[-h\sin(bt+\phi) + b\cos(bt+\phi)\right]\end{array}\right]\frac{J_0(r\lambda_n)}{J_1(a\lambda_n)}$$

(25)

Expression for flow rate is

$$Q(r,t) = 2\pi\int_0^a r\,u(r,t)\,dr$$ (26)

$$Q(r, t) = 4\pi\sum_{n=1}^{\infty}\frac{1}{\alpha^2\lambda_n^2 J_1(a\lambda_n)}\left[\begin{array}{l}\frac{1}{h}A_0 + e^{-ht}\left(-\frac{h}{1+h^2}A_1 - \frac{a_0}{h^2+b^2}(h\cos\phi + b\sin\phi) + \frac{1}{h}(A_1 + a_0\cos\phi)\right)\\ + \frac{1}{1+h^2}A_1(h\cos t + \sin t) + \frac{a_0}{h^2+b^2}\left[h\cos(bt+\phi) + b\sin(bt+\phi)\right]\end{array}\right]$$

(27)

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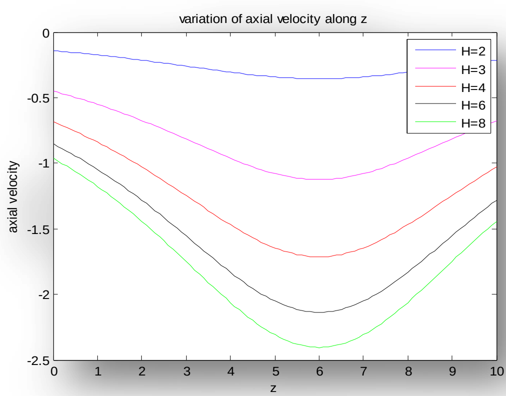
Expression for shear stress is

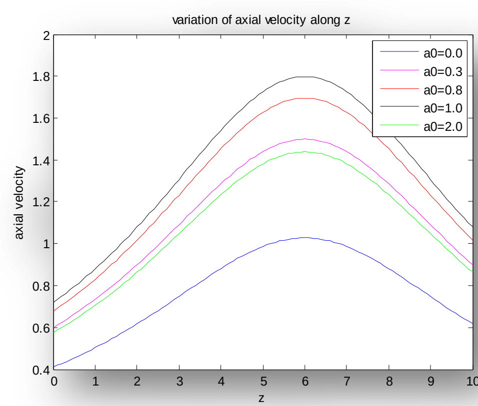
$$\tau(r,t)=\mu\frac{\delta u}{\delta r} \quad (28)$$

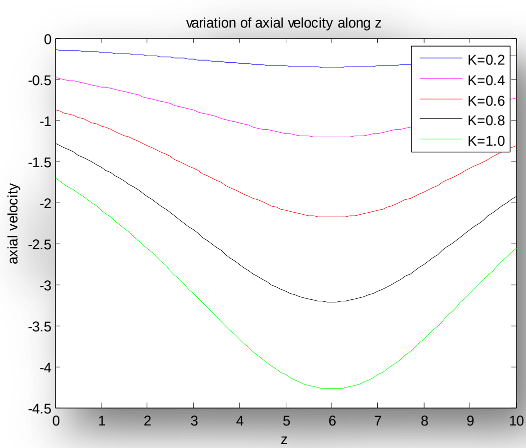
$$\tau(r,t)=-\frac{2}{a}\mu\sum_{n=1}^{\infty}\frac{J_1(r\lambda_n)}{\alpha^2 J_1(a\lambda_n)}\left[\begin{array}{l}\frac{1}{h}A_0+e^{-ht}\left(-\frac{h}{1+h^2}A_1-\frac{a_0}{h^2+b^2}(h\cos\phi+b\sin\phi)+\frac{1}{h}(A_1+a_0\cos\phi)\right)\\+\frac{1}{1+h^2}A_1(h\cos t+\sin t)+\frac{a_0}{h^2+b^2}\left[h\cos(bt+\phi)+b\sin(bt+\phi)\right]\end{array}\right] \quad (29)$$

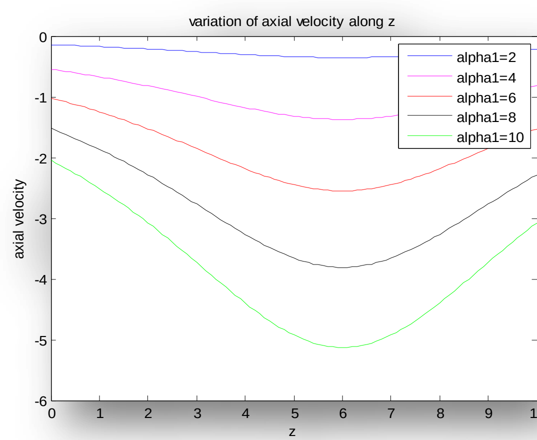
RESULTS

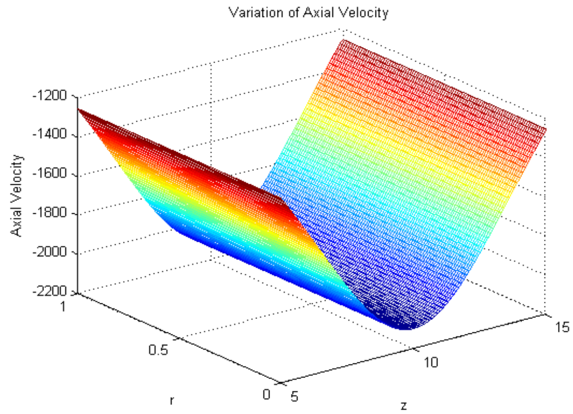
The velocity profile for pulsatile flow of blood through a porous medium with periodic body acceleration in the presence of magnetic field is interpreted by using the velocity expression for different values of Hartmann number H , permeability of the porous medium K, amplitude of body acceleration. It is observed that velocity decreases as the Hartmann number H increases and velocity increases as amplitude of body acceleration increases. We can observe that axial velocity profile is slightly elevated from its mean position i.e. maximum and minimum value of axial velocity exist just near the throught of the stenosis. But volumetric flow rate always showing the growing behaviour. On the other hand, shear stress exhibits the declining behaviour as our objectives.

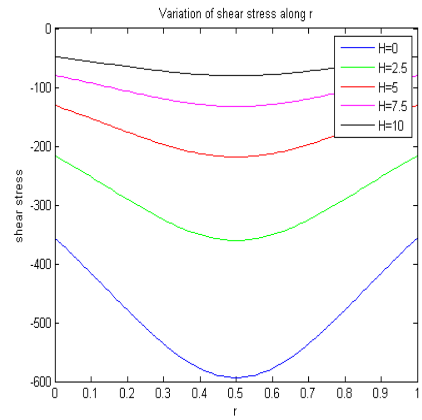


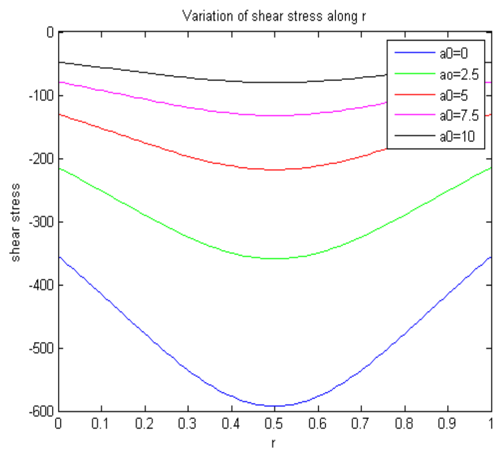


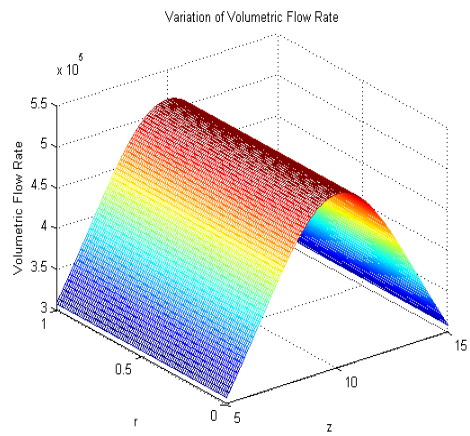


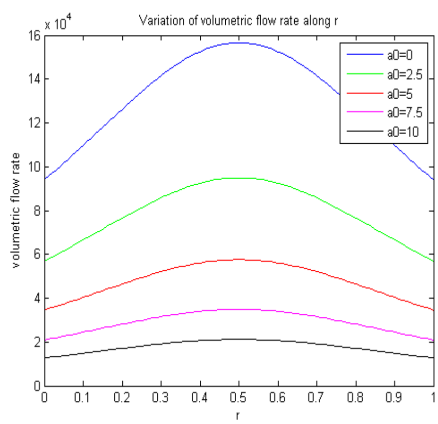


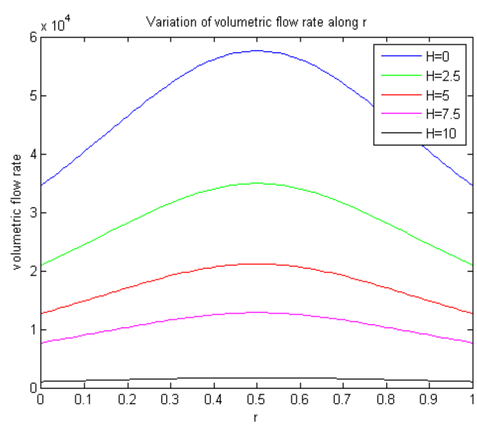












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